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FAST CIRCLE FIT WITH THE CONFORMAL MAPPING METHOD

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A slight modification of the conformal mapping method transforms circular tracks into parabolas with small curvature. A fast linear fit to these parabolas yields the original circle parameters.

1. Introduction

Fitting a circle through a number of measured points obtained e.g. from a drift chamber requires generally an iterative fit due to the nonlinearity of the problem.

Since iterative methods are too time consuming in most applications, one generally uses approximations which permit the linearization of the problem yielding a set of linear equations which can be solved in one single iteration.

An approximation introduced by Karimaki [1] is widely used in track analysis. Another method proposed by Crawford [2] can also be used, but in some circumstances the results are not good [3].

We propose here an alternative method which has the advantage that it can be easily combined with track-recognition.

2. Outline of the method

One of the standard methods of track recognition is the method of conformal mapping. In the normal version of that method one transforms the circle equation:

$$(x - a)^2 + (y - b)^2 = R^2 \quad (1)$$

into a straight line with the prescription:

$$u = \frac{x}{x^2 + y^2}, \quad v = \frac{y}{x^2 + y^2} \quad (2)$$

If one fixes R by imposing the relation

$$R^2 = a^2 + b^2, \quad (3)$$

one obtains straight lines of the form:

$$v = \frac{1}{2b} - u \frac{a}{b}. \quad (4)$$

These straight lines can then be used for pattern recognition by establishing search roads following the

points, to find out which points belong to a given straight line. A fit to such a straight line in (u, v) space will then yield the coordinates of the center of the circle, a and b , and together with eq. (3) the radius R is also determined. The values thus obtained however, are only approximate, since eq. (3) forces the circle to pass through the origin and the important third parameter determining a track, the impact parameter ϵ , is lost. The situation can be remedied by allowing for a small difference between R^2 and $a^2 + b^2$, which we will call δ .

$$\delta = R^2 - a^2 - b^2. \quad (5)$$

For δ much smaller than R^2 we then have, instead of a straight line, a parabola with a very small curvature:

$$v = \frac{1}{2b} \left(1 - \frac{\delta}{4b^2} \right) - u \frac{a}{b} \left(1 - \frac{\delta}{2b^2} \right) - u^2 \delta \frac{R^2}{2b^3}. \quad (6)$$

Here terms of the order of δ^2 and higher have been neglected.

Furthermore, we may usually set

$$1 - \frac{\delta}{4b^2} \quad \text{and} \quad 1 - \frac{\delta}{2b^2}$$

in the above equation equal to one (in the case of the OPAL detector used for testing the algorithm δ/b^2 is of the order of 10^{-4} or smaller). The equation for the parabola then becomes:

$$v = \frac{1}{2b} - u \frac{a}{b} - u^2 \epsilon \left(\frac{R}{b} \right)^3, \quad (7)$$

where $\epsilon = R - \sqrt{a^2 + b^2} \cong \delta/2R$ is the impact parameter. This equation is identical to eq. (4) except for the term representing the curvature, which is proportional to the impact parameter. An ordinary parabola fit in (u, v) space will therefore yield the three circle parameters a , b , ϵ in (x, y) space.

Since a parabola is already linearized, it falls into the categories of fast, non-iterative fits mentioned above.

What one needs, then, for this type of fit to work, is a point known to lie close to the track, with respect to which the impact parameter ϵ is defined. In the analysis given above, this point was the origin. If the track does not pass close to origin, any other point like the last or first experimental point of the track will do. As long as the distance of this point to the track is small, the errors will be negligible. (This type of condition is also necessary for Karimaki's approximation.) *

Like for an ordinary parabola fit, it is best to rotate the track into a position where it is approximately parallel to the x -axis. This avoids singularities in the slope and, in the case considered here, minimizes the neglected terms of the order of δ/b^2 .

3. Error calculation

Following Gluckstern [4], one can give the estimated errors in the parameters for this type of fit.

The errors of the parameters A , B and C of a parabola

$$v = A + Bu + Cu^2$$

are for uncorrelated measurement errors:

$$\sigma_A^2 = \frac{\sum \sigma_n^2 R_n^2}{(\sum R_n)^2}, \quad \sigma_B^2 = \frac{\sum \sigma_n^2 P_n^2}{(\sum u_n P_n)^2},$$

$$\sigma_C^2 = \frac{\sum \sigma_n^2 Q_n^2}{(\sum u_n^2 Q_n)^2}.$$

$$\sigma_A \sigma_B = \frac{\sum \sigma_n^2 R_n P_n}{(\sum R_n) \sum u_n P_n}, \quad \sigma_B \sigma_C = \frac{\sum \sigma_n^2 P_n Q_n}{(\sum u_n P_n) \sum u_n^2 Q_n},$$

$$\sigma_A \sigma_C = \frac{\sum \sigma_n^2 R_n Q_n}{(\sum R_n) \sum u_n^2 Q_n},$$

* In fact, this trick can also be used in order to generalize the track recognition method so that it is able to find tracks not passing near the origin. For this, it suffices to perform a preliminary step in the track recognition procedure which searches for a point assumed to be on the track to be found. This point is then taken as the origin of a NEW coordinate system into which the other experimental points can be expressed by a simple translation. In this new system, the eq. (1), (2), (3) and (4) are valid provided the new origin is close to the track. The track recognition method is then again reduced to finding points lying on a straight line for a trial point (the new origin) assumed to belong to the track. By successively applying this straight line search for each trial point, the track recognition problem can be completely solved, yielding, at the same time, approximate values for the circle parameters which are simply related to the straight line parameters by the eq. (3) and (4).

where

$$R_n = f_n \left\{ \begin{vmatrix} F_2 & F_3 \\ F_3 & F_4 \end{vmatrix} - u_n' \begin{vmatrix} F_1 & F_2 \\ F_3 & F_4 \end{vmatrix} + u_n'^2 \begin{vmatrix} F_1 & F_2 \\ F_2 & F_3 \end{vmatrix} \right\},$$

$$P_n = f_n \left\{ - \begin{vmatrix} F_1 & F_3 \\ F_2 & F_4 \end{vmatrix} + u_n \begin{vmatrix} F_0 & F_2 \\ F_2 & F_4 \end{vmatrix} - u_n^2 \begin{vmatrix} F_0 & F_1 \\ F_2 & F_3 \end{vmatrix} \right\},$$

$$Q_n = f_n \left\{ \begin{vmatrix} F_1 & F_2 \\ F_2 & F_3 \end{vmatrix} - u_n \begin{vmatrix} F_0 & F_1 \\ F_2 & F_3 \end{vmatrix} + u_n^2 \begin{vmatrix} F_0 & F_1 \\ F_1 & F_2 \end{vmatrix} \right\},$$

with

f_n = weight for point v_n

and

$$F_j = \sum f_n (u_n)^j.$$

The σ_n used above are the errors on the points in u , v space. They are related to the errors on those same points in x , y space through:

$$\sigma_n^v = \frac{\sigma_n^x}{x^2 + y^2}.$$

In the fit as well as for the error estimation one must obviously use σ_n^v .

The relations between the parameters A , B and C and the circle parameters are (see eq. (7)):

$$b = 1/2A, \quad a = -B/2A,$$

$$\epsilon = -C(b/R)^3 = -C(1+B^2)^{-3/2}$$

with

$$R = \epsilon + \sqrt{a^2 + b^2} \cong \sqrt{a^2 + b^2}.$$

This yields then the following relations for the errors of the circle parameters:

$$\sigma_a^2 = \frac{1}{4A^4} (B^2 \sigma_A^2 + A^2 \sigma_B^2 - 2AB \sigma_A \sigma_B),$$

$$\sigma_b^2 = \frac{1}{4} \frac{\sigma_A^2}{A^4},$$

$$\sigma_a \sigma_b = \frac{1}{4A^4} (-B \sigma_A^2 + A \sigma_A \sigma_B),$$

$$\sigma_R^2 = \frac{1}{R^2} (b^2 \sigma_a^2 + a^2 \sigma_b^2 + 2ab \sigma_a \sigma_b).$$

One can also calculate the errors of the physically more interesting quantities: momentum, angle and impact parameter.

The relative momentum error is:

$$\sigma_P/P = \sigma_R/R,$$

the angular error is:

$$\sigma_\varphi = \frac{\sigma_B}{1+B} \varphi$$

(for tracks passing close to the origin so that $\varphi =$

Fig. 1. Comparison of results between the two fast track-fitting methods, Karimaki (triangles) and conformal mapping (circles). (a) Relative momentum resolution, (b) angular resolution, (c) impact parameter resolution.

$\arctan(a/b)$, and finally the error of the impact parameter:

$$\sigma_c = \left[\frac{1}{(1+B^2)^3} \left(\frac{9(BC)^2}{(1+B^2)^2} \sigma_B^2 + \sigma_c^2 - \frac{6BC}{(1+B^2)} \sigma_B \sigma_c \right) \right]^{1/2}.$$

If the track has been rotated so that it lies approximately parallel to the x -axis, B^2 will be in general small compared to 1, making the error of the impact parameter approximately equal to σ_c .

Naturally these error estimates are not valid if multiple scattering is present, for then the measurement errors are not uncorrelated.

The method has been tested with events generated through the OPAL detector simulation program called GOPAL110. In figs. 1a, 1b and 1c we show the results obtained with this method for the momentum, angular

and impact parameter resolution for events between 0.1 and 5 GeV. Also shown for comparison are points obtained with Karimaki's method.

4. Conclusion

In conclusion one can say that this circle fitting method works as well and as fast as the method of Karimaki, but with the added advantage that it can be more easily combined with the conformal mapping pattern recognition.

References

- [1] V. Karimaki, CERN UA1/TN-82-24.
- [2] J.F. Crawford, Nucl. Instr. and Meth. 211 (1983) 223.
- [3] H. Jeremie, CERN OPAL note july 86.
- [4] R.L. Gluckstern, Nucl. Instr. and Meth. 24 (1963) 381.