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Erratum

Vector Boson pair production via gluon fusion

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ERRATUM

Due to a computational error, the cross section for the process $gg \rightarrow WW$ shown in fig. 4 was overestimated by a factor 4. The correct figure is shown overleaf. We thank S. S. D. Willenbrock for drawing our attention to this error.

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Fig. 4.

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ABSTRACT

We study the production of Z -boson pairs at hadron colliders, produced by gluon fusion via an internal quark loop of arbitrary mass, both with and without the presence of a Higgs boson. As a background to Higgs boson production we find that the gluon fusion process, $gg \rightarrow ZZ$, is important, (60 – 70%), but never dominant over the $\bar{q}q \rightarrow ZZ$ process at the SSC. The Higgs boson signal and the continuum background interfere destructively for an invariant mass of the Z -boson pair larger than the Higgs boson mass. We use the results to give an estimate of the cross section $gg \rightarrow WW$ in the limit where up- and down-type quarks are degenerate and massless.

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One of the main purposes of the proposed high energy hadron colliders like the SSC ($\sqrt{s} = 40$ TeV) or LHC ($\sqrt{s} = 16$ TeV) is the discovery of the Higgs boson responsible for electroweak symmetry breaking. If the Higgs boson is heavy enough it will mainly decay into W or Z boson pairs [1]. The existence of the Higgs particle would then be signalled by a peak in the invariant mass distribution of the weak bosons. It is clearly necessary to have a complete understanding of the continuum production of vector boson pairs in order to get a realistic estimate of the signal to background ratio.

The most direct source of continuum production of vector boson pairs is the tree level process $\bar{q}q \rightarrow WW, ZZ$ [2]. Two other sources of continuum electroweak boson pair production are vector boson fusion [3] and gluon fusion [4-7]. In vector boson fusion the incoming quarks radiate two weak bosons which subsequently scatter off each other. In gluon fusion the vector bosons are produced via a quark loop. The contributing diagrams are shown in fig. 1. Because of the loop this process is of higher order in α_s , but it is still important because the gluon-gluon luminosity is much higher than the quark-antiquark luminosity for small values of x at hadron colliders. The Higgs boson contributes to both vector boson fusion and gluon fusion and these processes are the main mechanisms for producing a Higgs particle.

Vector boson fusion has been studied extensively in the literature [3]. The gluon fusion process has been studied much less. A very rough estimate has been given in ref. 4. In ref. 5 numerical results have been given for the background $gg \rightarrow ZZ$ process without the taking the Higgs contribution into account. It was found that near threshold $gg \rightarrow ZZ$ becomes more important than $\bar{q}q \rightarrow ZZ$. Subsequently in ref. 6 the interference effects between the Higgs triangle graph and the box graphs was studied. A positive interference at large invariant masses was claimed in this paper. In refs. 5 and 6 the cross sections were calculated by using Veltman's FORMF program.

Recently we have completed an analytic analysis of the complete $gg \rightarrow ZZ$ process [7]. We used the methods of ref. 8 to reduce all helicity amplitudes to the set of basic integrals described in ref. 9. This reduction was done algebraically using SCHOONSCHIP. Making the reduction algebraically rather than numerically has the advantage of making certain cancellations explicit. As a consequence some of the numerical problems mentioned in refs. 5 and 6 are under better control.

Let us first examine the importance of the gluon fusion process relative to quark-antiquark annihilation when the Higgs boson is absent. In fig. 2 we show the invariant mass distribution of the Z -boson pair for pp collisions at $\sqrt{s} = 16$ TeV and $\sqrt{s} = 40$ TeV. We used the structure functions of ref. 10 with $\Lambda = 0.2$ GeV evolved to a scale $Q^2 = \hat{s}/4$ and a strong coupling constant $\alpha_s = 12\pi/23 \log(Q^2/\Lambda^2)$. To give an idea of an observable cross section both Z -bosons are required to have a rapidity $|y_Z| < 1.5$. For the light quarks we use the limiting formulae for $m_q \rightarrow 0$ [7]. There is no problem in this limit since there are no mass singularities in the helicity amplitudes.

To illustrate the effect of the top quark, we show curves for $m_t = 40$ GeV and $m_t = 100$ GeV. We see that the dependence on the top quark mass is relatively small. This can be understood by comparing the cross section for $m_t = 0$ and $m_t = \infty$. For massless quarks the helicity amplitudes have the following coupling structure,

$$\mathcal{M} \sim \sum_q (v_q^2 + a_q^2), \quad (1)$$

where v_q (a_q) is the vector (axial vector) coupling of the Z -boson to the quarks. As is the case for $q\bar{q} \rightarrow ZZ$, the Z -bosons are mainly produced with transverse polarisations. As the quark mass becomes large, the quark decouples. The overall effect of the varying the top quark mass is then,

$$\frac{\sigma(m_t = 0)}{\sigma(m_t = \infty)} = \frac{\left(\sum_{q=1}^6 (v_q^2 + a_q^2)\right)^2}{\left(\sum_{q=1}^5 (v_q^2 + a_q^2)\right)^2} \sim 1.37, \quad (2)$$

and which is insensitive to the precise value of $\sin^2 \theta_W$. Therefore at small invariant mass, where the cross section is largest decreasing the top quark mass increases the cross section.

At large invariant mass however, the longitudinal amplitudes grow like,

$$\frac{a_q^2 m_q^2}{M_Z^2} \log^2\left(\frac{\hat{s}}{m_q^2}\right). \quad (3)$$

This is because at high energies, the longitudinal polarisation vector of the Z -boson is,

$$\epsilon_0^\mu = \frac{p_Z^\mu}{M_Z} + O\left(\frac{M_Z}{p_Z^0}\right), \quad (4)$$

and, since the axial current is not conserved for massive quarks, the first term gives a $\hat{s}/M_Z^2$ enhancement which cancels the $1/\hat{s}$ behaviour of the form factor. Hence at large invariant mass, increasing the top quark mass increases the cross section.

We notice that at the LHC the ratio $gg \rightarrow ZZ/\bar{q}q \rightarrow ZZ$ is only about 35-45%, while at the SSC it is 60-70%. This is due to the fact that the gluon structure functions fall faster with increasing x than the quark structure functions which also explains the gradual drop of the gluon fusion cross section relative to $q\bar{q}$ with increasing invariant mass. Even though the gluon fusion process is quite important at the SSC we find that it never dominates the $\bar{q}q$ process. We therefore disagree with the results of ref. 5, where the gluon fusion process becomes the dominant one at low invariant masses.

Of course these results are affected by the choice of structure functions that are used, as well as by higher order effects. The quark and gluon structure functions are not well known at small x , however, the uncertainties in Q^2 and Λ cancel to some extent in the ratio of quark and gluon luminosity. Some higher order calculations exist for $q\bar{q} \rightarrow ZZ$ [11], which indicate that effects of $O(30\%)$ may be present. Since the gluon fusion process

is $O(\alpha_s^2)$ it is much more sensitive to the value of Q^2 , but, due to the complexity of the lowest order no higher order corrections to the $gg \rightarrow ZZ$ process have been attempted.

In fig. 3 we compare the invariant mass distributions for $gg \rightarrow ZZ$ for $m_t = 100$ GeV with and without a Higgs of mass $M_H = 500$ GeV. The contributions for $gg \rightarrow Z_L Z_L$ are shown separately. With this value of the top quark mass, the Higgs resonance is clearly visible above the $q\bar{q} \rightarrow ZZ$ background. In particular, we note that beyond the Higgs pole, the $gg \rightarrow ZZ$ cross section is reduced, while there is an increase for $M_{ZZ} < M_H$. The Higgs decays almost completely into longitudinal Z -bosons and at the resonance interference between the signal and gluon fusion background is unimportant. At energies much larger than the Higgs mass the interference becomes important and is destructive as is clear from the $\mathcal{M}_{++00}$ amplitude at large $\hat{s}$,

$$\begin{aligned}\mathcal{M}_{++00}(\text{box}) &= -\frac{16a_q^2 \hat{s} m_q^2 C(\hat{s})}{M_Z^2}, \\ \mathcal{M}_{++00}(\text{triangle}) &= \frac{\hat{s}^2 m_q^2 C(\hat{s})}{M_Z^2 (\hat{s} - M_H^2 + i\Gamma_H M_H)},\end{aligned}\tag{5}$$

where $a_q = \pm 1/4$ and Γ_H is the total width of the Higgs boson. $C(\hat{s})$ is the basic triangle integral,

$$C(\hat{s}) = \int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)},\tag{6}$$

with p_1 and p_2 the gluon momenta, $p_1^2 = p_2^2 = 0$, and $\hat{s} = (p_1 + p_2)^2$. At large $\hat{s}$, ($\hat{s} \gg m_q^2$), $C(\hat{s})$ is given by,

$$C(\hat{s}) = \frac{1}{2\hat{s}} \left(\log^2\left(\frac{\hat{s}}{m_q^2}\right) - \pi^2 - 2i\pi \log\left(\frac{\hat{s}}{m_q^2}\right) \right).\tag{7}$$

The nature of the interference can be understood as follows. At large $\hat{s}$, the crossed box in fig. 1a does not contribute as is obvious from eq. (5). If we now make an $\hat{s}$ -channel cut on the other three graphs we have the process $t\bar{t} \rightarrow Z_L Z_L$. For this process, unitarity requires a cancellation between the Higgs graph and the other two graphs at large $\hat{s}$. The integration over loop momenta does not change this cancellation. We therefore disagree with the results of ref. 6, which finds a constructive interference at large $\hat{s}$.

Finally let us turn to the pair production of W -bosons via gluon fusion, $gg \rightarrow W^+ W^-$. In addition to the graphs of fig. 1a, there is also the $\hat{s}$ -channel exchange of a photon or Z -boson shown in fig. 1b. C -invariance tells us that the photon exchange graph is zero and that only the axial vector coupling of the Z contributes in the Z exchange graph. The box diagrams are now complicated by the fact that both up- and down-type quarks circulate in the loop, however, in the limit $m_u = m_d$ etc., the box diagrams reduce to those for $gg \rightarrow ZZ$. Moreover, in this limit, the contribution from the up and down quarks to the Z exchange graph cancel, and the complete $gg \rightarrow WW$ helicity amplitudes are given

by those for $gg \rightarrow ZZ$ with the appropriate couplings.

In fig. 4 we show the invariant mass distribution for WW pair production via gluon fusion for two and three light generations. For light quarks, each quark doublet contributes equally to the amplitude, and so, the invariant mass distribution for three massless generations is a factor $9/4$ larger than that for two massless generations. We show also the contribution from quark-antiquark annihilation. We see that in the approximation of massless degenerate quarks, the gluon fusion process is the dominant source of W -boson pairs at the SSC, and is certainly not negligible at the LHC. Although $m_t \neq m_b$, we might expect that the correct distribution lies somewhere between the two and three light generation curves. We note that for $m_t \neq m_b$ there is also a contribution from the cross terms $a_q v_q$ which could be large. For $m_t = m_b$, these terms vanish due to C -invariance.

In conclusion we have studied the production of vector boson pairs via gluon fusion and $\bar{q}q$ annihilation at pp colliders for arbitrary top quark mass. We found that the continuum production of ZZ pairs via gluon fusion is a relatively small effect (35 – 45%) at the LHC but is important, though not dominant, (60 – 70%) at the SSC. At small invariant mass decreasing the top quark mass increases the $gg \rightarrow ZZ$ cross section. The situation is reversed at large invariant mass, where the growth of the longitudinal amplitudes leads to a bigger cross section for larger m_t . The relative size of the gluon fusion cross section falls slowly with increasing M_{ZZ} due to the diminishing gluon luminosity.

We have also studied the effects of the Higgs boson in the gluonic production of Z -boson pairs. At large $\hat{s}$, there is a cancellation between the triangle and box graphs which is not explicitly required by unitarity. The process $t\bar{t} \rightarrow Z_L Z_L$ is directly related to $gg \rightarrow Z_L Z_L$ and in this case a cancellation is indeed required at large $\hat{s}$ to preserve unitarity. Hence, we found a negative interference at large $\hat{s}$ which is at variance with the literature [6].

Finally we presented an estimate of the production of W -boson pairs from gluon fusion. In the limit that up- and down-type quarks are degenerate the amplitudes for $gg \rightarrow WW$ are given by those for $gg \rightarrow ZZ$ with trivial coupling replacements. For light degenerate quarks, the gluon fusion process dominates the quark-antiquark annihilation process at the SSC.

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Figure Captions

- Fig. 1: Diagrams contributing to the process $gg \rightarrow VV$, where V is either a Z - or a W -boson. The diagrams in fig. 1a contribute to ZZ and to WW production. Those in fig. 1b contribute only to WW production.
- Fig. 2: The invariant mass distribution of Z boson pairs via $\bar{q}q \rightarrow ZZ$ and $gg \rightarrow ZZ$ for pp collisions at $\sqrt{s} = 40$ TeV and $\sqrt{s} = 16$ TeV. Curves are given for $m_t=40$ GeV and $m_t=100$ GeV. A rapidity cut on the Z -bosons of $|y_Z| < 1.5$ has been applied.
- Fig. 3: The invariant mass distribution for $gg \rightarrow ZZ$ in pp collisions at $\sqrt{s} = 40$ TeV. We took $m_t=100$ GeV and show curves both with a Higgs boson of mass $M_H = 500$ GeV (straight lines) and without the Higgs boson (dashed lines). We show curves for longitudinally polarised Z -boson pair production in addition to the sum over all Z -boson polarisations. A rapidity cut on the Z -bosons of $|y_Z| < 1.5$ has been applied.
- Fig. 4: The invariant mass distribution for WW pair production for pp collisions at $\sqrt{s} = 40$ TeV and $\sqrt{s} = 16$ TeV. In addition to the contribution from $q\bar{q} \rightarrow WW$, we show the contribution from $gg \rightarrow WW$ for two and three generations of degenerate massless quarks. A rapidity cut on the W -bosons of $|y_W| < 1.5$ has been applied.

(a)

(b)

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.