1 Introduction

One of the unsatisfactory features of the standard model is the lack of understanding of the strong CP-problem.\textsuperscript{1} The parameter $\theta$ which multiplies a $P$, $T$ violating term in the QCD Lagrangian has to be fine tuned to $< 10^{-9}$ in order to agree with non-observation of the neutron dipole moment. An elegant solution of this problem is to add a global Peccei-Quinn (PQ) $U(1)_{PQ}$ symmetry\textsuperscript{2} which breaks at a scale $f_a$ in the order of $10^8 - 10^{12}$ GeV according to the experimental, astrophysical, and cosmological considerations.\textsuperscript{3} This high-scale breakdown of $U(1)_{PQ}$ gives rise to an invisible pseudo-Goldstone boson, the invisible axion.\textsuperscript{4-6} There are two well-known minimal types of invisible axion models: (i) Dine-Fischler-Srednicki-Zhitnitskii (DFSZ)\textsuperscript{5} and (ii) Kim-Shifman-Vainshtein-Zakharov (KSVZ)\textsuperscript{6} types, in which the breaking scale $f_a$ is obtained by the large vacuum expectation value (VEV) of a complex Higgs-singlet $\chi$ field.

If neutrinos are not massless, one can achieve an understanding of the smallness of their masses within the framework of invisible axion models. This is done by introducing one singlet right-handed neutrino for each family. Then by identifying $f_a$ with the scale $\Lambda$ of the see-saw mechanism\textsuperscript{7} the following naive neutrino mass hierarchy

$$M_{\nu_3} : M_{\nu_2} : M_{\nu_1} \sim m_{\nu_3}^2 : m_{\nu_2}^2 : m_{\nu_1}^2,$$  \hspace{1cm} (1.1)

can be derived.\textsuperscript{8} It is common to assume a generation independent diagonal Dirac neutrino mass matrix and a unique right-handed Majorana mass for each generation.

In general, the neutrino mass hierarchy (NMH) of (1.1) need not hold true and it can be different for different structures of fermion mass matrices. However, in
\( t_1 = |t_1|^2 = |t_2| = |t_1 + t_2| = |t_1 + t_2| \)

\[
\begin{pmatrix}
  t_1 & 0 \\
  t_2 & t_1
\end{pmatrix}
\]

for quark mass matrices and must satisfy the following condition:

\[
\begin{pmatrix}
  t_1 & 0 \\
  t_2 & t_1
\end{pmatrix} = \mathcal{V}
\]

In order to have a Fitzpatrick-like matrix:

\[
\begin{pmatrix}
  t_1 & t_2 & t_1 + t_2 \\
  t_1 + t_2 & t_2 & t_1
\end{pmatrix} \equiv \mathcal{V} = \mathcal{Z}
\]

\[\text{Explicitly}\]

\[
\begin{pmatrix}
  t_1 & t_2 & t_1 + t_2 \\
  t_1 + t_2 & t_2 & t_1
\end{pmatrix} \equiv \mathcal{V} = \mathcal{Z}
\]

For the purpose of the matrix form change for the quarks can be written in a matrix form.

\[\begin{pmatrix}
  t_1 & t_2 & t_1 + t_2 \\
  t_1 + t_2 & t_2 & t_1
\end{pmatrix} \equiv \mathcal{V} = \mathcal{Z}
\]

\[\text{We now determine the horizontal family of each fermion family by requiring}\]

\[\text{denoted by } \mathcal{Z}\]

\[\text{respectively, and } i, j, \text{ and } k \text{ are the family indices. The fermionic } \mathcal{Z} \text{ and}]

\[\text{are the generic left-handed quark and lepton doublets.}\]

\[\begin{pmatrix}
  \mathcal{Z}_{i,j} \mathcal{Z} & \mathcal{Z}_{i,j} \mathcal{Z} \\
  \mathcal{Z}_{i,j} \mathcal{Z} & \mathcal{Z}_{i,j} \mathcal{Z}
\end{pmatrix} = \mathcal{Z}
\]

\[\chi_{i,j} \mathcal{Z} \mathcal{Z} \mathcal{Z} \mathcal{Z} \mathcal{Z} = \chi_{i,j} \mathcal{Z} \mathcal{Z} \mathcal{Z} \mathcal{Z} \mathcal{Z}
\]

\[\text{Symmetry. The Higgs and fermion fields have the following transformation}\]

\[\mathcal{H}(t_1 \mathcal{Z}) \times \mathcal{H}(t_2 \mathcal{Z}) \times \mathcal{H}(t_1 \mathcal{Z}) \times \mathcal{H}(t_2 \mathcal{Z}) \]

\[\text{and one single } X \text{ Higgs fields with}\]

\[\text{The model contains two doublets and one single } X \text{ Higgs fields with}\]

\[\text{The Model}\]

\[\text{with the experimental and cosmological bounds imposed on them. The condensation}\]

\[\text{study the fermion mass matrices. InSector } 2 \text{ we examine the high neutrino masses}\]

\[\text{The paper is organized as follows: in Sector 2, we present details of the model and}\]

\[\text{will explore the high neutrino masses in various cases.}\]

\[\text{We scale in Sector 2. In this paper, we will study in detail the DM model. Especially, we}\]

\[\text{the problem of the DM. The fermion mass matrices are suppressed at } U(1)_{\text{Fermion}}\]

\[\text{are unidentified determined. In their model, there is a unique natural current induced by}\]

\[\text{quark mass matrices and a global horizontal } \mathcal{P}_G \text{ symmetry exists and these changes}\]

\[\text{that the two Higgs doublets and one Higgs singlet minima model with Fitzpatrick-like}\]

\[\text{DSS and KZAS models. If this point is not by definition and wall (DW)}\]

\[\text{are no longer generation-independent. To incorporate this we have to beyond the}\]

\[\text{symmetry and thus the fermion mass matrices and the strong CP-}\]

\[\text{symmetry. Obviously, the } \mathcal{P}_G \text{ changes of the symmetry}\]

\[\text{model. It should be very interesting if this symmetry can be identified as a } \mathcal{P}_G\]

\[\text{In the KM matrix, they are placed on the operator such masses a way to dispose the}\]

\[\text{prominent quark mass matrices are the Fitzpatrick-like matrices which lead to the}\]

\[\text{is well-known that the most}\]

\[\text{Higgs bifundamentals (HBF) of quark mixings. It is well-known that the most}\]

\[\text{accounting the compactified } 6 \times 6 \text{ neutrino mass matrix in General. Naturally, one}\]

\[\text{practically, it is hard to get exact expression of } \mathcal{H}\text{ because of the difficulty in the}\]
and

\[ 2|Z_1|, 2|Z_2|, |Z_1 + Z_3| \neq 1 , \]  

(2.6)
because the U(1)_{PQ} charges for \( \phi_{1,2} \) are \( \pm 1 \). From (2.5) and (2.6), one finds that the only solutions are

\[ (Z_1, Z_2, Z_3) = \pm \left( \frac{5}{2}, \frac{-3}{2}, \frac{1}{2} \right) . \]  

(2.7)

The two solutions become equivalent when the signs of the \( \phi_i \)'s charges are redefined and we shall take the plus sign in (2.7) for the remaining paper. The PQ charges in (2.7) are precisely the one given by DW.\(^{11}\) It should be noted that the PQ charges for \( \phi_1 \) and \( \phi_2 \) must be opposite in order to have both up- and down-quark mass matrices be Fritzsch-type. It is easily seen that the assignment of PQ charges in (2.7) results in the same amount of color anomaly on \([SU(3)_C]^2 \cdot U(1)_{PQ} \) as in the DFSZ model.\(^5\)

We thus have the following Yukawa couplings:

\[
\mathcal{L}_Y = \left( h^a_{12} \phi^+_1 \phi^0_{1R} + h^a_{13} \phi^+_1 \phi^0_{1L} + h^a_{21} \phi^+_2 \phi^0_{2R} + h^a_{23} \phi^+_2 \phi^0_{2L} + h^a_{31} \phi^+_3 \phi^0_{3R} + h^a_{32} \phi^+_3 \phi^0_{3L} \right) + \left( u \rightarrow d, \phi^{(2)}_{1(2)} \rightarrow \phi^{(1)}_{1(2)} \right) + \left( q \rightarrow l, u \rightarrow v, d \rightarrow e \right) + \\
+ (h^u_{11} \nu_1^T C \nu_1^R + h^u_{12} \nu_2^T C \nu_1^R + h^u_{13} \nu_3^T C \nu_1^R + h^u_{21} \nu_1^T C \nu_2^R + h^u_{22} \nu_2^T C \nu_2^R + h^u_{23} \nu_3^T C \nu_2^R + h^u_{31} \nu_1^T C \nu_3^R + h^u_{32} \nu_2^T C \nu_3^R + h^u_{33} \nu_3^T C \nu_3^R + h.c. \right) ,
\]  

(2.8)

and the most general Higgs potential:

\[
V = \sum m_i^2 \phi_i^+ \phi_i + \sum c_{ij} (\phi_i^+ \phi_j)(\phi_j^+ \phi_j) + \sum h_{i3} (\phi_i^+ \phi_3)(\phi_3^+ \phi_i) + [\phi_1^+ \phi_2 \phi_3^2 + h.c.] .
\]  

(2.9)

We write the Higgs fields and their VEV's as follows:

\[
\phi_i = \begin{pmatrix} \phi_i^+ e^{i\theta_i} (v_i + R_i + i I_i) \end{pmatrix} , \quad \chi = \frac{1}{\sqrt{2}} e^{i\theta_3} (\Lambda + R_3 + i I_3) ,
\]  

(2.10)

\[
\langle \phi_i \rangle = \frac{1}{\sqrt{2}} e^{i\theta_i} v_i , \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} e^{i\theta_3} \Lambda ,
\]  

(2.11)

where \( v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2} G_F)^{-1/2} \approx 250 \text{ GeV} \). Spontaneous symmetry breaking takes place at two different scales based on the VEV's of the doublets and singlet Higgs field. The Higgs potential \( V \) shown in (2.9) is exactly the same as the DFSZ model.\(^5\) From (2.9)-(2.11), one finds that the axion field is

\[
a = \frac{1}{f_a} \left[ \frac{2v_1 v_2}{v^2} (v_2 I_1 - v_1 I_2) + \Lambda I_3 \right] ,
\]  

(2.12)

with the axion decay constant

\[
f_a = \frac{1}{2v} \left[ 4v_1^2 v_2^2 + v^2 \Lambda^2 \right]^{1/2} \approx \frac{\Lambda}{2} ,
\]  

(2.13)

for \( \Lambda \gg v \). In order to obtain an invisible familon-axion, \( f_a \) would have to be in the range of \( 10^8 - 10^{12} \text{ GeV} \). The axion acquires a small mass via the color anomaly, and is given by

\[
m_a = m_a \frac{f_a}{f_a} \frac{3 \sqrt{m_a m_d}}{f_s (m_u + m_d)} .
\]  

(2.14)

From the Yukawa couplings in (2.8), one obtains the following fermion mass matrices:

\[
M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} h^u_{21} v_1 e^{-i\theta_1} & 0 & h^u_{23} v_2 e^{-i\theta_2} & 0 \\
0 & h^u_{31} v_1 e^{-i\theta_1} & 0 & h^u_{32} v_2 e^{-i\theta_2} \\
0 & h^u_{32} v_2 e^{-i\theta_2} & 0 & h^u_{33} v_3 e^{-i\theta_3} \\
0 & 0 & h^u_{33} v_3 e^{-i\theta_3} & 0 \end{pmatrix} ,
\]  

\[
M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} h^d_{21} v_1 e^{i\theta_1} & 0 & h^d_{23} v_2 e^{i\theta_2} & 0 \\
0 & h^d_{31} v_1 e^{i\theta_1} & 0 & h^d_{32} v_2 e^{i\theta_2} \\
0 & h^d_{32} v_2 e^{i\theta_2} & 0 & h^d_{33} v_3 e^{i\theta_3} \\
0 & 0 & h^d_{33} v_3 e^{i\theta_3} & 0 \end{pmatrix} ,
\]  

\[
M_e = M_e^D , \quad M_e = M_e \begin{pmatrix} h^e_{11} & h^e_{12} \end{pmatrix} ,
\]  

\[
M^N = \frac{1}{\sqrt{2}} \begin{pmatrix} h^N_{21} e^{i\theta_1} & 0 & h^N_{23} e^{i\theta_2} & 0 \\
0 & h^N_{31} e^{i\theta_1} & 0 & h^N_{32} e^{i\theta_2} \\
0 & h^N_{32} e^{i\theta_2} & 0 & h^N_{33} e^{i\theta_3} \\
0 & 0 & h^N_{33} e^{i\theta_3} & 0 \end{pmatrix} ,
\]  

(2.15)
(3.10) \[ \mu \approx \eta \mu \]

\[ \text{and so} \]

\[ \left( \mu \frac{\alpha}{\nu} + 1 \right) (\mu = \eta \mu) = \eta \mu \]

Running mass for the first order QCD correction is

\[ \mu \approx \eta \mu \]

charged Higgs contribution respectively. The relation between the physical mass and

106 GeV without the charged Higgs effect and to 106 GeV with the

\[ \mu \approx \eta \mu \]

of models the AMEGO result requires f-rho mass in the range of 96 GeV with the

\[ \mu \approx \eta \mu \]

a large charged Higgs is suggested that the large mixing results from a large

\[ \mu \approx \eta \mu \]

The 

\[ \mu \approx \eta \mu \]

The neutralino mass hierarchy is

\[ \nu \mu \approx \eta \mu \]

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Following the discussion in Ref. 13, one estimates the three light neutralino masses to

3. Light Neutralino Masses

The neutralino mass hierarchy is

\[ \nu \mu \approx \eta \mu \]

and

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be

Following the discussion in Ref. 13, one estimates the three light neutralino masses to
Taking \( m_i \) (1 GeV) \( \sim \) 150 GeV, \( m_e \) (1 GeV) \( \sim \) 1.3 GeV, \( m_s \) (1 GeV) \( \sim \) 5 MeV, and
\( A \approx 2 f_a \), we get
\[
M_{\nu_e} \approx 5.7 \times 10^{-2} \text{ eV} \cdot \left( \frac{10^7 \text{ GeV}}{h^N f_a} \right) = 5.7 \times 10^{-7} \left( \frac{10^{12} \text{ GeV}}{h^N f_a} \right) \text{ eV},
\]
(3.11a)
\[
M_{\nu_e} \approx 6.0 \text{ eV} \left( \frac{10^7 \text{ GeV}}{h^N f_a} \right) = 6.0 \times 10^{-4} \left( \frac{10^{12} \text{ GeV}}{h^N f_a} \right) \text{ eV},
\]
(3.11b)
and
\[
M_{\nu_e} \approx 6.4 \text{ eV} \left( \frac{10^7 \text{ GeV}}{h^N f_a} \right) = 64 \left( \frac{10^{12} \text{ GeV}}{h^N f_a} \right) \text{ eV}.
\]
(3.11c)

The current experimental and cosmological constraints on the neutrino masses are given by\(^2\)
\[
M_{\nu_e} < 18 \text{ eV}, \quad M_{\nu_\mu} < 250 \text{ keV}, \quad M_{\nu_\tau} < 35 \text{ MeV},
\]
(3.12a)
and\(^2\)
\[
\sum M_{\nu} < 65 \text{ eV},
\]
(3.12b)
for stable neutrinos and\(^2\)
\[
M_{\nu_e} > 1 \text{ MeV},
\]
(3.12c)
for unstable \( \nu_i \), respectively. From Eqs. (3.12a,c) we see that the only possible unstable neutrino is \( \nu_\tau \) with
\[
1 \text{ MeV} < M_{\nu_\tau} < 35 \text{ MeV}.
\]
(3.13)

Now we consider the following two cases,

(a) Stable \( \nu_e \):

From (3.11c) and (3.11b), one finds that
\[
h^N f_a > 10^{12} \text{ GeV},
\]
(3.14)
which requires \( h^N > 10^4 \) for \( f_a \sim 10^8 \text{ GeV} \) and \( h^N > 1 \) for \( f_a \sim 10^{12} \text{ GeV} \). With such unnatural large Yukawa coupling of Majorana right-handed neutrinos, we obtain
\[
M_{\nu_e} < 2.9 \times 10^{-7} \text{ eV}, \quad M_{\nu_\mu} < 6.0 \times 10^{-4} \text{ eV} \quad \text{and} \quad M_{\nu_\tau} < 64 \text{ eV}.
\]
(3.15)

With the above mass ranges we can understand the solar-neutrino puzzle via Mikheyev-Smirnov-Wolfenstein (MSW) mechanism\(^2\) due mainly to the \( \nu_e - \nu_\mu \) oscillations.

(b) Unstable \( \nu_e \):

In this case, the \( \nu_e \) mass range of (3.12) imposes the following constraint:
\[
6.4 \times 10^7 \text{ GeV} > h^N f_a > 1.8 \times 10^9 \text{ GeV}.
\]
(3.16)

On the other hand, the value of \( M_{\nu_e} \) which must be less than 65 eV implies
\[
h^N f_a > 9.2 \times 10^8 \text{ GeV}.
\]
(3.17)
Combining (3.16) and (3.17), we find that
\[
6.4 \times 10^7 \text{ GeV} > h^N f_a > 9.2 \times 10^8 \text{ GeV},
\]
(3.18)
and the Yukawa coupling: \( 0.64 > h^N > 0.092 \) for \( f_a \sim 10^8 \text{ GeV} \) and \( 6.4 \times 10^{-5} > h^N > 9.2 \times 10^{-6} \) for \( f_a \sim 10^{12} \text{ GeV} \), respectively, since the range of \( h^N f_a \) in (3.17) is now small, we have a better prediction for neutrino masses which are given by
\[
8.9 \times 10^{-3} \text{ eV} < M_{\nu_e} < 6.2 \times 10^{-2} \text{ eV},
\]
(3.19)
\[
9.3 \text{ eV} < M_{\nu_\mu} < 65 \text{ eV},
\]
and
\[
1 \text{ MeV} < M_{\nu_\tau} < 7 \text{ MeV}.
\]

It should be noted that to have a natural value of Yukawa couplings \( h \), cases (a) and (b) correspond to the two extreme PQ breaking scales \( 10^{12} \text{ GeV} \) and \( 10^8 \text{ GeV} \) respectively.
References


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cord Council of Canada.

I. My research topic is the mechanism of protein folding, which is a complex biological process. The study of protein folding is important because it helps us understand the function of proteins and how they interact with other molecules. The mechanism of protein folding involves several steps, including denaturation, refolding, and maturation, and each step is critical for the proper functioning of the protein.

In this paper, we have studied the refolding process in the horseradish peroxidase (HRP) system. HRP is a protein that is widely used as a model system due to its simple structure and well-characterized properties. We have investigated the refolding process under various conditions to understand the factors that influence the efficiency of protein refolding.

Our results show that the refolding process is influenced by several factors, including temperature, pH, and concentration. We have found that the refolding efficiency increases with increasing temperature and pH, and decreases with increasing concentration. These findings are consistent with previous studies and support the hypothesis that the refolding process is a thermodynamically driven process.

In conclusion, our study provides new insights into the refolding process of HRP and contributes to the understanding of protein folding mechanisms. Further studies are needed to elucidate the underlying mechanisms and to develop strategies for improving protein refolding efficiency.

Concluding Remarks

The results of our study are summarized in the following sections:

1. Introduction
2. Methods
3. Results
4. Discussion
5. Conclusion

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