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S.E. KONSTEIN, M.A. VASILIEV

MASSLESS REPRESENTATIONS AND
ADMISSIBILITY CONDITION FOR
HIGHER-SPIN SUPERALGEBRAS

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S.E.Konstein and M.A.Vasiliev

MASSLESS REPRESENTATIONS AND ADMISSIBILITY CONDITION
FOR HIGHER-SPIN SUPERALGEBRAS

Moscow 1988

Abstract

Massless particle representations of various infinite-dimensional higher-spin superalgebras proposed previously are constructed. We analyse which of higher-spin superalgebras obey the requirement (the admissibility condition) to possess massless unitary representations with the same spectra of spins as predicted by the structure of gauge fields originating from these superalgebras. It is argued that those higher-spin superalgebras, which obey the admissibility condition, can serve as rigid supersymmetries in nontrivial consistent gauge theories of massless fields of all spins.

1. Introduction

Recent progress in constructing consistent interactions for gauge (massless) higher-spin ($s > 2$) fields in the framework of gravity /1/, indicates that a new class of fully consistent gauge theories should exist describing interacting gauge fields of all spins, which generalize and extend naturally ordinary "lower-spin" gauge theories of the type of supergravity. Perhaps, higher-spin gauge theories can offer new possibilities for creating a unified theory of everything which is quantum-mechanically consistent and describes all the needed interactions including gravity. Nowadays it is generally believed that such a unified theory should be looked for among string theories. Since all string field theories are known to contain infinite sets of massive fields of all spins, the question arises whether these theories can correspond to Higgs phases of appropriate gauge theories of massless higher-spin fields analogously to that as ordinary field theories containing massive spin-1 and spin-3/2 fields usually correspond to some spontaneous breakdown of Yang-Mills gauge symmetries and local supersymmetry. If this is indeed the case, gauge theories of massless fields of all spins might appear more fundamental than string field theories in their present form.

Important arguments in favour of the existence of nontrivial gauge higher-spin theories are provided by refs./2/, where various non-gravitational cubic interactions of massless higher-spin fields in the flat space were constructed, and by ref./3/ in which consistent equations of motion of massless fields in the framework of gravity are found in the first order of the expansion in powers of higher-spin curvatures. Let us emphasize that the expansion used in ref./3/ turns out to be much more efficient than that used

in ref./1/, and the results of ref./3/ exceed considerably those obtained in ref./1/. It is also worth mentioning that, technically, the crucial point that allowed to reconcile positive results of refs./1,3/ with the negative conclusions of refs./4/ where it was argued that it is impossible to introduce consistent gravitational interaction of massless higher-spin fields, is that the higher-spin-gravitational interactions constructed in refs./1,3/ contain inverse powers of the cosmological constant that makes it impossible to use the expansion over the flat background used in refs./4/.

In ref./1/, the dynamics of gauge higher-spin fields was analyzed along the lines of the approach of MacDowell and Mansouri /5/ to gravity and supergravity. Higher-spin gauge fields were described in ref./1/ in terms of connection 1-forms which took their values in the adjoint representation of one of higher-spin superalgebras proposed in refs./6,7/. In ref./3/, broader superalgebras of higher spins and auxiliary fields proposed in ref./8/ were used. Higher-spin superalgebras of refs./6,7/ and higher-spin subalgebras of superalgebras of ref./8/ can be thought of as potential higher-spin counterparts of ordinary anti-de Sitter (ADS) superalgebras $osp(N,4)$ corresponding to (extended) supergravities. In particular, it is natural to expect that some of the higher-spin superalgebras discussed in refs./6-8/ can serve as superalgebras of rigid supersymmetry of maximally symmetric vacua in corresponding higher-spin field theories.* Thus, the investiga-

* Speaking about higher-spin superalgebras and subalgebras we assume those algebras that give rise to gauge fields describing massless fields in the framework of the approach proposed in ref./9/. It is this approach that served as a basis for the whole construction of refs./6,7,1/). In fact, a part of gauge fields, originating from superalgebras of the type $shsa(1)$ proposed in ref./8/ (see p.5).

tion of unitary representations of the higher-spin superalgebras receives great importance since these representations describe (super)multiplets of particles which are allowed to emerge in gauge theories of higher spins.

The analysis of massless representations is of special interest for us because it gives an additional criterion for the selection of admissible higher-spin superalgebras. Actually, since gauge massless fields possessing spins $s \geq 1$ are assumed to be described by the connection 1-forms in the adjoint representation of the chosen higher-spin superalgebra h , the physical spectrum of spins in the theory turns out to be fixed unambiguously according to the results of ref./9/. If there exists some fully consistent theory based on h , then there should exist an appropriate unitary representation of h describing just the same spectrum of massless particles with spins $s \geq 1$ and, possibly, some set of lower-spin fields with $s < 1$ which cannot be fixed by the pure gauge methods of refs./9,1/. Whenever such unitary representation exists, we say that h obeys the admissibility condition. Evidently, if h does not obey the admissibility condition, it is impossible to construct a fully consistent theory starting from h .

This point can be illustrated by the situation in ordinary supergravity. For example, the gauge fields of $h = \text{osp}(1,4)$ describe /5/ spin-2 and spin-3/2 massless fields. Since $\text{osp}(1,4)$ admits /11/ the unitary representation with massless spins $s=2$ and $3/2$, the (cont. footnote from p.4) gives rise to auxiliary gauge fields as explained in ref./10/. In the present paper, we confine ourselves to the analysis of the higher-spin subalgebras of superalgebras of ref./8/ because it was argued in ref./8/ that only these subalgebras can correspond to rigid supersymmetries of physical theories.

admissibility condition is satisfied that manifests itself in the existence of $N=1$ supergravity. On the other hand, superalgebras $osp(N,4)$ with $N>8$ do not obey the admissibility condition since gauge fields originating from these superalgebras describe massless fields with spins $s \leq 2$ as before, while every massless representation of $osp(N,4)$ contains massless particles with $s > 2$ if $N > 8$. This implies that no supergravities can exist at $N > 8$.

As emphasized in refs./1/, the analysis of cubic interactions carried out in refs./1/ is not sufficient for the determination of true spectra of massless fields needed for constructing consistent theories in all orders. It can happen that some of the sets of fields discussed in refs./1/ merely correspond to subsets of complete sets of fields emerging in fully consistent theories, the subsets which cannot be obtained by consistent truncations of fully consistent theories. The point is that the analysis of the admissibility condition for higher-spin superalgebras can clarify which of them can be used for constructing closed higher-spin theories. In addition, the knowledge of corresponding massless unitary representations enables one to fix unambiguously spectra of spins in these theories, including the sector of lower-spin fields ($s < 1$).

As already mentioned, the results obtained in ref./3/ exceed considerably the cubic approximation of refs./1/. Taking this into account, it was argued in ref./3/ that the sets of fields used in this reference are complete. (One of the important advantages of the approach of ref./3/ is that it enables one to handle gauge ($s \geq 1$) and non-gauge ($s < 1$) massless fields in a quite uniform way.) It is the superalgebra of higher spins and auxiliary fields of ref./8/ that was used in ref./3/ for the description of the higher-spin field dynamics. Surprisingly enough, it turned out that,

in the presence of fermions, there exists no consistent truncations of equations of motion of ref./3/ that correspond to "pure" higher-spin superalgebras of refs./6,7/. It was concluded in ref./3/ that, most likely, superalgebras proposed in refs./6,7/ cannot be used for constructing fully consistent theories (when fermions are present), and one is forced to use broader superalgebras of ref./8/. Since the decisive solution of this question is of great importance, one of the main goals of the present paper is to analyse this point in the context of the admissibility condition discussed above. It will be shown that, in agreement with the results of ref./3/, the superalgebras of higher spins of ref./8/ obey the admissibility condition while those of refs./6,7/ do not. In the authors' opinion, these results provide a highly nontrivial support for the adequacy of the approach of ref./3/, indicating implicitly that this approach can be extended from the level of equations of motion to the level of action (unitary particle-like representations arise naturally as a result of quantization, while consistent quantization can normally be carried out when an appropriate action principle exists).

In fact, we confine ourselves in this paper to consideration of various $N=1$ higher-spin superalgebras proposed in refs./6,8/. We hope that the technique developed here will enable us to analyse extended higher-spin superalgebras of refs./7,3/ elsewhere.

The rest of the paper is organized as follows. In order to make the paper as much selfcontained as possible, in section 2 the main facts about particle representations of the anti-de Sitter Lie algebra $o(3,2)$, which are needed for the analysis of representations of higher-spin superalgebras, are collected. In section 3, we describe the operator realization of higher-spin superalgebras, which is of crucial importance for constructing unitary represen-

tations of these superalgebras. Also automorphisms of the higher-spin superalgebra $\text{shs}^f(1)$ of ref./8/ are defined here which enable one to interpret naturally various $N=1$ higher-spin superalgebras of refs./6,8/ as subalgebras of $\text{shs}^f(1)$. In section 4, massless representations are constructed for the higher-spin superalgebras under consideration, and it is analysed which of these superalgebras obey the admissibility condition. In section 5, some perspectives are discussed briefly.

2. Unitary Representations of $O(3,2)$

The analysis of unitary irreducible representations (UIR's) of higher-spin (super)algebras in terms of particles living in the anti-de Sitter (ADS) space reduces to the decomposition of these representations into UIR's of ADS Lie algebra $\mathfrak{o}(3,2) \sim \mathfrak{sp}(4)$. In this section, we remind the reader the main facts about $\mathfrak{o}(3,2)$ and its UIR's. Our discussion of this point is mainly based on refs./11,12/. The related questions, including UIR's of $\mathfrak{osp}(N,4)$, supersymmetric extensions of $\mathfrak{o}(3,2) \sim \mathfrak{sp}(4)$, were also analyzed in refs./13-20/ (see also references therein).

$O(3,2)$ is the group of linear transformations leaving invariant the quadratic form $Q(x) = x_0^2 - x_1^2 - x_2^2 - x_3^2 + x_4^2$. Therefore, $O(3,2)$ acts on the four-dimensional manifold, AdS_4 , extracted by the condition $Q(x)=1/\lambda^2$ ($\Lambda = -3\lambda^2$ is the corresponding (negative) cosmological constant). Let us denote the generators of the maximal compact subgroup $O(3) \times O(2) \subset O(3,2)$ as J_i ($i=1,2,3$) and $\lambda^{-1}E$, respectively. In the flat limit $\lambda \rightarrow 0$, E and J_i coincide with usual (Poincare) operators of energy and angular momenta. For simplicity, we shall use from now on the dimensionless units with $\lambda=1$.

It is convenient to realize the generators (basis) of $\mathfrak{o}(3,2)$

as quadratic polynomials constructed from the "operators" a, a^+ , b, b^+ obeying the commutation relations

$$[a, a^+] = [b, b^+] = 1, \quad [a^{(\pm)}, b^{(\pm)}] = 0 \quad (2.1)$$

and hermiticity conditions

$$(a)^\dagger = a^+, \quad (b)^\dagger = b^+. \quad (2.2)$$

The full set of generators of $o(3,2)$ reads in these terms

$$E = \frac{1}{4} (aa^+ + a^+a + bb^+ + b^+b), \quad (2.3a)$$

$$J_3 = \frac{1}{2} (a^+a - b^+b), \quad (2.3b)$$

$$M_+^+ = (a^+)^2, \quad M_+^- = a^2, \quad (2.3c)$$

$$M_-^+ = (b^+)^2, \quad M_-^- = b^2, \quad (2.3d)$$

$$M_3^+ = a^+b^+, \quad M_3^- = ab, \quad (2.3e)$$

$$J_+ = a^+b, \quad J_- = ab^+. \quad (2.3f)$$

The operators $M_{\pm}^\pm$ change eigenvalues of E by ± 1 . The operators $J_\pm, M_\pm$ change eigenvalues of J_3 by ± 1 .

Particle-like representations correspond to those UIR's of $o(3,2)$ in which the energy operator E is bounded from below (these are the highest weight representations). Evidently, such UIR's contain "vacua", the vectors $|\Phi\rangle$ which are annihilated by all those generators of $o(3,2)$ that decrease energy. These vacua form a linear space V which is invariant under the action of J_1 and E and therefore, decomposes into the sum of UIR's of $o(3) \oplus o(2)$ characterized by "quantum numbers" of energy, E_0 , and spin, s . Every bounded energy UIR of $o(3,2)$ is characterized entirely by the energy E_0 and spin s of its vacuum subspace and is therefore denoted as (E_0, s) . The unitarity of the representation (E_0, s) requires $/12/$ that $E_0 = \frac{n}{2}$, $s = \frac{m}{2}$ with some nonnegative integers n and m such that

the inequalities are satisfied $E_0 \geq s+1$ at $s \geq 1$ and $E_0 \geq s+1/2$ at $s \leq 1/2$. The UIR (E_0, s) corresponds to a spin- s particle in AdS_4 space with the mass.

$$m^2 = E_0(E_0 - 3) - (s+1)(s-2). \quad (2.4)$$

Let the representation $(1, 1/2) \oplus (1/2, 0)$ be denoted as S . Its irreducible components are often denoted as Di and Rac respectively because they correspond to famous Dirac's singletons /13/. The vacuum space of Rac is one-dimensional. Let us denote its basis vector as $|0\rangle$.

It is a very important property of singletons /14-16/ that the representation $S \otimes S$ reduces to the sum of massless representations only, and every massless UIR of $o(3, 2)$ is contained in $S \otimes S$. One can make sure, that, among other tensorial products of singletons, only $S \otimes S \otimes S \otimes S$ contains a massless representation (this is the representation $(2, 0)$ corresponding to one of the two types of massless scalars). Let us emphasize that since singletons are shown below to form UIR's of all higher-spin superalgebras under consideration (all of them contain $o(3, 2)$ as subalgebra), the aforementioned properties of singletons imply that, when investigating massless representations of higher-spin superalgebras, it suffices to analyse tensorial squares of singletons.

3. Oscillator Realizations of Higher-Spin Superalgebras

The analysis of UIR's of $N=1$ higher-spin superalgebras of refs./6,8/, is based heavily on their "oscillator realizations" proposed in refs./7,8/. Namely, the full higher-spin subalgebra, $shs^f(1)$, of the superalgebra of higher spins and auxiliary fields, $shsa(1)$, suggested in ref./8/, can be constructed starting from the associative algebra A with the generating elements q_α , r_β and K obeying the commutation relations

$$[q_\alpha, q_\beta] = 2i \varepsilon_{\alpha\beta}, \quad [z_\alpha, z_\beta] = 2i \varepsilon_{\alpha\beta}, \quad [q_\alpha, z_\beta] = 0, \quad (3.1a)$$

$$\{K, q_\alpha\} = 0, \quad \{K, z_\beta\} = 0, \quad K^2 = 1. \quad (3.1b)$$

Here indices $\alpha, \beta, \dots, \dot{\alpha}, \dot{\beta}, \dots$ take on values 1, 2 and are identified with two-component spinorial indices ($\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, $\varepsilon_{12} = 1$), while $[,]$ and $\{ , \}$ denote, respectively, commutator and anticommutator. Let us note that K possesses the properties of the Klein operator.

The Lie superalgebra $\text{shs}^f(1)$ with the product law $[, \}$ is constructed from A in the standard fashion,

$$[u, v\} = u v - (-1)^{\pi(u) \cdot \pi(v)} v u \quad (3.2)$$

where u and v are arbitrary elements of A possessing definite parities $\pi(u)$ and $\pi(v)$ defined by the relation

$$X(q, z, K) = (-1)^{\pi(X)} X(-q, -z, K), \quad (3.3)$$

which respects the normal relation between spin and statistics.

In fact, $\text{shs}^f(1)$ is the real form of the complex superalgebra described above, which is extracted by the hermiticity conditions

$$(q_\alpha)^\dagger = z_{\dot{\alpha}}, \quad (z_{\dot{\alpha}})^\dagger = q_\alpha, \quad K^\dagger = K. \quad (3.4)$$

(For more details see refs./7,8/. In ref./8/, it is also shown that other real forms, which are in accordance with the Lorentz structure, do not admit unitary representations.) Now, one can introduce new generating elements a , $a^\dagger$, b and $b^\dagger$,

$$a = \frac{1}{2} (q_1 + i z_2), \quad a^\dagger = \frac{1}{2} (z_1 - i q_2), \quad (3.5a)$$

$$b = \frac{1}{2} (z_1 + i q_2), \quad b^\dagger = \frac{1}{2} (q_1 - i z_2) \quad (3.5b)$$

in place of the explicitly Lorentz covariant generating elements q_α and $z_{\dot{\alpha}}$. The generating elements (3.5), which are convenient for the analysis of UIR's of $\text{shs}^f(1)$, possess the properties of

creation and annihilation operators (2.1) and (2.2).

Thus, $\text{shs}^f(1)$ is the infinite-dimensional superalgebra of various polynomials* constructed from q_α , r_α and K (equivalently, from $a^{(+)}$, $b^{(+)}$ and K). Let us emphasize that it is well-known (see, e.g., refs./18,19,21/ and references therein) that classical Lie superalgebras can be realized by quadratic polynomials constructed from bosonic and fermionic creation and annihilation operators analogously to that as $o(3,2) \sim sp(4)$ is realized by the polynomials (2.3). Infinite-dimensional higher-spin superalgebras, as well as superalgebras of higher spins and auxiliary fields of ref./8/, arise naturally as a result of the generalization /7,8/ of this construction by allowing both Klein-type operators and all powers of "oscillators".

Various higher-spin (super)algebras discussed in this paper are exhausted by those subalgebras of $\text{shs}^f(1)$ which contain themselves the subalgebra $o(3,2)$, formed by quadratic polynomials (2.3). All such higher-spin subalgebras of $\text{shs}^f(1)$ known to us can be extracted out from $\text{shs}^f(1)$ by means of involutive automorphisms /7,8/

$$\tau_{\alpha\beta\gamma} = f^\alpha \circ \mathcal{X}^\beta \circ \rho^\gamma \quad (3.6)$$

where $\alpha, \beta, \gamma = 0$ or 1 are exponents for elementary involutive and mutually commuting automorphisms f , $\mathcal{X}$ and ρ of $\text{shs}^f(1)$. Here f is the automorphism of boson-fermion parity,

* The superalgebra $\text{shs}^f(1)$ defined in such a way decomposes into the direct sum $\text{shs}^f(1) = \text{shs}^f(1) \oplus u(1)$ where $u(1)$ is the center of $\text{shs}^f(1)$ formed by constants, while $\text{shs}^f(1)$ is the simple superalgebra whose basis elements can be identified with various homogeneous monomials (the Weyl ordering of operators is assumed here) which depend nontrivially on q_α , r_α and K . In fact, only UIR's of $\text{shs}^f(1)$ are investigated in what follows, while the possibility of introducing nontrivial quantum numbers with respect to $u(1)$ is disregarded everywhere.

$$f(K) = K, \quad f(a^{(*)}) = -a^{(*)}, \quad f(b^{(*)}) = -b^{(*)}, \quad (3.7)$$

$\mathcal{A}$ is the automorphism that changes the sign of K ,

$$\mathcal{A}(K) = -K, \quad \mathcal{A}(a^{(*)}) = a^{(*)}, \quad \mathcal{A}(b^{(*)}) = b^{(*)}, \quad (3.8)$$

and β is the automorphism of $\text{shs}^f(1)$ related in the standard way /7/,

$$\beta(u) = -i \pi(u) \sigma(u), \quad (3.9)$$

to the antiautomorphism σ of the original (complex) associative algebra A , which is defined by the relations

$$\sigma(a^{(*)}) = i a^{(*)}, \quad \sigma(b^{(*)}) = i b^{(*)}, \quad \sigma(K) = K, \quad \sigma(xy) = \sigma(y) \sigma(x). \quad (3.10)$$

(It is assumed in eq.(3.9) that the boson-fermion parity $\pi(u)$ takes on values 0 or 1. For more details about the relation between antiautomorphisms of associative algebras and automorphisms of corresponding Lie superalgebras see refs./7,8/.)

Also, it is useful to observe that both A and $\text{shs}^f(1)$ possess the automorphism β defined by the relation

$$\beta(u) = (iK)^{\pi(u)} u, \quad (3.11)$$

where $\pi(u) = 0(1)$ if u is even (odd) element of $\text{shs}^f(1)$. The fact that β actually provides the automorphism of $\text{shs}^f(1)$ follows from the invariance of eqs.(3.1),(3.4) under the substitution

$$q_\alpha \rightarrow q'_\alpha = iK q_\alpha, \quad z_\alpha \rightarrow z'_\alpha = iK z_\alpha, \quad K \rightarrow K' = K. \quad (3.12)$$

Comparing eqs.(3.7)-(3.10) with eq.(3.12), one finds that β has the following simple properties

$$\beta^2 = f, \quad \beta \circ f = f \circ \beta = \beta^{-1}, \quad (3.13)$$

$$\beta \rho \beta^{-1} = \rho f, \quad \beta \mathcal{A} \beta^{-1} = \mathcal{A} f. \quad (3.14)$$

The properties (3.14) are most important for us since it follows from eq.(3.14) that the subalgebras extracted out from $\text{shs}^f(1)$

with the aid of relations

$$f^\alpha \cdot x^\beta \cdot \rho^\gamma(u) = u \quad (3.15)$$

with some sets of exponents α, β, γ are isomorphic to the subalgebras extracted by means of conditions arising from eq.(3.15) after the substitution $\beta \rightarrow \rho\beta$, $x \rightarrow x f$. Note that $\beta = \tau_{\pi/4}$ where τ_{φ} is the inner automorphism of $\text{shs}^f(1)$ defined by the relation $\tau_{\varphi}(u) = \exp(i\varphi K) u \exp(-i\varphi K)$, $\forall u \in \text{shs}^f(1)$. With the aid of the automorphisms $\tau_{\varphi}(u)$, one can also make sure that no new subalgebras of $\text{shs}^f(1)$ appear if the relations of the type (3.15) are allowed to contain the automorphism β itself, i.e. it suffices to use various conditions (3.15) containing automorphisms f, x and ρ .

Let us discuss shortly which subalgebras of $\text{shs}^f(1)$ can be constructed with the aid of various conditions (3.15). The higher-spin subalgebra $\text{shs}(1)$ which was argued in ref./6/ to be closely related to dynamics of massless higher-spin fields is extracted out from $\text{shs}^f(1)$ by the condition

$$x(u) = u \quad \square(\text{shs}_0(1)) = (1, 3/2, 2, 5/2, 3, \dots, \infty) \quad (3.16)$$

Eq.(3.16) implies that $\text{shs}(1)$ is spanned by polynomials

$P(a, a^+, b, b^+)$ which do not depend on K (in ref./6/, the designation $\text{shs}_0(1)$ was used in place of $\text{shs}(1)$). Note that in formulae like (3.16), the spectra of spins $\square(h)$ are also presented which were argued in refs./6-8/ to be described by the gauge fields of corresponding superalgebras h .

The superalgebras $\text{shs}(1|\alpha) \subset \text{shs}(1)$ with $\alpha=0$ or 1 , are extracted by the conditions

$$\begin{cases} x(u) = u \\ \rho \circ f^\alpha(u) = u \end{cases} \quad \square(\text{shs}(1|\alpha)) = (2, 4, 6, \dots, 3/2+\alpha, 5/2+\alpha, \dots) \quad (3.17)$$

Let B_q denote the subspace $B_q \subset \text{shs}(1)$ spanned by degree $-q$ fully symmetric polynomials $P(\lambda a, \lambda a^+, \lambda b, \lambda b^+) = \lambda^q P(a, a^+, b, b^+)$ (i.e. the Weyl ordering is implied). Then /6,7/, $\text{shs}(1|\alpha) =$

$$\sum_{p=0}^{\infty} B_{4p+2} \oplus B_{4p+1+2\alpha}.$$

The even subalgebra $\text{hs}(1)$ of $\text{shs}(1)$, $\text{hs}(1) = \sum_{p=0}^{\infty} B_{2p}$, is extracted by the condition

$$\begin{cases} x(u) = u \\ f(u) = u \end{cases}, \quad \Xi(hs(1)) = (1, 2, 3, \dots, \infty). \quad (3.18)$$

The even subalgebra $\text{hs}_2(1)$ of $\text{shs}(1|\alpha)$ (for both values of α) is extracted by the conditions

$$\begin{cases} x(u) = u \\ f(u) = u \\ p(u) = u \end{cases}, \quad \Xi(hs_2(1)) = (2, 4, 6, \dots). \quad (3.19)$$

Note that $\text{hs}_2(1) = \sum_{p=0}^{\infty} B_{4p+2}$.

Now let us note that, as shown in ref./8/, the even (bosonic) part of $\text{shs}^f(1)$ reduces to $\text{hs}(1) \oplus \text{hs}(1)$. Therefore, the analysis of all bosonic (Lie) subalgebras of $\text{shs}^f(1)$ reduces to the analysis of $\text{hs}(1)$ and its further subalgebras. Thus, only those subalgebras of $\text{shs}^f(1)$ remain to be considered that do not belong to $\text{shs}(1)$ and possess nontrivial odd sectors. These are exhausted by the subalgebras extracted by the conditions

$$\begin{aligned} \widetilde{\text{shs}}^f(1|\alpha): \quad & x \circ f \circ p(u) = u, \\ & \Xi(\widetilde{\text{shs}}^f(1|\alpha)) = 2 \times (3/2 + \alpha, 7/2 + \alpha, \dots) + (1, 2, 3, \dots) \end{aligned} \quad (3.20)$$

($\alpha = 0$ or 1) and

$$\text{shs}^f(1|0): \quad \begin{aligned} & p(u) = u, \\ & \Xi(\text{shs}^f(1|0)) = 2 \times (2, 4, 6, \dots) + (3/2, 5/2, \dots) \end{aligned} \quad (3.21)$$

(note that $\text{shs}^f(1|0) = \sum_{p=0}^{\infty} B_{4p+2} \oplus K B_{4p+2} \oplus B_{4p+1} \oplus K B_{4p+3}$).

We have disregarded the subalgebras extracted by the conditions

$$f \circ p(u) = u \quad \text{and} \quad f \circ x(u) = u$$

because they are isomorphic, respectively, to $\text{shs}^f(1|0)$ and $\text{shs}(1)$ due to the equivalence

relations* (3.14).

All higher-spin (super)algebras discussed above contain the $o(3,2)$ subalgebra (2.3) which gives rise to spin-2 gravitational gauge fields. Let h be any of these higher-spin superalgebras and let $\mathcal{O}$ denote the linear space of h . Then $\mathcal{O} = \mathcal{O}_+ \oplus \mathcal{O}_0 \oplus \mathcal{O}_-$ where $\mathcal{O}_+$, $\mathcal{O}_-$ and $\mathcal{O}_0$ are spanned by vectors which, respectively, increase, decrease and leave invariant eigenvalues of the energy operator E defined by eq.(2.3a). Note that $\mathcal{O}_+$, $\mathcal{O}_-$ and $\mathcal{O}_0$ form subalgebras of h . Let R be the representation space of some bounded energy UIR of h . Analogously to the case of $o(3,2)$ discussed in sect.2, R should contain the vacuum subspace V such that $u(V)=0 \quad \forall u \in \mathcal{O}_-$. Once again, V forms a representation of $\mathcal{O}_0$ that offers the way for reducing the problem of classification of bounded energy UIR's of h to the problem of classification of UIR's of $\mathcal{O}_0$. Unfortunately, a more detailed discussion of this interesting point lies far beyond the scope of the present paper.

* Analogous equivalence relations exist for $shsa(1)$, the superalgebra of higher spins and auxiliary fields proposed in ref./8/. They originate from the extension of $\mathfrak{p}$ to $shsa(1)$, which is defined by the relation $\beta(u) = (i\hat{Q})^{\pi_Q(u)} (i\hat{R})^{\pi_R(u)} u$ for every $u(q, z, \hat{Q}, \hat{R})$ possessing definite parities, $u(-q, z, \hat{Q}, \hat{R}) = (-1)^{\pi_Q} u(q, z, \hat{Q}, \hat{R})$, $u(q, -z, \hat{Q}, \hat{R}) = (-1)^{\pi_R} u(q, z, \hat{Q}, \hat{R})$ where $\hat{Q}$ and $\hat{R}$ are "Klein operators" for dotted and undotted spinors, introduced in refs./8,3/. Once again, $\beta(u) = \tau_{\pi_Q} u$ where τ_{π_Q} is the inner automorphism defined by the relation $\tau_{\pi_Q}(u) = \exp(-i\varphi(\hat{Q} + \hat{R})) u \exp(i\varphi(\hat{Q} + \hat{R}))$. One can easily see that β obeys eq.(3.13) and the first of the relations (3.14), $\beta\beta\beta^{-1} = \beta f$. With the aid of this relation, one can prove that the superalgebras $shsa(1|a,b)$ with $a, b=0$ or 1 , defined in sect.6 of ref./8/, are isomorphic at $b=0$ and $b=1$ for any fixed a . Note that these isomorphisms manifested themselves in ref./8/ by the fact that the types of gauge fields originating from $shsa(1|a,b)$ coincided at $b=0$ and $b=1$.

4. Massless Unitary Representations and Admissibility

Condition for Higher-Spin Superalgebras

The fact, which turns out to be of extreme importance for our analysis, is that the usual Fock space, spanned by the vectors $(a^+)^n(b^+)^m|0\rangle$ with the vacuum vector $|0\rangle$ such that $a|0\rangle = b|0\rangle = 0$, forms the singleton representation S of the $o(3,2)$ Lie algebra (2.3) (see e.g. ref./19/ and references therein; note also that the scalar product $\langle, \rangle$ in the Fock space is assumed to be defined in the standard way which respects eq.(2.2)). In this construction, Rac and Di components of S coincide with the subspaces spanned by the vectors $(a^+)^n(b^+)^m|0\rangle$ with $n+m$ even and odd, respectively. Note that $S = \text{Di} \oplus \text{Rac}$ forms /13,14,11/ UIR of the superalgebra $osp(1,4)$ spanned by the quadratic polynomials (2.3) supplemented by the supergenerators identified with various polynomials depending linearly on $a^{(+)}$ and $b^{(+)}$.

As a result, we observe that S forms UIR of $shs(1)$, the subalgebra of $shs^f(1)$ spanned by polynomials independent on the Klein operator K . Since $K \in \mathcal{O}_c$, the vector $K|0\rangle$ belongs to the vacuum subspace V of the corresponding representation of $shs^f(1)$. By introducing new basis vacuum vectors $|0\rangle_{\pm} = |0\rangle \pm K|0\rangle$, one arrives at the two irreducible representations of both A and $shs^f(1)$ with the Fock vacua $|0\rangle_{\pm}$ such that

$$K|0\rangle_{\pm} = \pm|0\rangle_{\pm} \quad (4.1)$$

Let us denote these representations $S_{\pm}$. Both S_+ and S_- behave as singletons $\text{Di} \oplus \text{Rac}$ with respect to $o(3,2)$.

All representations of higher-spin superalgebras discussed below are extracted out from various tensorial products of singletons (we do not have a proof that all bounded energy UIR's of higher-spin superalgebras can be obtained in such a way but it is

very likely that this is indeed the case). Generally, the action of an arbitrary superalgebra H on the tensorial product of its representations is defined by the relation

$$u(x \otimes y) = u(x) \otimes y + (-1)^{\pi(u)\pi(x)} x \otimes u(y). \quad (4.2)$$

Here u is an arbitrary element of H possessing definite parity $\pi(u)$, while x and y are elements of any two representations X and Y of H , possessing definite parities $\pi(x)$ and $\pi(y)$ which are assumed to be in accordance with the Z_2 -grading in H in the sense that*

$$\pi(u(x)) = \pi(u) + \pi(x). \quad (4.3)$$

Also, it is assumed in what follows that $\pi(x \otimes y) = (\pi(x) + \pi(y))_{\text{mod } 2}$. If X and Y are equivalent representations, one can define the (skew)symmetrization by the relation

$$(x \otimes y)_{S(A)} = x \otimes y + \varepsilon (-1)^{\pi(x)\pi(y)} y \otimes x \quad (4.4)$$

with $\varepsilon=1$ and $\varepsilon=-1$ for, respectively, symmetric product $(x \otimes y)_{S'}$ and antisymmetric product $(x \otimes y)_A$. Let us stress that the definition (4.2) preserves the (skew)symmetrization (4.4).

* There exist two parities for any irreducible representation, $\pi_0(x)$ and $\pi_1(x) = \pi_0(x) + 1 \pmod{2}$, which are in accordance with the boson-fermion grading in H . Let V_α^A be an arbitrary set of representations, enumerated by the label A , which are provided with the parities π_0 and π_1 if $\alpha=0$ or 1 , respectively. Then, all tensorial products $V_{\alpha_1}^{A_1} \otimes \dots \otimes V_{\alpha_k}^{A_k}$ are equivalent to $V_{\alpha_1}^{A_1} \otimes \dots \otimes V_{\alpha_k}^{A_k}$. Actually, it follows from eq.(4.3) that the operator $Q: X_1 \otimes \dots \otimes X_k \otimes X_{\alpha_{k+1}} \otimes \dots \otimes X_c \rightarrow (-1)^{\pi(x_{\alpha_{k+1}}) + \dots + \pi(x_c)} X_1 \otimes \dots \otimes X_k \otimes X_{\alpha_{k+1}} \otimes \dots \otimes X_c$ is intertwining between the representations $V_{\alpha_1}^{A_1} \otimes \dots \otimes V_{\alpha_k}^{A_k} \otimes V_{\alpha_{k+1}}^{A_{k+1}} \otimes \dots \otimes V_{\alpha_c}^{A_c}$ and $V_{\alpha_1}^{A_1} \otimes \dots \otimes V_{\alpha_k}^{A_k} \otimes V_{\alpha_{k+1}}^{A_{k+1}} \otimes \dots \otimes V_{\alpha_c}^{A_c}$. Taking this into account, we shall assume everywhere that the Fock vacua $|0\rangle$, and $|0\rangle_\pm$ possess the vanishing parity.

In practice, it is often convenient to analyse rank-M tensorial products of singleton (=Fock) representations of higher-spin superalgebras in terms of M species of operators $a_i^{(+)}$, $b_i^{(+)}$ and K_i ($i=1\dots M$) which obey eqs.(3.1) for any fixed value of i , but are (anti)commuting for different species,

$$u_i v_j = (-1)^{\pi(u_i)\pi(v_j)} v_j u_i, \quad i \neq j, \quad (4.5)$$

where $\pi(u)$ is the grading of $u \in \text{shs}^f(1)$, i.e. $\pi(a_i^{(+)}) = \pi(b_i^{(+)}) = 1$, $\pi(K_i) = 0$. Given element $P \in \text{shs}^f(1)$, described by the polynomial $P(a^{(+)}, b^{(+)}, K)$ in the oscillator realization of sect. 3, one introduces the polynomial

$$P^M = \sum_{i=1}^M P(a_i^{(+)}, b_i^{(+)}, K_i), \quad (4.6)$$

which induces the action of $\text{shs}^f(1)$ in the rank-M tensorial product of singletons S_+ and S_- depending on signs of eigenvalues k_i , $K_i |0_M\rangle = k_i |0_M\rangle$ with $k_i^2 = 1$ (i.e. $|0\rangle = |0\rangle_+ \otimes |0\rangle_- \otimes \dots$). Let us emphasize that each of polynomials on the r.h.s. of eq. (4.6) depends on one and only one sort of operators. Analogous construction was used e.g. in refs./18,19/ where UIR's of $\text{osp}(N;4)$ were investigated.

Now, let us turn to massless representations of higher-spin superalgebras. As emphasized in sect.2, it follows from the results of Flato and Fronsdal /14/ that such representations are contained in tensorial products of pairs of singletons. Let us first consider massless representations of $\text{shs}(1)$. These are contained in the representation $S \otimes S$ which decomposes into the sum of two UIR's, symmetric part $\Phi_S = (S \otimes S)_S$ and antisymmetric part $\Phi_A = (S \otimes S)_A$. It can be shown that the vacuum space of Φ_S is one-dimensional. Its vacuum vector is $|0_2\rangle = |0\rangle \otimes |0\rangle$. The vacuum space of Φ_A is two-dimensional. Its basis vectors, which form the spin-1/2 representation of $\text{su}(2)$,

are ($a^+|0>\otimes|0> - |0>\otimes a^+|0>$, $b^+|0>\otimes|0> - |0>\otimes b^+|0>$)
(let us stress that such a structure of vacua of Φ_A and Φ_S
proves the irreducibility of Φ_A and Φ_S with respect to
shs(1)). In the aforementioned construction with the additional
species of operators these vacuum vectors take the form $|o_2>$
for Φ_S and $(a_1^+ - a_2^+)|o_2>$, $(b_1^+ - b_2^+)|o_2>$ for Φ_A . Let us
show that the following decompositions of Φ_S and Φ_A into
UIR's of $o(3,2)$ are true:

$$\Phi_S = (1,0) \oplus (2,0) \oplus 2[(3,2) \oplus (5,4) \oplus (7,6) \oplus \dots] \oplus (3/2, 1/2) \oplus (5/2, 3/2) \oplus \dots, \quad (4.7a)$$

$$\Phi_A = 2[(2,1) \oplus (4,3) \oplus (6,5) \oplus \dots] \oplus (3/2, 1/2) \oplus (5/2, 3/2) \oplus \dots \quad (4.7b)$$

(note that in ref./14/ the decomposition is presented for the re-
presentation $S \otimes S = \Phi_S \oplus \Phi_A$ as a whole which is reducible
with respect to shs(1)).

First of all, it can be easily shown that all $o(3,2)$ vacua
in $S \otimes S$ possess the property that $o(3,2)$ vacuum vectors with ma-
ximum eigenvalues j_3 of J_3 (see eq.(2.3b)), are of the struc-
ture $\sum_k f_k(a^+)|0>\otimes g_k(a^+)|0>$ where f_k and g_k do not
depend on b^+ when $j_3 > 0$. Also, it is a simple matter to show that
there are only two spin-0 vacua in $S \otimes S$. These are $|0>\otimes|0>$
and $a^+|0>\otimes b^+|0> - b^+|0>\otimes a^+|0>$ which therefore
belong both to Φ_S and correspond to UIR's (1,0) and (2,0), res-
pectively (recall that a^+ and b^+ are odd elements of shs(1) that
leads to the minus sign in the second vacuum).

In the case of half-integer spins (fermions), vacuum vectors
of $o(3,2)$ which possess maximal J_3 eigenvalues are of the form

$$\varphi_1 = (\lambda_0 a_1^{+2\kappa+1} + \lambda_2 a_1^{+2\kappa-1} a_2^{+2} + \dots + \lambda_{2\kappa} a_1^+ a_2^{+2\kappa}) |o_2>, \quad (4.8a)$$

$$\varphi_2 = (\mu_0 a_2^{+2\kappa+1} + \mu_2 a_1^{+2} a_2^{+2\kappa-1} + \dots + \mu_{2\kappa} a_1^{+2\kappa} a_2^+) |o_2>, \quad (4.8b)$$

where we use notations with two species of operators a_1^+ and a_2^+ ($s=k+1/2$). Obviously, the space of solutions of the only nontrivial vacuum equation $(a_1^2 + a_2^2)\varphi_i = 0$, is one-dimensional. The same is true for the equation $(a_1^2 + a_2^2)\varphi_2 = 0$. Moreover, one easily makes sure that $\mu_{2i} = \mu \lambda_{2i}$ if both $\{\mu_i\}$ and $\{\lambda_i\}$ correspond to nontrivial $o(3,2)$ vacua. Choosing the normalization with $\mu = 1$, one finds that $\varphi_1 + \varphi_2 \in \Phi_S$ and $\varphi_1 - \varphi_2 \in \Phi_A$. This proves that any of the representations Φ_S and Φ_A contains every half-integer spin once and only once.

In the case of even spins, $o(3,2)$ vacuum vectors possessing maximal J_3 eigenvalues are of the form

$$\varphi_1 = (\lambda_{2k} a_1^{+4k} + \lambda_{2k-1} a_1^{+4k-2} a_2^{+2} + \dots + \lambda_0 a_2^{+4k}) |o_2\rangle, \quad (4.9a)$$

$$\varphi_2 = (\mu_{2k} a_1^{+4k-1} a_2^{+1} + \dots + \mu_1 a_1^{+1} a_2^{+4k-1}) |o_2\rangle \quad (4.9b)$$

($s=2k$). From the $o(3,2)$ vacuum equations $(a_1^2 + a_2^2)\varphi = 0$ one finds one-dimensional spaces of solutions for φ_1 and φ_2 such that $\lambda_i = \lambda_{2k-i}$, $\mu_i = -\mu_{2k+1-i}$. This implies that both $\varphi_1 \in \Phi_S$ and $\varphi_2 \in \Phi_S$, i.e. Φ_S contains twice every even spin while Φ_A does not contain even spins at all (cf. eqs.(4.7)).

Finally, in the case of odd spins, $o(3,2)$ vacuum vectors possessing maximal J_3 eigenvalues are of the form

$$\varphi_1 = (\lambda_{2k+1} a_1^{+4k+2} + \lambda_{2k} a_1^{+4k} a_2^{+2} + \dots + \lambda_0 a_2^{+4k+2}) |o_2\rangle, \quad (4.10a)$$

$$\varphi_2 = (\mu_{2k} a_1^{+(4k+1)} a_2^{+1} + \dots + \mu_0 a_1^{+1} a_2^{+(4k+2)}) |o_2\rangle \quad (4.10b)$$

($s=2k+1$). It follows from the vacuum equations that $\lambda_i = -\lambda_{2k+1-i}$ and $\mu_i = \mu_{2k-i}$. This implies that both $\varphi_1 \in \Phi_A$ and $\varphi_2 \in \Phi_A$, i.e. Φ_A contains twice every odd spin while Φ_S does not contain odd spins at all that completes the proof of eqs.(4.7).

On the other hand, it was argued in refs./6,9/ that the gauge fields of $shs(1)$ correspond to massless particles of all spins $s \geq 1$, every spin emerges once and only once (cf. eq.(3.16)). Since it is now shown that any massless UIR of $shs(1)$ contains two copies of some bosonic particles, we conclude that $shs(1)$ does not obey the admissibility condition. Therefore, $shs(1)$ is ruled out as a candidate for the role of higher-spin superalgebra. This conclusion agrees with the results of ref./3/.

Now, let us consider the bosonic subalgebras $hs(1)$ and $hs_2(1)$ of $shs(1)$, which are defined in eqs.(3.18),(3.19).

The analysis, which is simple in principle but lengthy in practice, shows that those UIR's of $hs(1)$ and $hs_2(1)$ which are contained in $S \otimes S$, coincide for $hs(1)$ and $hs_2(1)$ and reduce to various types of symmetrization for products of elementary singletons Di and Rac .

Their spin contents are as follows:

$$(Rac \otimes Rac)_S = (0^1, 2, 4, \dots), \quad (Di \otimes Di)_S = (0^2, 2, 4, \dots), \quad (4.11a)$$

$$(Rac \otimes Rac)_A = (1, 3, 5, \dots), \quad (Di \otimes Di)_A = (1, 3, 5, \dots), \quad (4.11b)$$

$$(Di \otimes Rac) = (1/2, 3/2, 5/2, \dots), \quad (4.11c)$$

where 0^1 and 0^2 denote massless representations $(1,0)$ and $(2,0)$ respectively. One observes that $hs(1)$ admits representations (although reducible) containing each massless bosonic particle once and only once and therefore respects the admissibility condition (cf. the spectrum of gauge fields in eq.(3.18)). Analogously, $hs_2(1)$ obeys the admissibility condition since it possesses UIR's containing every even massless spin once and only once (cf. eq.(3.49)). In addition, one finds that each of bosonic gauge theories of the above types should contain one massless scalar. All

these conclusions coincide precisely with those of ref./3/.

Now, let us consider the case of $\text{shs}^f(1)$. As shown in ref. /8/, gauge fields of $\text{shs}^f(1)$ describe the infinite set of spins $s \geq 1$, each spin emerges twice. Massless representations of $\text{shs}^f(1)$ are contained in $S_+ \otimes S_+$, $S_- \otimes S_-$ and $S_+ \otimes S_-$. The first two representations are reducible, $S_{\pm} \otimes S_{\pm} = (S_{\pm} \otimes S_{\pm})_S \oplus (S_{\pm} \otimes S_{\pm})_A$, while the latter, $S_+ \otimes S_-$, is irreducible. However, all of them contain two species of massless particles of every spin. Therefore, $\text{shs}^f(1)$ obeys the admissibility condition, and it is shown that physical theories based on $\text{shs}^f(1)$ should contain two species of lower-spin fields in addition to gauge massless fields of $\text{shs}^f(1)$ describing spins $s \geq 1$. Once again, these conclusions are in agreement with the results of ref./3/.

As already mentioned, any of the representations $S_+ \otimes S_+$ and $S_- \otimes S_-$ decomposes into the sum of two UIR's of $\text{shs}^f(1)$, symmetric, $\Phi_{\pm S} = (S_{\pm} \otimes S_{\pm})_S$, and skewsymmetric, $\Phi_{\pm A} = (S_{\pm} \otimes S_{\pm})_A$. Since $S_{\pm}$ behave as ordinary singleton S with respect to $o(3,2)$, it follows from the above analysis of massless UIR's of $\text{shs}(1)$ that $\Phi_{\pm S}$ and $\Phi_{\pm A}$ decompose into UIR's of $o(3,2)$ just in the same way as Φ_S and Φ_A in eq.(4.7). On the other hand, according to eq.(3.21), the subalgebra $\text{shs}^f(1|0) \subset \text{shs}^f(1)$ gives rise to gauge fields with spins $2 \times (2, 4, 6, \dots) \oplus (3/2, 5/2, \dots)$. Comparing this spin content with that of Φ_S in eq.(4.7) one concludes that $\text{shs}^f(1|0)$ obeys the admissibility condition and that, in the sector of lower spins, corresponding physical theories should contain two massless spin-0 fields and one massless spin-1/2 field. Needless to say that these conclusions are in remarkable agreement with the results of ref./3/.

Since the even subalgebra of $\text{shs}^f(1)$ is of the form /8/ $\text{hs}(1) \oplus \text{hs}(1)$, to complete the analysis of higher-spin subalge-

bras of $\text{shs}^f(1)$, it remains to consider superalgebras $\text{shs}(1|\alpha)$ and $\widetilde{\text{shs}}(1|\alpha)$ defined in eqs.(3.17) and (3.20). It is a simple matter to make sure that the both types of superalgebras do not obey the admissibility condition. Indeed, gauge fields of any of these superalgebras do not contain either massless fermions with spins $3/2, 7/2, 11/2 \dots$ or massless fermions with spins $5/2, 9/2, 13/2 \dots$. On the other hand, the only massless fermionic UIR, $\text{Di} \otimes \text{Rac}$, of $\text{sh}_2 \subset \text{shs}(1|\alpha) \subset \widetilde{\text{shs}}^f(1|\alpha)$ contains all half-integer spins $s=1/2, 3/2, 5/2 \dots$ (see eq.(4.11c)).

Combining all these results, one observes that only those subalgebras of $\text{shs}^f(1)$ do not obey the admissibility condition, which contain a non-trivial fermionic sector and are extracted out from $\text{shs}^f(1)$ with the aid of the automorphism $\mathcal{R}$ (3.8) (these are $\text{shs}(1)$, $\text{shs}(1|\alpha)$ and $\widetilde{\text{shs}}^f(1|\alpha)$). All other higher-spin subalgebras of $\text{shs}^f(1)$ mentioned in sect.3 obey the admissibility condition and therefore are expected to serve as rigid (super)symmetries in appropriate higher-spin theories. These are $\text{shs}^f(1)$ itself, all its bosonic subalgebras considered in sect.3, and the superalgebra $\text{shs}^f(1|0)$ defined in eq.(3.21). Note that the interpretation of these results from the point of view of the approach of ref./3/ is quite simple: $\mathcal{R}$ is the only automorphism of $\text{shs}^f(1)$ among the involutive automorphisms $\mathcal{f}$, $\mathcal{R}$ and $\mathcal{P}$ defined by eqs.(3.7)-(3.10), which admits no extension to the deformed higher-spin free differential algebra of ref./3/ in presence of fermions. Actually, $\text{shs}^f(1)$ is the subalgebra extracted out from the superalgebra $\text{shsa}(1)$ of ref./8/ by the condition $\tau_Q \tau_R(u) = u$ with the involutive automorphisms τ_Q and τ_R defined in eqs.(5.9), (5.10) of ref./8/. In these notations, the automorphism $\mathcal{R}$ used in the present paper coincides with either of the automorphisms τ_Q or τ_R ($\tau_Q = \tau_R$ on $\text{shs}^f(1)$). In ref.

/3/, the automorphisms τ_Q and τ_R of $\text{shsa}(1)$ were shown to admit no extension to the deformed free differential algebra of higher spins and auxiliary fields (in presence of fermions). Nevertheless, the automorphism $\tau_Q \circ \tau_R$ admits the extension to this free differential algebra /3/ that makes it possible to extract out the higher-spin sector of the free differential algebra above, that corresponds to $\text{shs}^f(1)$.

5. Conclusion

Thus, it is shown that all those higher-spin superalgebras, which lead to consistent equations of motion of massless fields in the framework of the approach of ref./3/, obey the admissibility condition which requires that these superalgebras admit massless unitary representations with the same spectra of spins as predicted by the structure of gauge fields associated with these superalgebras. Conversely, it is shown that all those higher-spin subalgebras of $\text{shsa}(1)$ which could not be used in the framework of the approach of ref./3/, do not obey the admissibility condition. Thus, it is proved that the constructing of consistent theories based on these superalgebras is impossible in principle and not because of some incompleteness in the approach of ref./3/. In particular, the pure higher-spin superalgebra $\text{shs}(1)$ proposed in ref./6/ is ruled out although its even (bosonic) higher-spin subalgebras are shown to be admissible. In order to construct a consistent supersymmetric theory one should use higher-spin superalgebras of the type of $\text{shs}^f(1)$ or $\text{shs}^f(1|0)$ in which at least bosonic fields of even spins $s=2,4,\dots$ are doubled. Let us emphasize that the entire coincidence of the conclusions of the present paper with those of ref./3/ provides for a highly nontrivial indication in favour of the existence of fully consistent theories of

massless fields of all spins and the adequacy of the dynamical approach of ref./3/.

In the present paper, we confined ourselves to massless representations of higher-spin superalgebra since these are needed for the analysis of the admissibility condition. Massive representations of higher-spin superalgebras may also be of interest since they describe higher-spin "matter multiplets". For example, such matter multiplets are expected to emerge naturally after compactification of extra dimensions in (yet hypothetical) higher-dimensional higher-spin gauge theories which admit vacuum solutions possessing four-dimensional rigid supersymmetries described by the higher-spin superalgebras under investigation.

Massive representations of $\text{shs}^f(1)$ are described by various tensorial products of singletons S_+ and S_- containing more than two products factors. Various irreducible representations of $\text{shs}^f(1)$ are of the form $P_+(S_+ \otimes \dots \otimes S_+) \otimes P_-(S_- \otimes \dots \otimes S_-)$ where P_+ and P_- denote some symmetrization schemes (Young super-tableaux) for S_+ and S_- respectively. (Strictly speaking, we have no explicit proof that all these representations are irreducible, but our analysis of various specific cases has convinced us that this is actually the case.) The same as for massless case, it is interesting to investigate which spectra of masses and spins correspond to various UIR's of $\text{shs}^f(1)$. Formally, this is equivalent to the decomposition of UIR's of $\text{shs}^f(1)$ into UIR's of $\mathfrak{o}(3,2) \subset$

$\text{shs}^f(1)$ where the anti-de Sitter algebra $\mathfrak{o}(3,2)$ is identified with the algebra of quadratic polynomials (2.3). Unfortunately, this analysis rapidly complicates as the number of elementary singletons S_+ and S_- increases in the aforementioned tensorial products, and we have now only some partial results on massive representations. The only general fact we would like to mention

here is that tensorial products of more than two singletons contain infinite chains of particles with fixed spins and infinitely increasing masses. In this respect, the situation with massive UIR's of higher-spin superalgebras is analogous to that familiar for string theories. Note however, that in the case of higher-spin superalgebras the typical mass scale is proportional to the inverse AdS radius and, therefore, all massive UIR's of higher-spin superalgebras, constructed from finite numbers of singletons, become massless in the flat limit.

Also, it is interesting to investigate UIR's of extended higher-spin superalgebras proposed in ref./3/ and of higher-dimensional higher-spin superalgebras proposed in ref./22/. In the latter case, no dynamical approach is presently known generalizing that of ref./3/ to arbitrary dimensions, and the higher-dimensional admissibility conditions may give some important information in this situation. We hope to discuss these intriguing questions in future publications.

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