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Ultraviolet finite models with generalized scale invariance

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Abstract

We construct the models with generalized scale invariance /classical equations of motion are invariant under the scale transformations, the action is invariant only under simultaneous change of the scale and coupling constant/. The generalized scale invariance leads to the well defined perturbation theory. For generalized Gross-Neveu model we find ultraviolet fixed point for $n=2$ and $n=3$ space-time.

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I. At present the superstring theories are the most popular candidates for the unification of all known interactions/I/. The main reason is the hope /partly proved/ that such theories are ultraviolet finite. Strings are extended objects and are equivalent /at least free strings/ to the infinite number of local fields with an arbitrary large spin and mass. Probably the property of the ultraviolet finiteness is closely related to the existence of the infinite number of fundamental fields. Therefore it is of interest to consider the Lagrangians with the infinite number of fundamental local fields. In ref./2/ we suggested to use the infinite component fields with higher order derivatives in the interaction to obtain the ultraviolet finite models. Further development of these ideas contains in ref./3/. In ref./4/ we proposed another approach for construction of the ultraviolet finite models based on the use of the analytical continuation procedure. However the proposed approach based on the use of "ad hoc" analytical continuation looks rather artificial.

In this paper we construct the models with generalized scale invariance /classical equations of motion are invariant under the scale transformations, the action is invariant only under simultaneous change of the scale and coupling constant/. The generalized scale invariance severely restricts the form of the Lagrangians. Possible local counterterms are prohibited that leads to the ultraviolet finiteness /the perturbation theory is well defined/. For generalized Gross-Neveu model we find ultraviolet fixed point for $n=2$ and $n=3$ space-time.

2. To illustrate the main ideas let us consider the φ^4 model in n -dimensional space-time. The action has the form

$$S = \frac{1}{g} \int d^n x \left[\frac{1}{2} \partial_\mu \varphi \partial_\mu \varphi - \varphi^4 \right] \quad (1)$$

The equation of motion

$$\frac{\delta S}{\delta \varphi} = \square \varphi + 4 \varphi^3 = 0 \quad (2)$$

is invariant under scale transformation

$$\varphi(x) \rightarrow K \varphi(Kx), \quad (3)$$

however the action is not invariant (for $n \neq 4$) under the scale transformation (3), namely

$$S \rightarrow K^{4-n} S$$

Therefore the symmetry (3) holds only for the classical solutions of the eq.(2) and it is broken at quantum level /the generating functional

$$W(J) = \ln \left[\int d\varphi \exp(iS + \int J\varphi d^n x) \right]$$

is not invariant under the transformation (3)/. The action (I) is invariant under the generalized scale transformation /the simultaneous change of the scale and coupling constant/

$$\begin{aligned} \varphi(x) &\rightarrow K \varphi(Kx), \\ g &\rightarrow K^{4-n} g \end{aligned} \quad (4)$$

The model (I) is nonrenormalizable for $n \neq 4$. The generalized scale invariance (4) prohibits all local L-loop counterterms

$$\Delta S_L = g^{L-1} \int d^n x \left[(\Box)^{p_1} \varphi \dots (\Box)^{p_m} \varphi \right] \quad (5)$$

for irrational n that leads to the well defined perturbation theory. This fact is well known in dimensional regularization /5/ where the analytical continuation procedure from the ultraviolet finite $\text{Re } n > 2$ region to the ultraviolet divergent $n > 3$ region is used to obtain the ultraviolet finite Feynman integrals. On the language of the symmetry (4) it means that we have to subtract all ultraviolet infinities at zero momenta, therefore the symmetry (4) gives us the unambiguous prescription for removing the ultraviolet divergences.

3. Let us consider the model which describes the selfinteracting scalar field in five-dimensional space-time. The action has the form

$$S_5 = \frac{1}{g} \int d^5 x \left[\frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} ((\partial_5)^\delta \varphi)^2 M_0^{2-2\delta} - \left((-\frac{\partial_5}{M_1})^{\frac{\alpha}{2}} (\frac{\partial_5}{M_1})^{\frac{\alpha}{2}} \varphi \right)^4 \right], \quad (6)$$

$$\begin{aligned} (\partial_5)^\beta F(x_5) &\equiv \int e^{iKx_5} (iK)^\beta \tilde{F}(K) dK, \\ F(x_5) &= \int e^{iKx_5} \tilde{F}(K) dK, \\ (-\partial_5)^{\alpha/2} (\partial_5)^{\alpha/2} F(x_5) &= \int e^{iKx_5} (K^2)^{\frac{\alpha}{2}} \tilde{F}(K) dK, \\ \alpha > 0, \delta > 0; \mu &= 1, 2, 3, 4. \end{aligned}$$

The equation of motion is invariant under the transformation

$$\varphi(x_\mu, x_5) \rightarrow K^{1-\frac{2\alpha}{\delta}} \varphi(Kx_\mu, K^{1/\delta} x_5) \quad (7)$$

The action (6) is invariant under the generalized scale transformations

$$\begin{aligned} \varphi(x_\mu, x_5) &\rightarrow K^{1-\frac{2\alpha}{\delta}} \varphi(Kx_\mu, K^{1/\delta} x_5), \\ g &\rightarrow K^{-4\alpha/\delta - 1/\delta} \end{aligned} \quad (8)$$

The action (6) is invariant only under four-dimensional Poincare group and it is local only in four-dimensional space-time. For the action (6) Feynman rules have the standard form, the single peculiarity consists in the use of the propagator

$$D_0 = \frac{1}{p_\mu^2 - (\frac{p_5^2}{M_0^2})^\delta M_0^2 + i\varepsilon} \quad (9a)$$

and the vertex

$$V_4 = \prod_{K=1}^4 \left(\frac{p_{5,K}^2}{M_1^2} \right)^{\frac{\alpha}{2}} \delta \left(\sum_{K=1}^4 p_K \right) \quad (9b)$$

The generalized scale symmetry (8) prohibits all local L-loop counterterms

$$\Delta S_L = g^{L-1} \int d^5 x \left[\partial_M^{k_1} \varphi \dots \partial_N^{k_n} \varphi \right] \quad (10)$$

for irrational $\frac{\alpha}{\beta}$ and α that leads to the well defined perturbation theory. Note that according to standard counting rules/6/ the model (6) is nonrenormalizable. The generalized scale symmetry (8) selects the renormalization prescription unambiguously /the subtraction at zero momenta in full analogy with dimensional regularization case/. Unfortunately the model (6) is not asymptotically free, the perturbation theory "blows up" and is not applicable in the ultraviolet region.

For the torus compactification

$$M_5 \rightarrow M_4 \times S_1 \quad (11)$$

the boundary condition

$$\varphi(x_\mu, x_5 + L) = \varphi(x_\mu, x_5) \quad (12)$$

holds. The boundary condition (12) leads to the expansion

$$\varphi(x_\mu, x_5) = \sum_{n=-\infty}^{n=\infty} \exp\left(\frac{2\pi i n}{L} x_5\right) \varphi_n(x_\mu) \quad (13)$$

After the substitution of the expansion (13) into five dimensional action (6) and integration over dx_5 we obtain the four dimensional action which contains the infinite number of the local scalar fields

$$g S_4 = \int d^4 x \left[\frac{1}{2} \partial_\mu \varphi_n \partial^\mu \varphi_{-n} + \frac{1}{2} (2\pi n)^2 M_0^{2-2\delta} \varphi_n \varphi_{-n} - \right.$$

$$\left. \left(\frac{n_1}{M_1} \frac{n_2}{M_1} \frac{n_3}{M_1} \frac{n_4}{M_1} \right)^\alpha (2\pi)^{4\alpha} \delta^{KP}(n_1 + n_2 + n_3 + n_4) \varphi_{n_1} \varphi_{n_2} \varphi_{n_3} \varphi_{n_4} \right],$$

$$\delta^{KP}(x) = \begin{cases} 1, & x=0 \\ 0, & x \neq 0 \end{cases},$$

$$L \equiv 1$$

One can obtain that the effective action S_4 also leads to the ultraviolet finite theory since in compactification we make the replacement

$$\int dp_5 \rightarrow \sum_{p_5=-\infty}^{p_5=\infty},$$

therefore the compactification being the infrared operation does not change the ultraviolet properties of the theory. The one-loop proof of the ultraviolet equivalence of the five dimensional φ^4 -model and its compactified version contains in ref./7/.

4. To test our ideas it is useful to consider strictly soluble model /the generalized Thirring model/ which describes the selfinteracting fermion field in two-dimensional space-time. The action of the model is

$$S_2 = \int d^2x \left[i \bar{\psi} \gamma^\mu \partial_\mu \psi - \alpha \left(\left(1 - \frac{\partial_1}{M_1} \right)^{\delta/2} \left(\frac{\partial_1}{M_1} \right)^{\delta/2} j_\mu \right) \left(\left(1 - \frac{\partial_1}{M_1} \right)^{\delta/2} \left(\frac{\partial_1}{M_1} \right)^{\delta/2} j^\mu \right) \right], \quad (14)$$

$$j_\mu = \bar{\psi} \gamma_\mu \psi$$

The classical equation of motion is invariant under the scale transformation

$$\psi(x) \rightarrow K^{1/2-\delta} \psi(Kx) \quad (15)$$

that leads at quantum level to the well defined perturbation theory. The simplest way to solve the model (14) consists in the use of the bosonization rules/8/

$$i \bar{\psi} \hat{\partial} \psi \sim \frac{1}{2} \partial_\mu \phi \partial^\mu \phi, \quad \bar{\psi} \gamma_\mu \psi \sim \frac{1}{\sqrt{\pi}} \varepsilon_{\mu\nu} \partial^\nu \phi, \quad (16)$$

here ϕ is the scalar field. In the bosonization representation the action is represented in the form

$$S_2^B = \int d^2x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\alpha}{\pi} \left(\frac{\partial_1}{M_1} \right)^\delta \partial_\mu \phi \left(\frac{\partial_1}{M_1} \right)^\delta \partial^\mu \phi \right], \quad (17)$$

The action (17) describes free scalar particle with the propagator

$$D_0(p_0, p_1) = -\frac{1}{p_0^2 - p_1^2 + i\varepsilon} \left(1 + \frac{2\alpha}{\pi} \left(\frac{p_1^2}{M_1^2} \right)^\delta \right)^{-1}, \quad (18)$$

For $\alpha > 0$ the imaginary part of the propagator D_0 is nonnegative. However for $\alpha < 0$ the imaginary part of the propagator D_0 is negative for $p_1^2 > (-\frac{\pi}{2\alpha})^{1/\delta}$, that means the appearance of ghost states in the spectrum. $\overline{H}_1^2$

5. The main shortcoming of the scalar model (6) is the asymptotic "antifreedom". Thus it is of interest to construct the model with controllable within perturbation theory ultraviolet behaviour. To this aim consider the model with the action

$$S' = \frac{1}{g'} \int \mathcal{L} d^{n+1}x, \quad \mathcal{L}' = \sum_{a=1}^N \bar{\psi}^a i \hat{\partial}' \psi^a + \frac{1}{2N} \left(\sum_{a=1}^N \left(\frac{\partial_{n+1}}{M} \right)^{\frac{\alpha}{2}} \bar{\psi}^a \left(\frac{\partial_{n+1}}{M} \right)^{\frac{\alpha}{2}} \psi^a \right)^2, \quad (19)$$

$$\hat{\partial}' = \sum_{\mu=1}^n \not{x}^{\mu} \partial_{\mu} + \not{x}^{n+1} \left(-\frac{\partial_{n+1}}{M_0} \right)^{\delta/2} \left(\frac{\partial_{n+1}}{M_0} \right)^{\delta/2},$$

$$g' = \mu^{(2-n-\frac{2\alpha}{\delta}-1/\delta)} g$$

The model (19) describes the selfinteracting N-component spinor field in (n+1)-dimensional space-time. For $\delta=1$ and $\alpha=0$ our model coincides with the Gross-Neveu model [8].

The action (19) is invariant (for $n+1=2k$; $k=1,2,\dots$) under the transformations

$$\psi^a \rightarrow \gamma_5 \psi^a, \quad \bar{\psi}^a \rightarrow -\bar{\psi}^a \gamma_5 \quad (20)$$

The action (19) is invariant under the generalized scale transformations

$$\psi(x) \rightarrow K^{1/2-\frac{\alpha}{\delta}} \psi(Kx_{\mu}, K^{1/\delta} x_{n+1}),$$

$$g \rightarrow K^{2-n-\frac{2\alpha}{\delta}-1/\delta} g, \quad (21)$$

that leads to the well defined perturbation theory for irrational $\frac{\alpha}{\delta}$ and δ . The Lagrangian $\mathcal{L}'$ is equivalent to the Lagrangian

$$\mathcal{L}'' = \sum_{a=1}^N \bar{\psi}_a i \hat{\partial}' \psi_a - \frac{N}{2} \phi^2 + \phi \sum_{a=1}^N \left(\frac{\partial_{n+1}}{M} \right)^{\frac{\alpha}{2}} \bar{\psi}^a \left(\frac{\partial_{n+1}}{M} \right)^{\frac{\alpha}{2}} \psi^a \quad (22)$$

In the leading order in $\frac{1}{N}$ the effective charge is determined by the propagator of the ϕ -field

$$\mathbb{D} = iN \int e^{ipx} \langle 0 | T(\phi(x)\phi(0)) | 0 \rangle d^{n+1}x =$$

$$\frac{g}{1 + \Sigma} \quad (23)$$

In the leading order on $\frac{1}{N}$ only one-loop diagram (fig.I) contributes. For $p_{n+1}=0$ the calculations are trivial and we obtain

$$\Sigma = g K \left[\left(-\frac{p^2}{\mu^2} \right)^{\frac{n}{2}-1+\frac{\alpha}{\delta}+\frac{1}{2\delta}} - 1 \right], \quad (24)$$

$$K = \frac{2^{\left[\frac{n+1}{2}\right]}}{4\pi\delta (4\pi)^{n/2}} \Gamma\left(\frac{\alpha}{\delta}+\frac{1}{2\delta}\right) \Gamma\left(2-\frac{\alpha}{\delta}-\frac{1}{2\delta}-\frac{n}{2}\right)$$

$$\frac{\Gamma^2\left(\frac{n}{2}-1+\frac{\alpha}{\delta}+\frac{1}{2\delta}\right)}{\Gamma\left(n-2+\frac{2\alpha}{\delta}+\frac{1}{\delta}\right)},$$

Here we put $M_0 = M = \mu$ and used the renormalization condition

$$\Sigma(-p_\mu^2 = \mu^2, p_{n+1}=0, \mu, g, \alpha, \delta) = 0 \quad (25)$$

For $g = \frac{1}{K}$ we have fixed point the ϕ -propagator $\mathbb{D}$ scales with an anomalous dimension

$$\mathbb{D} \sim \left(-\frac{p^2}{\mu^2} \right)^{-\varepsilon}, \quad \varepsilon = \frac{n}{2}-1+\frac{\alpha}{\delta}+\frac{1}{2\delta} \quad (26)$$

From the Kallen-Lehman representation

$$\mathbb{D} = \int_0^\infty \frac{\rho(t)}{t - p^2 - i\varepsilon} dt, \quad \rho(t) \geq 0, \quad (27)$$

we find that $\varepsilon < 1$. Therefore the dimension of space-time has to be less than

$$d = (n+1) < 5 \quad (28)$$

For $g > \frac{1}{K}$ the ϕ -propagator develops a space like pole (a tachyon) that means the unstability of vacuum. Really, the renormalized effective potential

$$V = \frac{N\phi^2}{2\bar{g}(\frac{\phi^2}{\mu^2}, g)} = \frac{N\phi^2}{2} \left[\frac{1}{g} + K \left(\frac{\phi^2}{\mu^2} \right)^\varepsilon - K \right],$$

has nontrivial vacuum solution $\langle \phi \rangle \neq 0$ for $g > \frac{1}{K}$ and spontaneous symmetry breaking occurs in full analogy with Gross-Neveu model case. As a result the fermions acquire a mass. For $g < \frac{1}{K}$ we have ordinary theory with massless fermions.

In compactification of our model over x_{n+1} -coordinate we obtain n-dimensional Poincare invariant model with ultraviolet fixed point which describes the selfinteracting infinite component fermionic theory. The fact that in three-dimensional space-time we obtain the theory with ultraviolet fixed point is rather untrivial since according to standard counting rules all selfinteracting fermionic theories are nonrenormalizable in three dimensional space-time.

6. Concluding, we have constructed several models with generalized scale invariance /classical equations of motion are invariant under scale transformations, the action is invariant only under simultaneous change of the scale and coupling constant/. Generalized scale invariance leads to the well defined perturbation theory in full analogy with dimensional regularization case. We have found the model with ultraviolet fixed point. After compactification the model describes the infinite component selfinteracting fermionic field in $n=2$ or in $n=3$ space-time. It would be extremely interesting to extend our approach to the gauge fields and especially to the gravity. There are a lot of unsolved problems /chiral fermions, gravity, compactification,.../. Nevertheless the suggested approach opens new ways in the "struggle" with ultraviolet divergences and nonrenormalizable field theories.

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Figure I

One-loop correction to the 6-propagator

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