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DIMENSIONAL REDUCTION OF TYPE-II SUPERSTRINGS^{*)}

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ABSTRACT

As a prototype of a type-II superstring compactified on a (2,2) superconformal field theory, we give the explicit expression of the $SU(3)$ -invariant reduction of 10D, type-IIA supergravity to $D = 4$ dimensions. It is shown, as expected from general arguments, that this Lagrangian has a standard $N = 2$ supergravity form as dictated by the $N = 2$ tensor calculus.

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Recently, type-II superstrings [1], compactified on a (2,2) superconformal field theory [2], have been used to exploit some properties of the moduli space [3] (flat-directions) of (stringy) Calabi-Yau compactifications [4].

The additional information [5, 6] on the moduli space is due to the fact that (2,2) superconformal systems, when used for compactification of type-II superstrings, give rise to an enhanced $N = 2$ space-time supersymmetry [7] with respect to heterotic superstrings [8].

As in the case of heterotic superstrings, the natural question that arises is whether the effective 4D supergravity Lagrangian has a standard form [9], as predicted by the tensor calculus. Witten has shown [10] that for the case of heterotic superstrings, with $N = 1$ space-time supersymmetry, the effective Lagrangian for the massless fields has indeed a standard supergravity form [11].

The above result has been extended [12] to all toroidal-type compactifications, such as orbifolds, for the case of heterotic strings, also including the case of $N = 2$ space-time supersymmetry.

The main difference here is that the two supersymmetries come one from the left-moving sector and one from the right-moving sector, in contrast to the heterotic case where, for $N = 2$, the two supersymmetries come from the left-moving (world-sheet supersymmetric) sector^{*)}.

The result of this is, for example, that the universal sector of the superconformal field theory [the identity operator of the (2,2) system] that contains the 4D dilaton and antisymmetric tensor fields belongs to an $N = 2$ hypermultiplet rather than to an $N = 2$ vector multiplet [5, 6]. If the supergravity tensor calculus applies [9], then from this fact it follows that hypermultiplet scalars cannot be coupled to vector fields, in contrast to vector multiplet scalars. On the other hand, one is tempted to think [5] that since the dilaton is coupled to vectors, superstrings give rise to new couplings that cannot be explained by the usual tensor calculus for $N = 2$ supergravity.

In Ref. [6] it was shown that this is not the case. This was proven in two ways: by use of the hidden symmetries of the $N = 8$ supergravity Lagrangian [13], and by a direct string calculation. In both approaches it follows that the 4D dilaton (sometimes called the S field) does not couple to vectors. The point is that the 4D dilaton is not the same as the 10D dilaton (which indeed couples to 10D vectors), but is a mixture of the dilaton and the determinant of the internal metric.

It should be emphasized that the use of supergravity techniques [6] is adequate for this problem. Indeed, the universal hypermultiplet (containing the S field) is independent of the particular conformal field theory, so one can use any theory, e.g. a trivial toroidal compactification, to study its properties. Then one makes use of the fact that toroidal compactifications of 10D type-II superstrings reduce, in $D = 4$, to $N = 8$ superstrings, and the low-energy Lagrangian is uniquely fixed to be the Cremmer-Julia theory [13]. For example, all (scalar) untwisted configurations of $T_6/\mathbb{Z}_N$ orbifolds [14, 15] can be studied by making appropriate $\mathbb{Z}_N$ projections of the original $E_{7(7)}/SU(8)$ coset space [13]. In particular, all orbifolds with $\mathbb{Z}_N \subset SU(3)$ will have in common the $SU(3)$ singlet states. Then, as an oversimplified example, one may use a toy model made up of the universal sector and the $SU(3)$ singlet states. This was the sector considered in Ref. [10] for heterotic superstrings. We will consider the same sector for type-IIA superstrings.

We will show, in terms of the original 10D fields, the appropriate combinations that give rise to a standard supergravity form in four dimensions. The starting point is to consider $N = 2$, 10D supergravity, rather than $N = 8$ supergravity in $D = 4$. The reason for this is that in the Cremmer-Julia theory [13], all vectors have the same parity [as required by $SO(8)$ invariance], whilst the $N = 2$ supergravity Lagrangian, as obtained by a $SU(3)$ reduction from $D = 10$, has two vectors

*) The moduli space of $N = 2$ heterotic strings is related to $(4,p)$ internal superconformal systems.

of opposite parities, in agreement with an $N = 2$ supergravity coupling of a matter vector multiplet specified by a holomorphic function which is a cubic monomial^{*)} [6].

Type-IIA 10D supergravity can be obtained by dimensional reduction [16] from $D = 11$ to $D = 10$ of the maximal supergravity of Cremmer, Julia and Scherk [17].

The 11D supergravity (bosonic) fields^{**)} are the vierbein $\hat{e}_\mu^{\hat{a}}$ and the three-form $\hat{A}_{\hat{\mu}\hat{\nu}\hat{\rho}}$. Dimensional reduction to $D = 10$ is obtained by fixing a triangular gauge for $\hat{e}_\mu^{\hat{a}}$

$$\hat{e}_\mu^{\hat{a}} = \begin{pmatrix} e_\mu^a & e_\mu'' \\ 0 & e_\mu'' \end{pmatrix} \hat{a}, \hat{\mu} = 1 \dots 11, \quad a, \mu = 1 \dots 10 \quad (1)$$

with

$$e'' = \phi, \quad e_\mu'' = \phi Z_\mu \quad (2)$$

Furthermore, from the three-form we define the two 10D gauge fields^{***)}:

$$B_{\mu\nu} = \hat{A}_{\mu\nu 11}, \quad H_{\mu\nu\rho} = 3 \partial_{[\mu} B_{\nu\rho]} \quad (3)$$

$$A_{\mu\nu\rho} = \hat{A}_{\mu\nu\rho} - 3 Z_{[\mu} B_{\nu\rho]} \quad (4)$$

with $F_{a_1 a_2 a_3 a_4} = \hat{F}_{a_1 a_2 a_3 a_4} - 6 Z_{[a_1 a_2} B_{a_3 a_4]}$. It is trivial to show that $B_{\mu\nu}$, Z_μ , and $A_{\mu\nu\rho}$ transform as 10D gauge fields. In the language of 10D, $N = 1$ supergravity, ϕ and $B_{\mu\nu}$ belong to the gravity multiplet, whilst Z_μ and $A_{\mu\nu\rho}$ belong to the multiplet of the second gravitino.

The $D = 11$ (bosonic part of the) Lagrangian is [17]

$$\mathcal{L}_{11} = -\frac{1}{2} \hat{e} \hat{R} - \frac{1}{48} \hat{e} (\hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_4})^2 + \frac{\sqrt{2}}{(12)^4} \varepsilon^{\hat{\mu}_1 \dots \hat{\mu}_8} \hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_4} \hat{F}_{\hat{\mu}_5 \dots \hat{\mu}_8} \hat{A}_{\hat{\mu}_9 \hat{\mu}_{10} \hat{\mu}_{11}} \quad (5)$$

Using the definition given for Eqs. (1) to (4), we get for the terms in Eq. (5) the following Lagrangian in $D = 10$:

$$-\frac{1}{2} \hat{e} \hat{R} \rightarrow -\frac{1}{2} \phi e R - \frac{1}{8} e \phi^3 Z_\mu^2 \quad (6)$$

$$-\frac{1}{48} \hat{e} (\hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_4})^2 \rightarrow -\frac{1}{48} \phi e \left\{ (F_{\mu_1 \dots \mu_4} + 6 Z_{[\mu_1 \mu_2} B_{\mu_3 \mu_4]})^2 + 4 \phi^{-2} (H_{\mu_1 \mu_2 \mu_3})^2 \right\} \quad (7)$$

*) Here we will work out only the bosonic sector of the Lagrangian, the rest being determined by supersymmetry.

**) The two formulations are related by a duality transformation [13], which is not needed here.

***) Under the B gauge transformation A is not inert, i.e. $\delta B = d\eta$, $\delta A = Z \wedge d\eta$.

$$\frac{\sqrt{2}}{(12)^4} \varepsilon^{\hat{\mu}_1 \dots \hat{\mu}_{10}} \hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_4} \hat{F}_{\hat{\mu}_5 \dots \hat{\mu}_8} \hat{A}_{\hat{\mu}_9 \dots \hat{\mu}_{10}} \rightarrow \frac{\sqrt{2}}{(48)^2} \varepsilon^{\mu_1 \dots \mu_{10}} \{ (F_{\mu_1 \dots \mu_4} + 6 Z_{\mu_1 \mu_2} B_{\mu_3 \mu_4}) F_{\mu_5 \mu_6} B_{\mu_7 \mu_8} + 12 Z_{\mu_1 \mu_2} Z_{\mu_3 \mu_4} B_{\mu_5 \mu_6} B_{\mu_7 \mu_8} B_{\mu_9 \mu_{10}} \} \quad (8)$$

In order to have a canonically normalized Einstein action, we perform a Weyl rescaling by using the general formula, valid in any dimension D:

$$-\sqrt{g} \Omega^{D-2} R \rightarrow -\sqrt{g} R - \sqrt{g} (D-1)(D-2) (\partial_\mu \log \Omega)^2$$

$$g_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu} \quad (e^\alpha_\mu \rightarrow \Omega^{-1} e^\alpha_\mu) \quad (9)$$

For D = 10, then, $\Omega = \phi^{1/8}$, and we get from formulae (6) and (7),

$$-\frac{1}{2} \hat{e} \hat{R} \rightarrow e \left(-\frac{1}{2} R - \frac{9}{16} (\partial_\mu \log \phi)^2 - \frac{1}{8} \phi^{3/4} Z_{\mu\nu}^2 \right) \quad (10)$$

$$-\frac{1}{48} \hat{e} (\hat{F}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}})^2 \rightarrow -\frac{1}{48} e \phi^{3/4} (F_{\mu\nu\rho\sigma} + 6 Z_{[\mu\nu} B_{\rho\sigma]})^2 - \frac{1}{12} e \phi^{-3/2} (H_{\mu\nu\rho})^2 \quad (11)$$

the topological term being unaffected by the Weyl rescaling. Formulae (8), (10), and (11) give the bosonic sector of type-IIA supergravity in D dimensions.

To perform the dimensional reduction to D = 4, following ref. [10] we keep only the SU(3) singlets in the internal indices:

$$g_{\mu\nu} \rightarrow g_{\mu\nu} \quad , \quad g_{i\bar{j}} = e^{\sigma} \delta_{i\bar{j}}$$

$$B_{\mu\nu} \rightarrow B_{\mu\nu} \quad , \quad B_{i\bar{j}} = i a \delta_{i\bar{j}} \quad (12)$$

$$Z_\mu \rightarrow Z_\mu \quad , \quad \phi \rightarrow \phi$$

$$A_{\mu i\bar{j}} = i A_\mu \delta_{i\bar{j}} \quad , \quad A_{i\bar{j}k} = G \varepsilon_{i\bar{j}k} \quad (13)$$

The new terms, with respect to the N = 1, D = 10 supergravity, are those coming from the Z_μ vector field and the three-form $A_{\mu\nu\rho}$. In D = 4 they reduce to

$$-\frac{1}{8} e \phi^{3/4} Z_{\mu\nu}^2 \rightarrow -\frac{1}{8} e \phi^{3/4} e^{3\sigma} Z_{\mu\nu}^2 \quad (14)$$

which is Weyl-invariant,

$$-\frac{1}{48} e \phi^{3/4} (F_{\mu\nu\rho\sigma} + 6 Z_{[\mu} B_{\rho\sigma]})^2 \rightarrow$$

$$-\frac{3}{4} e \phi^{3/4} e^{\sigma} (\bar{F}_{\mu\nu} + a Z_{\mu\nu})^2 - e \phi^{3/4} e^{-3\sigma} |\partial_{\mu} C|^2 - \frac{1}{48} e \phi^{3/4} e^{9\sigma} (F_{\mu\nu\rho\sigma} + 6 Z_{[\mu} B_{\rho\sigma]})^2 \quad (15)$$

after a Weyl rescaling $e \rightarrow \Omega^{-1} e$, with $\Omega = e^{3\sigma/2}$, which brings the Einstein term to the canonical normalization $-1/2 e R$.

Finally, the topological term (which is Weyl-invariant) reduces to

$$-\frac{i}{\sqrt{2}6} \epsilon^{\mu\nu\rho\sigma} \bar{C} \partial_{\mu} C H_{\nu\rho\sigma} + \frac{1}{2\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} (a^3 Z_{\mu\nu} Z_{\rho\sigma} + 3a^2 Z_{\mu\nu} F_{\rho\sigma} + 3a F_{\mu\nu} F_{\rho\sigma}) \quad (16)$$

Formulae (14), (15), and (16) must be added to the part of the $N = 1, D = 10$ bosonic reduction (worked out in Ref. [10]), which is

$$-\frac{1}{2} e R - 3e (\partial_{\mu} \sigma)^2 - \frac{9}{16} e (\partial_{\mu} \log \phi)^2 - \frac{3}{2} e \phi^{-3/4} e^{-2\sigma} (\partial_{\mu} a)^2$$

$$- \frac{1}{12} e \phi^{-3/4} e^{6\sigma} (H_{\mu\nu\rho})^2 \quad (17)$$

with $H_{\mu\nu\rho} = 3 f_{\mu\nu\rho}$ (ref. 10), $B_{\mu\nu} = a_{\mu\nu}$ (ref. 10)

If we now perform a duality transformation on the $B_{\mu\nu}$ field, this amounts to adding a Lagrange multiplier

$$\frac{1}{3} \phi^{-3/4} e^{6\sigma} H_{\mu\nu\rho} = e \epsilon_{\mu\nu\rho\lambda} (\partial^{\lambda} D + \frac{i}{3\sqrt{2}} \bar{C} \partial^{\lambda} C) \quad (18)$$

and to integrating over $H^{\mu\nu\rho}$ treated as a Lagrange multiplier. Owing to the topological term [Eq. (16)], this gives an interference between the D and C fields.

If we now define the complex field

$$S = \phi^{-3/4} e^{3\sigma} + i 3\sqrt{2} D + \bar{C} C, \quad A_{\mu}^0 = \sqrt{2} Z_{\mu}$$

$$iT = \phi^{3/4} e^{\sigma} - i\sqrt{2} a, \quad A_{\mu}' = 2A_{\mu}, \quad \frac{X'}{X^0} = T \quad (19)$$

we get, for the scalar kinetic terms, a Kähler manifold with the Kähler potential given by

$$K = -3 \log i(T - \bar{T}) - \log (S + \bar{S} - 2 \bar{C} C) \quad (20)$$

In agreement with Ref. [6], this is a product of an $N = 2$ Kähler manifold [18] $SU(1,1)/U(1)$ and a quaternionic manifold^{*)} [19, 20] $SU(2,1)/SU(2) \times U(1)$, which corresponds to the ‘universal sector’ of any (2,2) conformal system in type-II superstrings. The vector kinetic terms depend only on the T field, and they are in a standard supergravity form if, in the language of the tensor calculus [9–18], one defines the holomorphic F function to be

$$F(T) = i T^3 = i \frac{X^3}{X^0} = (X^0)^{-2} F(X^1, X^0) \quad (21)$$

In terms of Eq. (21), the normalization of the vector kinetic term and of the topological $F\bar{F}$ terms is given by the real and the imaginary part of the kinetic matrix [9] ($I, J = 0, 1$):

$$\mathcal{N}_{IJ} = \frac{1}{4} \bar{F}_{IJ} - \frac{(NX)_I (NX)_J}{(X^L N_{LK} X^K)} \quad (22)$$

with $F_{IJ} = \partial_I \partial_J F$, $N_{IJ} = \frac{1}{4} (F_{IJ} + \bar{F}_{IJ})$

The entries 0, 1 refer to the $\mathbb{Z}_\mu$ and A_μ fields, respectively. The three entries of the $\mathcal{N}^2$ matrix are given by

$$\begin{aligned} \mathcal{N}_{00} &= \frac{i}{32} (T - \bar{T})^3 - \frac{3i}{32} (T - \bar{T})(T + \bar{T})^2 - \frac{i}{16} (T + \bar{T})^3 \\ \mathcal{N}_{01} &= \frac{3i}{16} (T - \bar{T})(T + \bar{T}) + \frac{3i}{16} (T + \bar{T})^2 \\ \mathcal{N}_{11} &= -\frac{3i}{8} (T - \bar{T}) - \frac{3i}{4} (T + \bar{T}) \end{aligned} \quad (23)$$

By using the formula

$$\frac{1}{4} F_{\mu\nu}^+ \mathcal{N}_{IJ} F_{\mu\nu}^{+J} + h.c. \quad (F_{\mu\nu}^+ = \text{self-dual combination})$$

one easily reproduces the terms given in formulae (15) and (16). We note, in particular, the Peccei–Quinn symmetry [6],

$$a \rightarrow a + c, \quad A_\mu \rightarrow A_\mu - c Z_\mu \quad (24)$$

under which the Lagrangian is invariant, up to a total derivative induced by the topological term in Eq. (16). Formulae (24) tell us that the Peccei–Quinn symmetry is a rotation between the graviphoton and the matter vector. In the language of the $N = 2$ superconformal tensor calculus, it is a rotation between matter vector multiplet and the compensating vector multiplet.

^{*)} The hypermultiplet manifold can be written in a standard form by introducing a $U(1)$ auxiliary vector multiplet [19].

In conclusion, we see that no coupling is present between the four-dimensional (dilaton) S and the vector fields. Therefore no unusual couplings are generated in $N = 2$ supergravity theories, even when the two supersymmetries come one from the left-movers and one from the right-movers.

From the general analysis given in Ref. [6] it is rather obvious that these results are model-independent and are properties of the effective Lagrangian of type-II superstrings compactified on an arbitrary (2,2) superconformal system.

We explicitly worked out the case of type-IIA superstrings. In type-IIB superstrings [1] the same truncation gives rise to a theory with no matter vector multiplet and a quaternionic manifold [6] $G_2/SO(4)$ with quaternionic dimension 2. The graviphoton is in this case the $SU(3)$ -invariant vector $A_{\mu ij k} = A_{\mu} \epsilon_{ijk}$, where $A_{\mu\nu\rho\sigma}$ is the four-form of chiral 10D supergravity, and the two hypermultiplets correspond to the fields $g_{i\bar{i}}$, $b_{i\bar{i}}^c, A_{\mu\nu i\bar{i}}, \phi^c$, where $b_{\mu\nu}^c$ and ϕ^c are the complex two-form and the complex dilaton of chiral 10D supergravity.

Using the $SU(1,1)$ covariance [1] of the equations of motion and the self-duality of the field strength of $A_{\mu\nu\rho\sigma}$, it can immediately be seen that, also in this case, no scalar couples to the graviphoton, according to $N = 2$ tensor calculus rules.

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