RAMOND STRING SCATTERING IN THE GROUP THEORETIC APPROACH TO STRING THEORY

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ABSTRACT

We use the group theoretic method to determine the tree-level N-string vertex for Ramond strings and the corresponding integration measure. As an explicit example we show how to recover the scattering amplitude for four Ramond states. Our calculation is manifestly Lorentz covariant. It appears that at no stage is it essential to introduce ghosts, and for the four- and six-point functions there is no advantage in doing so. For higher-point functions, however, the inclusion of ghosts will probably lead to calculational simplifications.

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In the last two years considerable effort has gone into the development of a number of new approaches to string theory. These approaches – the group theoretic approach,^[1–4] the “new dual model”^[5–13] and Grassmannians^[14–17] – have somewhat different starting points, but essentially the end result is the same in each case, namely a vertex describing the scattering of N arbitrary on-shell string states together with an appropriate integration measure. In the group theoretic approach, which will be our concern here, the vertex is determined by an extremely natural overlap relation for operators of a given conformal weight. From these overlap relations one can derive so-called integrated identities;^[1–5, 18–20] these correspond precisely to the subsequently found conserved charges^[15, 16] which follow from the conformal invariance of the Polyakov path integral.

Most of the work on these approaches has concentrated on the relatively straightforward cases of the bosonic and Neveu-Schwarz strings, apart from the addition of ghosts^[19, 11, 13] to the fermion emission vertex,^[21] but recently some attention has been given to the problems associated with the scattering of Ramond strings. Neveu and West showed how the group theoretic method could be used in principle to construct a vertex for Ramond strings,^[4] and Alvarez-Gaumé et al^[17] and Petersen et al^[6] have considered Ramond vertices in the framework of Grassmannians and the new dual model respectively. In this letter we show explicitly how to construct the tree-level vertex and integration measure for the Ramond case, and as an example we show how to reproduce the well-known 4-string scattering amplitude. It should be noted that our calculation is manifestly Lorentz covariant. Furthermore it is at no point essential to introduce ghosts, and for the four- and six-point functions there appears to be no advantage in doing so. For higher-point functions, however, it probably is better to include them.

Let us begin by recalling briefly the basic principles of the group theoretic method; a more detailed review can be found in ref.[22] For the bosonic string the essential idea is to take the world-sheet of the string and to choose coordinate patches around each Koba-Nielsen point, where a string is emitted. In these coordinate patches we can then introduce operators of definite conformal weight for each external string. At any point lying in an overlap region between coordinate patches there will in general be several such operators defined, one for each corresponding string. These operators act in different Fock spaces and so cannot be compared directly, but it does make sense to compare them when they act on the vertex which describes string scattering. In fact, we shall take it as the defining property of the vertex that these different operators, when referred to a common coordinate system, should

each give the same result when acting on the vertex. This is precisely the content of the overlap conditions written down by Neveu and West in ref.[1].

An equivalent way in which to view this picture of string scattering is to imagine starting with a number of free external strings and then to glue the world sheets of these strings together using conformal transformations. To extend this method to Neveu-Schwarz-Ramond strings it is then natural to consider gluing together copies of the super complex plane using appropriate superconformal transformations to form a super Riemann surface. For Ramond strings the obvious choice is to take superconformal transformations given by exponentiating the generators L_n, F_m of the Ramond algebra. For example, one such possibility would be to take supercoordinates (ξ_i, α_i) for the i th string to be given in terms of an underlying copy of the super complex plane with coordinates (z, θ) by

$$(\xi_i, \alpha_i) = e^{\theta_i F_i} e^{z_i L_{-1}}(z, \theta), \quad (1)$$

i.e.

$$\begin{aligned} \xi_i &= z - z_i + \theta_i \theta (z - z_i)^{1/2}, \\ \alpha_i &= \theta - \theta_i (z - z_i)^{-1/2}. \end{aligned} \quad (2)$$

Having chosen some such set of coordinates we could then write down an overlap identity in superspace and use it to construct the vertex, but in practice it is somewhat easier to proceed slightly differently. Any superconformal transformation T generated by the Ramond algebra can be written in the factorized form

$$T = \exp\left(\sum \theta_n F_n\right) \exp\left(\sum a_n L_n\right), \quad (3)$$

where the a_n are commuting parameters and the θ_n are anticommuting. From the general theory of the group theoretic approach it is easy to see that, if we know the vertex V_0 in the case when all the anticommuting parameters θ_n are zero, the complete vertex is given by

$$V_0 \prod_i \exp\left(\sum \theta_n F_n^i\right). \quad (4)$$

Although the vertex thus depends upon an infinite number of anticommuting parameters, the dependence on most of these disappears when we consider the scattering of on-shell physical

states. We know, for example, that F_n annihilates physical states for $n \geq 0$, so we can restrict the sum in (4) to run over $n < 0$, and in fact physical amplitudes will depend on only a few of the remaining anticommuting parameters. This point will be discussed in more detail later in this paper.

We shall begin by constructing the tree level vertex V_0 for the scattering of $2N$ Ramond strings $\Gamma^{j\mu}$. In the coordinates of ref.[23] this will satisfy the integrated overlap

$$V_0 \sum_{j=1}^{2N} \oint_{z=z_j} dz \phi(z) \frac{\Gamma^{j\mu}(z - z_j)}{\sqrt{z - z_j}} = 0. \quad (5)$$

Here $z - z_j$ is a coordinate for the j th string and $\phi(z)$ is a tensor field of rank $1/2$ with a branch cut structure such that $\phi(z) \Gamma^{j\mu}(z - z_j)/\sqrt{z - z_j}$ is single-valued around $z = z_j$ for each j . We can write ϕ in the form

$$\phi = \left(\prod_1^{2N} \sqrt{z - z_j} \right) f(z) \equiv Q f(z), \quad (6)$$

where now f is a meromorphic function of z with poles only at Koba-Nielsen points. In order for the tensor field ϕ not to have a pole at infinity (where we choose in general not to place a Koba-Nielsen point) we must have that ϕ is at most of order $1/z$ as $z \rightarrow \infty$.

In terms of modes we can expand the Ramond field Γ as

$$\frac{\Gamma^{j\mu}(z - z_j)}{\sqrt{z - z_j}} = \sum_{r \in \mathbb{Z}} d_r^{j\mu} (z - z_j)^{-r-1/2}. \quad (7)$$

A particular case of the identity (5) is given by taking the function f to have single poles at precisely $N+1$ of the Koba-Nielsen points and to be regular elsewhere. There are precisely N linearly independent functions of this form, and a convenient basis is given by

$$f^p = \frac{1}{(z - z_p)(z - z_{N+1})(z - z_{N+2}) \cdots (z - z_{2N})} \quad p = 1, \dots, N. \quad (8)$$

The corresponding tensor field $\phi^p(z)$ then has the form

$$\phi^p = \frac{1}{z - z_p} \frac{(z - z_1)^{1/2}(z - z_2)^{1/2} \cdots (z - z_N)^{1/2}}{(z - z_{N+1})^{1/2}(z - z_{N+2})^{1/2} \cdots (z - z_{2N})^{1/2}} \quad p = 1, \dots, N. \quad (9)$$

In a neighbourhood of $z = z_j$ we can expand ϕ^p as

$$\phi^p = a_0^{pj} (z - z_j)^{-1/2} + \sum_{n=1}^{\infty} a_n^{pj} (z - z_j)^{n-1/2}, \quad (10)$$

where

$$a_n^{pj} = \frac{1}{n!} \frac{\partial^n}{\partial z^n} \left[(z - z_j)^{1/2} \phi^p(z) \right] \Big|_{z=z_j}. \quad (11)$$

More explicitly we have, for example,

$$a_0^{pj} = \begin{cases} \frac{\prod_{\substack{k=1 \\ k \neq p}}^N (z_p - z_k)^{1/2}}{\prod_{k=N+1}^{2N} (z_p - z_k)^{1/2}}, & \text{if } j = p, \\ \frac{1}{(z_j - z_p)^{1/2}} \frac{\prod_{\substack{k=1 \\ k \neq p}}^N (z_j - z_k)^{1/2}}{\prod_{\substack{k=N+1 \\ k \neq j}}^{2N} (z_j - z_k)^{1/2}}, & \text{if } j = N+1, \dots, 2N, \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Substituting the expression for ϕ into eqn (5) leads to the N identities

$$V_0 \left\{ \sum_{j=1}^{2N} a_0^{pj} d_0^{j\mu} + \sum_{n>0} \sum_j a_n^{pj} d_n^{j\mu} \right\} = 0. \quad (13)$$

It will be convenient to introduce a symbol to stand for the combination of zero-modes appearing in eqn (13). Let us write $A_0^{pj} = a_0^{pj}/a_0^{pp}$, $A_n^{pj} = \frac{1}{\sqrt{2}} a_n^{pj}/a_0^{pp}$ for $n > 0$, and then define

$$\begin{aligned} D^p &\equiv \frac{1}{\sqrt{2}} \sum_{j=1}^{2N} A_0^{pj} d_0^j \\ &= \frac{1}{\sqrt{2}} \left(d_0^p + \sum_{j=N+1}^{2N} A_0^{pj} d_0^j \right) \quad p = 1, \dots, N. \end{aligned} \quad (14)$$

(Here and in what follows the spacetime index μ will be omitted unless this is likely to lead to confusion.) The identity (13) then becomes

$$V_0 \left\{ D^p + \sum_{n>0} \sum_j A_n^{pj} d_n^j \right\} = 0. \quad (15)$$

There are some integrability conditions which follow by taking the anti-commutator of

this equation with itself; these constrain the coefficients A_0^{pj} to satisfy

$$\delta^{p,q} + \sum_{j=N+1}^{2N} A_0^{pj} A_0^{qj} = 0. \quad (16)$$

Given the N operators D^p it is then possible to find a further set of N oscillators S^p such that

$$\begin{aligned} \{S^p, D^q\} &= \delta^{p,q} \\ \{S^p, S^q\} &= 0; \end{aligned} \quad (17)$$

a convenient choice is to take S^p to be given by

$$S^p = \frac{1}{\sqrt{2}} \left(d_0^p - \sum_{j=N+1}^{2N} A_0^{pj} d_0^j \right). \quad (18)$$

Eqns (14) and (18) can readily be inverted, using the integrability condition (16), to give

$$d_0^q = \begin{cases} \frac{1}{\sqrt{2}}(S^q + D^q), & q = 1, \dots, N, \\ \frac{1}{\sqrt{2}} \sum_{p=1}^N (S^p - D^p) A_0^{pq}, & q = N+1, \dots, 2N. \end{cases} \quad (19)$$

As explained in ref.[4] we also find identities of the form

$$V_0 \left\{ d_{-m}^j + \sum_k \sum_{n \geq 0} c_{mn}^{jk} d_n^k \right\} = 0, \quad m = 1, \dots \quad (20)$$

Using equations (19) and (15) we can rewrite this in terms of S^p , giving

$$V_0 \left\{ d_{-m}^j + \sum_p f_{m0}^{jp} S^p + \sum_k \sum_{n > 0} f_{mn}^{jk} d_n^k \right\} = 0, \quad m = 1, \dots, \quad (21)$$

for certain coefficients f_{mn}^{jk} . There are some integrability conditions following from these identities, namely

$$\begin{aligned} f_{m0}^{jp} + A_m^{pj} &= 0 \\ f_{mn}^{jk} + f_{nm}^{kj} &= 0. \end{aligned} \quad (22)$$

We are now in a position to write down the vertex V_0 . In order to do this we first choose

a vacuum state $\langle\phi|$ that satisfies

$$\begin{aligned}\langle\phi|D^{p\mu} &= 0 & p = 1, \dots, N; \quad \mu = 1, \dots, 10 \\ \langle\phi|d_n^{j\mu} &= 0 & n > 0.\end{aligned}\tag{23}$$

We shall see in a moment how to write down such a state in a manifestly Lorentz covariant way. In terms of such a vacuum state the vertex V_0 is given by

$$V_0 \propto \langle\phi| \exp \left\{ \sum_{n>0} S^p A_n^{pj} d_n^j + \frac{1}{2} \sum_{m,n>0} d_m^j d_n^k f_{mn}^{jk} \right\}; \tag{24}$$

in deriving this expression it is necessary to make use of the integrability conditions (22).

We now turn to the problem of the construction of the vacuum $\langle\phi|$ using the zero-mode conditions

$$\langle\phi|D^{p\mu} = 0, \quad p = 1, \dots, N. \tag{25}$$

The solution to these identities can be given in a non-covariant form by making a decomposition of the $d^{i\mu}$ into creation and annihilation operators $d^{i1} \pm id^{i2}, \dots, d^{i9} \pm id^{i10}$ and then choosing a state $\langle 0|$ which is annihilated by all of the operators $d^{i,2p-1} + id^{i,2p}$. It is then clear that, up to a normalisation factor, the solution to eqn (25) is given by

$$\langle\phi| = \langle 0| \prod_{p=1}^N \prod_{a=1}^5 \left(\frac{1}{\sqrt{2}} (D^{p,2a-1} - iD^{p,2a}) \right). \tag{26}$$

Before showing how to rewrite this in a covariant way let us consider the normalization of the vertex V_0 —this is not determined by the overlaps. In order to fix this we need to consider the action of the Virasoro generators L_{-1}^k on the vertex. For the simple cycling used here we know that^[23]

$$\frac{\partial V_0}{\partial z^k} = V_0 (L_{-1}^k + c^k). \tag{27}$$

To find c^k we apply a suitable state to both sides of eqn (27). A suitable choice is

$$|\phi\rangle = \prod_{p,a} \left(\frac{1}{\sqrt{2}} (S^{p,2a-1} + iS^{p,2a}) \right) |0\rangle \tag{28}$$

where the $S^{p\mu}$ are given above. Applying $|\phi\rangle$ to the left hand side of eqn (27) we find that

$$\begin{aligned} \frac{\partial V_0}{\partial z^k} |\phi\rangle &= \frac{1}{2} \sum_{p,a} \left\{ \frac{\partial(D^{p,2a-1} - iD^{p,2a})}{\partial z^k}, S^{p,2a+1} + iS^{p,2a} \right\} \\ &= -\frac{5}{2} \sum_{p,j} \frac{\partial A_0^{pj}}{\partial z^k} A_0^{pj} \\ &= 0. \end{aligned} \tag{29}$$

On the other hand we have

$$\begin{aligned} V_0 L_{-1}^k |\phi\rangle &= \frac{1}{2} V_0 \psi_{-1}^k \psi_0^k |\phi\rangle \\ &= -\frac{1}{2} V_0 \left(\sum_p f_{1,0}^{kp} S^p \right) \psi_0^k |\phi\rangle \\ &= -\frac{1}{2} \sum_{p,\mu} f_{1,0}^{kp} \{ S^{p\mu}, \psi_0^{k\mu} \}, \end{aligned} \tag{30}$$

where we have used $V_0 |\phi\rangle = 1$. Using the expression (18) for S^p we then find

$$V_0 L_{-1}^k |\phi\rangle = \begin{cases} \frac{5}{\sqrt{2}} A_1^{kk}, & 1 \leq k \leq N, \\ \frac{5}{\sqrt{2}} \sum_p A_1^{pk} A_0^{pk}, & N+1 \leq k \leq 2N. \end{cases} \tag{31}$$

An explicit calculation then shows that we can write c_k as

$$c^k = -\frac{5}{4} \partial_k \ln \frac{\prod_{1 \leq p < q \leq N} (z_p - z_q) \prod_{N+1 \leq i < j \leq 2N} (z_i - z_j)}{\prod_{\substack{1 \leq p \leq N \\ N+1 \leq j \leq 2N}} (z_p - z_j)}; \tag{32}$$

the fact that c_k can be written as a derivative in this way follows from the integrability condition derived from eqn (27). It will be convenient to define a normalized vertex

$$V = V_0 \left\{ \frac{\prod_{1 \leq p < q \leq N} (z_p - z_q) \prod_{N+1 \leq i < j \leq 2N} (z_i - z_j)}{\prod_{\substack{1 \leq p \leq N \\ N+1 \leq j \leq 2N}} (z_p - z_j)} \right\}^{5/4}; \tag{33}$$

V then satisfies $V L_{-1}^i = \partial_i V$ for all i .

Now we want to write V in a manifestly Lorentz covariant form. One way to do this which appears to be quite natural physically for the problem in hand is to introduce a new vector $\langle\psi|$ satisfying

$$\langle\psi|(d^{i,\mu} + id^{i+N,\mu}) = 0, \quad i = 1, \dots, N. \quad (34)$$

It is then rather easy to see that the state $\langle\psi|\prod D^{\mu\mu}$, which evidently satisfies the conditions (25), is given by

$$\langle\psi|\prod D^{\mu\mu} = \det(\delta^{pq} + iA^{p,q+N})^5 \langle\phi|. \quad (35)$$

Let us now return to the incorporation of anticommuting parameters into the vertex. It is explained in ref.[24] that string vertices are induced representations of copies of the (super) Virasoro group, with the isotropy group being given by the integrated overlaps for the (super) Virasoro operators. For the Virasoro operators $L(z) = \sum_n L_{-n} z^n$ the integrated overlaps for the bosonic and super string are the same and are given in ref.[5]. For the Ramond string the tree vertex V satisfies, in addition, identities for the super Virasoro generators F ,

$$V \sum_{j=1}^{2N} \oint d\xi_j \phi \frac{F^j(\xi_j)}{(\xi_j)^{3/2}} = 0, \quad (36)$$

where ϕ has the same branch cut and analyticity properties as for the Γ overlap (5) but must behave at most like z as $z \rightarrow \infty$. Taking ϕ to be given by

$$\begin{aligned} \phi^{(p)} &= Q(z - z_p)^{-1} \prod_{j=1}^{j=N-2} (z - z_j)^{-1}, \quad p = N-1, \dots, 2N \\ &= \begin{cases} \hat{a}_{-1}^{(p)j} (z - z_j)^{-1/2} + \sum_{n=0}^{\infty} \hat{a}_n^{(p)j} \xi_j^{n+1/2} & j = p \text{ or } j = 1, \dots, N-2 \\ \sum_{n=0}^{\infty} \hat{a}_n^{(p)j} \xi_j^{n+1/2} & \text{otherwise} \end{cases} \end{aligned} \quad (37)$$

implies identities of the form

$$V \left\{ F_{-1}^p + \sum_{j=1}^{N-2} \hat{c}_{1,-1}^{pj} F_{-1}^j + \sum_{n=0}^{\infty} \sum_{j=1}^{2N} \hat{c}_{1n}^{pj} F_n^j \right\} = 0, \quad p = N-1, \dots, 2N. \quad (38)$$

For suitable functions ϕ one also finds the identities

$$V \left\{ F_{-n}^j + \sum_{m=-1}^{\infty} \sum_{k=1}^{2N} \hat{c}_{nm}^{jk} F_m^k \right\} = 0, \quad n \geq 2, \quad (39)$$

where by eqn (38) we may take $\hat{c}_{n,-1}^{jk}$ to be zero for $k = N-1, \dots, 2N$. The vertex at any

point of the coset may be obtained from the vertex V by boosting with the coset generators. The overlaps of eqns (15) and (21) determine uniquely the vertex V at the coset point which involves no super boosts, i.e. no anticommuting parameters. Analysing eqns (38) and (39) tells us that the only relevant super boosts in the coset are any $N-2$ of the $2N$ F_{-1}^j 's, (as well as F_n^j , $n \geq 0$). We will choose these to be F_{-1}^j , $j = 1, \dots, N-2$. After the θ integration the vertex is given by

$$W = V F_{-1}^1 \dots F_{-1}^{N-2}, \quad (40)$$

plus possible terms with fewer F_{-1} 's. These extra terms can arise if the integration measure depends on θ ; in this case the integral over θ will result in terms containing fewer than $N-2$ F_{-1} 's. Such terms can arise only when more than 6 external legs are present and we shall not consider them further here. The scattering amplitude is then given by saturating the desired physical states against

$$\int dz_i f(z_i) W. \quad (41)$$

It should be noted that we could have given a precisely equivalent discussion of the inclusion of anticommuting parameters in the vertex by starting with an expression of the form (4). The physical state conditions enable us to drop terms F_n with $n \geq 0$, and the identities (38) and (39) enable us to eliminate all but $N-2$ of the F_{-1} 's from the vertex. The expression that remains depends on $2N-3$ commuting parameters z_i (taking into account $Sl(2; C)$ invariance) and $N-2$ anticommuting parameters; these are precisely the parameters describing a super Riemann surface of genus zero with $2N$ Ramond punctures.

We now illustrate how to compute the measure, following the general procedure of the group-theoretic method. Any state of the form $(L_{-1} + \frac{1}{2} F_{-1} F_0)|\Omega\rangle$, with $F_n|\Omega\rangle = (L_n + \delta_{n,0})|\Omega\rangle = 0$, for $n \geq 0$, is physical and has zero norm and must decouple from the vertex W of eqn (40). This means that when we apply this state to the vertex we should get at most a total derivative in those Koba-Nielsen variables which are to be integrated.

We find that for $1 \leq j \leq N-2$

$$\begin{aligned} 0 &= W(L_{-1}^j - 1/2 F_{-1}^j F_0^j)|\Omega\rangle \\ &= \int f(z_i) \left\{ \left(\frac{\partial V}{\partial z^j} + \frac{1}{2} c_{2,-1}^{jj} \right) F_{-1}^1 \dots F_{-1}^{N-2} + \text{terms with fewer factors of } F_{-1} \right\}. \end{aligned} \quad (42)$$

The terms with different numbers of factors of F_{-1} must cancel separately, so considering the

term with $N-2$ such factors and integrating by parts we find that the measure f obeys

$$\frac{\partial \ln f}{\partial z_j} - \frac{1}{2} \hat{e}_{2,-1}^{jj} = 0, \quad j = 1, \dots, N-2. \quad (43)$$

When $N-1 \leq j \leq 2N$ the zero-norm state does not sit against an F_{-1} factor, and one must rearrange the F_{-1} 's using the relation (38). Considering as before the term with $N-2$ factors of F_{-1} we find that

$$\frac{\partial \ln f}{\partial z_j} + \frac{1}{2} \hat{e}_{1,0}^{jj} = 0, \quad j = N-1, \dots, 2N. \quad (44)$$

Eqs (43) and (44) can readily be integrated (as a consequence of integrability conditions following from eqns (38),(39)) to give

$$f = \left\{ \frac{\prod_{\substack{1 \leq j \leq N-2 \\ N-1 \leq p \leq 2N}} (z_j - z_p)}{\prod_{N-1 \leq p < q \leq 2N} (z_p - z_q) \prod_{1 \leq i < j \leq N-2} (z_i - z_j)} \right\}^{1/4}. \quad (45)$$

We must also take into account $Sl(2; C)$ gauge invariance; if we choose to fix z_{2N-2} , z_{2N-1} and z_{2N} the full integration measure is

$$\prod_{i=1}^{2N-3} dz_i (z_{2N-2} - z_{2N-1})(z_{2N-1} - z_{2N})(z_{2N} - z_{2N-2}) f. \quad (46)$$

For the amplitudes with 2, 4 or 6 external fermionic strings there is at most one factor of F_{-1} in the vertex, and the above computation determines the measure completely. For more than 6 fermionic strings one can have contributions to W of the form V multiplied by fewer than $N-2$ factors of F_{-1} , as we discussed earlier. To calculate the coefficients of these terms we need to use the identity which eliminates L_{-2} in terms of L_n , $n \geq -1$, integrate by parts the derivatives on V coming from L_{-1} and use the equations above for the derivative of the measure. In general these coefficients will not be zero, but when ghosts are included it is found that only the leading term with $N-2$ factors of F_{-1} is present. It is for this reason that in the framework presented here the inclusion of ghosts leads to calculational simplifications for amplitudes with more than 6 external fermionic strings.

By way of example, we now indicate how to calculate the amplitude with 4 external fermionic ground states. Making a choice of signs for the various square roots occuring in D^1 and D^2 we have

$$D^1 = \frac{1}{\sqrt{2}}(d^1 + x d^3 + y d^4), \quad D^2 = \frac{1}{\sqrt{2}}(d^2 + y d^3 - x d^4), \quad (47)$$

where

$$\begin{aligned} x &= \frac{(z_1 - z_4)^{1/2}(z_2 - z_3)^{1/2}}{(z_1 - z_2)^{1/2}(z_3 - z_4)^{1/2}} \\ y &= \frac{(z_1 - z_3)^{1/2}(z_4 - z_2)^{1/2}}{(z_1 - z_2)^{1/2}(z_3 - z_4)^{1/2}}. \end{aligned} \quad (48)$$

As in eqn (37) we choose a state $\langle\psi|$ such that

$$\langle\psi|(d^1 + i d^2) = \langle\psi|(d^3 + i d^4) = 0. \quad (49)$$

We then find

$$\langle\psi| \prod_{r=1}^{p\mu} D^{p\mu} = y^{10} \sum_{r=1}^{10} \left[\frac{x - i}{y} \right]^r \langle\psi| (i\sqrt{2}d^3)^{\mu_1 \dots \mu_r} (i\sqrt{2}d^4)_{\mu_{r+1} \dots \mu_{10}}. \quad (50)$$

Acting on this vertex with the tensor product state $|u_1\rangle_1 |u_3\rangle_3 |u_2\rangle_2 |u_4\rangle_4$, where now we restrict attention to the case where the u_i all have the same chirality, we obtain the following expression for the matrix element:

$$y^{10} \sum_{r=0}^{10} \left[\frac{x - i}{y} \right]^r \bar{u}_1 \gamma^{\mu_1 \dots \mu_r} u_3 \bar{u}_2 \gamma_{\mu_{r+1} \dots \mu_{10}} u_4. \quad (51)$$

Some simple γ -matrix algebra reduces this to

$$4ixy^3(x-i)^5 \left\{ \frac{2}{y^2}(x^2 - 1) \bar{u}_1 \gamma^\mu u_3 \bar{u}_2 \gamma_\mu u_4 + \bar{u}_1 \gamma^{\mu\nu\rho} u_3 \bar{u}_2 \gamma_{\rho\nu\mu} u_4 \right\}, \quad (52)$$

and combining this with the normalization factors from eqns (36) and (46) gives an answer which agrees with the known result.^[28]

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REFERENCES

1. A. Neveu and P. West, Phys. Lett. 193B (1987) 187
2. A. Neveu and P. West, Phys. Lett. 194B (1987) 200.
A. Neveu and P. West, Comm. Math. Phys. 114 (1988) 613
3. A. Neveu and P. West, Phys. Lett. 200B (1987) 275
4. A. Neveu and P. West, "Group theoretic approach to the superstring and its supermoduli," to be published in Nucl. Phys. B
5. A. Neveu and P. West, Nucl. Phys. B278 (1986) 601
6. P. Di Vecchia, R. Nakayama, J.L. Petersen, J. Sidenius, and S. Sciuto, Phys. Lett. 182B (1986) 164,
P. Di Vecchia, R. Nakayama, J.L. Petersen, J. Sidenius, and S. Sciuto, Nucl. Phys. B287 (1987) 621,
P. Di Vecchia, M. Frau, A. Lerda and S. Sciuto, Nucl. Phys. B298 (1988) 527,
J.L. Petersen and J.R. Sidenius, Nucl. Phys. B301 (1988) 247
7. P. Di Vecchia, K. Hornfeck, M. Frau, A. Lerda and S. Sciuto, Phys. Lett. 205B (1988) 250,
P. Di Vecchia, K. Hornfeck, M. Frau, A. Lerda and S. Sciuto, "N-string, g-loop vertex for the fermionic string," preprint NORDITA 88/19 P,
J.L. Petersen, J.R. Sidenius and A.K. Tollsten, "Covariant loop calculus for the Neveu-Schwarz string," Niels Bohr Institute preprint NBI HE 88-30
8. J.L. Petersen, J.R. Sidenius and A.K. Tollsten, "Covariant super reggeon calculus for superstrings," Niels Bohr Institute preprint NBI HE 88-29
9. A. Clarizia and F. Pezzella, Nucl. Phys. B298 (1988) 636
10. U. Carow-Watamura and S. Watamura, Nucl. Phys. B302 (1988) 149
11. U. Carow-Watamura and S. Watamura, Nucl. Phys. B301 (1988) 132
12. A. LeClair, Nucl. Phys. B297 (1988) 603

13. A. LeClair, Nucl. Phys. B303 (1988) 189
14. L. Alvarez-Gaumé, C. Gomez and C. Reina, Phys. Lett. 190B (1987) 55,
L. Alvarez-Gaumé, C. Gomez and C. Reina, “New methods in string theory,” in proceedings of Spring School on Superstrings, Trieste 1987, ed. R. Iengo and E. Sezgin, World Scientific 1987,
L. Alvarez-Gaumé, C. Gomez G. Moore and C. Vafa, Nucl. Phys. B303 (1988) 455
15. C. Vafa, Phys. Lett. 190B (1987) 47
16. E. Witten, Comm. Math. Phys. 113 (1988) 529
17. L. Alvarez-Gaumé, C. Gomez, P. Nelson, G. Sierra and C. Vafa, “Fermionic strings in the operator formalism,” Boston preprint BUHEP 88-11
18. A. Neveu and P. West, Phys. Lett. 179B (1986) 235
19. Y. Kazama, A. Neveu, H. Nicolai and P. West, Nucl. Phys. B278 (1986) 833
20. P. Di Vecchia, K. Hornfeck and M. Yu, Phys. Lett. 195B (1987) 557
21. E. Corrigan and D. Olive, Nuovo Cim. 11A (1972) 749
22. P. West, “A brief review of the group theoretic approach to string theory,” Pres. at the 23rd Rencontre de Moriond - Electroweak interactions and unified theories, Les Arcs, 6 - 13 Mar 1988 and at the International conference on conformal field theories and related topics, Annecy-le-Vieux, 14 - 16 Mar 1988, CERN preprint CERN TH-5059/88
23. A. Neveu and P. West, “Cycling, twisting and sewing in the group theoretic approach to strings,” submitted to Comm. Math. Phys.
24. P. West, “String vertices and induced representations,” CERN preprint CERN TH 4954-88
25. J.H. Schwarz and C.C. Wu, Phys. Lett. 47B (1973) 453;
S. Mandelstam, Nucl. Phys. B69 (1974) 77;
D. Olive and J. Scherk, Nucl. Phys. B64 (1973) 334;
L. Brink, D. Olive, C. Rebbi and J. Scherk, Phys. Lett. 45B (1973) 379;
D. Bruce, E. Corrigan and D. Olive, Nucl. Phys. B95 (1975) 427.