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Chiral Fermions in the Instanton Vacuum at Finite Temperature

M.A. Nowak[†], J.J.M. Verbaarschot[†] and I. Zahed[†]

CERN[†]
Theory Division
CH1211 Geneva 23
Switzerland

Physics Department[†]
State University of New York at Stony Brook,
Stony Brook, New York, 11794.

ABSTRACT

We investigate the effects of temperature in the instanton description of the QCD vacuum with light quarks. Using variational arguments, a mean-field approximation and a semiquenched fermion determinant, we find a chiral phase transition for more than one massless flavor. Results for the chiral condensate and the constituent mass are presented.

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1. Introduction

It is generally believed that QCD undergoes a phase transition at sufficiently high temperatures or densities. The nature and the character of this transition are still debated. Its existence, however, has been partly supported by lattice Monte-Carlo simulations [1,2]. This phase transition is believed to have prevailed in the early stages of the formation of the universe and is hoped to be achieved using relativistic heavy-ion colliders. It is therefore important to understand the nature and properties of QCD at high temperatures or densities.

In the quenched approximation, lattice Monte-Carlo simulations [3,4] indicate that at finite temperature QCD undergoes a deconfining phase transition. Recent calculations [5,6] suggest that in the presence of light quarks, the phase transition is mostly a chiral transition with dynamically confined chiral multiplets. This is suggestive of the fact that confinement and chiral symmetry breaking need not follow from each other, implying perhaps the existence of two dynamical scales in QCD (chiral symmetry breaking and confinement). At zero temperature, the instanton approach to the QCD ground state provides some understanding of the spontaneous breakdown of chiral symmetry [7,8,9], putting aside the issue of confinement.

The primary motivation behind the instanton description of the QCD vacuum is to describe the non-perturbative gauge fluctuations at small distance scales [10]. The idea is to saturate the Euclidean QCD partition function by instantons and anti-instantons at short distance scales. The hope is that large scale gauge fluctuations responsible for confinement decouple.

A qualitative description of the instanton vacuum yields most of the features of strong interaction physics at low energy. The long-standing infrared problem has been recently addressed by Dyakonov and Petrov [11]. They have shown that quantum disorder can stabilize instantons and anti-instantons in a liquid state. In the latter, chiral symmetry is spontaneously broken following the delocalization

of the zero modes. Light quarks propagate coherently in the liquid state, hopping from one instanton to the other [12,13].

Recently, Kanki [14] and also Dyakonov and Merlin [15] have considered the effects of temperature on the instanton vacuum. Using variational arguments, they have shown that with increasing temperatures instantons and anti-instantons are boiled off the vacuum, providing a mechanism for chiral symmetry restoration. In this paper, we would like to extend their analysis to include light quarks. This question has also been addressed recently by Ilgenfritz and Shuryak [16] using different arguments. In section 2 we derive the temperature dependence of the average size and density of the instantons in the presence of fermions, and present numerical results for these quantities. The temperature dependence of the chiral condensate and the constituent quark mass is obtained in section 3. Concluding remarks are made in section 4.

2. Properties of the instanton vacuum at finite temperature.

The grand partition function for a gas of N_+ instantons and N_- antinstantons can be approximated by [17,15,18,19]

$$Z = \sum_{N_+, N_-} \frac{1}{N_+! N_-!} \int \prod_I^N d\Omega_I d_0(\rho_I, T) \exp(-\beta(\bar{\rho})u_{int}) \det(i\gamma_\mu \hat{\nabla}_\mu + im). \quad (2-1)$$

For a given number $N = N_+ + N_-$ of pseudoparticles we have $4NN_c$ collective variables¹. In the numerical results presented below, we will use $N_c = 3$. At finite temperature T , the integration along the Euclidean time-direction is restricted to a strip of length $1/T$, and the spatial integration to a box of volume V_3 . The thermodynamical limit follows from $V_3 \rightarrow \infty$ for a constant density of pseudoparticles.

¹ $4N$ for the location Z_I of the pseudoparticles, N for their sizes ρ_I and $N(3N_c - 5)$ for their global color orientations.

In deriving (2-1), it was assumed that the classical gauge field is a linear superposition of single instanton and anti-instanton field configurations. This assumption is justified if the system is dilute, a point to be checked *a posteriori*. The two-body interaction term u_{int} depends on this assumption. The coefficient $\beta(\bar{\rho})$ is equal to $8\pi^2/g^2(\bar{\rho})$ with $g(\bar{\rho})$ the running coupling constant. To one-loop order $\beta(\bar{\rho})$ is given by $-b \log(\bar{\rho}\Lambda)$ with $b = \frac{11}{3}N_c - \frac{2}{3}N_f$ and $\bar{\rho}$ the average size of the instantons to be specified later. To avoid confusion with the inverse temperature $\beta \equiv 1/T$, we will always quote $\beta(\bar{\rho})$ with its argument. $\hat{\nabla}$ is the covariant derivative. The mass matrix in the fermion determinant is diagonal in flavor space with diagonal matrix elements equal to the current quark masses m_f . At finite temperature, the pseudoparticle density is given by [17,19]

$$d_0(\rho, T) = C \rho^{-5} \left(\frac{8\pi^2}{g^2} \right)^{2N_c} (M/\Lambda)^{\frac{2}{3}N_f} (\rho\Lambda)^{\frac{11}{3}N_c} \exp\left[-\frac{1}{3}\lambda^2(2N_c + N_f) + B(\lambda)\right], \quad (2-2)$$

where

$$B(\lambda) = \left(1 + \frac{1}{6}(N_c - N_f)\right)(-\log(1 + \lambda^2/3) + 0.15(1 + 0.16\lambda^{-\frac{3}{2}})^{-8}), \quad (2-3)$$

and C is equal to $4.60 \exp(-1.68N_c)/(\pi^2(N_c - 1)!(N_c - 2)!)$. For convenience we have introduced the dimensionless variable $\lambda \equiv \pi\rho T$. The cut-off Λ is equal to Λ_{PV} and M is the renormalization mass. The temperature dependence of $B(\lambda)$ was obtained numerically by Gross, Pisarski and Yaffe [19]. At the one-loop level the coupling constant in the pre-exponential factor $8\pi^2/g^2$ is not renormalized. The value of this constant is chosen such that the phenomenological value of the gluon condensate is saturated by instantons (the same approach was followed in ref. [16]).

The fermion determinant in (2-1) can be split into a low-momentum contribution ($\equiv \det_{low}$) and a high-momentum contribution ($\equiv \det_{high}$). Each factor is regularized with a mass M and normalized by the noninteracting fermion determi-

nant. For a full discussion we refer to the work of Dyakonov and Petrov [17]. The high-momentum factor can be obtained in a standard way. It is given by [7,17]

$$\det_{high}(i\gamma_\mu \hat{\nabla}_\mu + im) = \prod_{I,f} (1.339 M_1 \rho_I) (M \rho_I)^{-\frac{2}{3}}, \quad M_1 \rho_I \ll 1, \quad (2-4)$$

where M_1 is an arbitrary splitting mass in the fermion determinant. The product is over each flavor f and pseudoparticle I . Of course the full fermion determinant should be insensitive to the arbitrary mass M_1 . The low-momentum factor is dominated by the set of zero modes

$$\det_{low}(i\gamma_\mu \hat{\nabla}_\mu + im) = \prod_f \det(m_f^2 + D^\dagger D) / \det(M_1^2 + D^\dagger D), \quad (2-5)$$

where the matrix elements of D are given by the overlaps between the finite temperature zero modes ϕ_I ,

$$D_{IJ} = \int d^4x \phi_I(x) i\gamma_\mu \partial_\mu \phi_J(x). \quad (2-6)$$

The right-handed finite temperature zero modes are given by [19]

$$\phi^R = \frac{1}{2\pi\rho} \Pi^{\frac{1}{2}} \gamma_\mu \partial_\mu \left(\frac{\phi}{\Pi} \right) \epsilon^R, \quad (2-7)$$

where

$$\Pi = 1 + \frac{\pi\rho^2}{\beta r} \frac{\sinh 2\pi r/\beta}{\cosh 2\pi r/\beta - \cos 2\pi t/\beta}, \quad \phi = (\Pi - 1) \frac{\cos \pi t/\beta}{\cosh \pi r/\beta}, \quad (2-8)$$

and the Euclidean coordinates are denoted by $(\mathbf{r}, t)$, $r \equiv |\mathbf{r}|$. The spin and color structure of the zero modes is contained in ϵ^R . The left-handed zero modes correspond to $\epsilon^R \rightarrow \epsilon^L$.

To evaluate the low-momentum part of the fermion determinant eq. (2-5) we use a cumulant expansion [20]. The positions and the orientations of the pseudoparticles are assumed to be distributed according to their invariant measure. Because of the formation of instanton-anti-instanton molecules [12,13], this assumption does not hold for a very dilute system. In the chirally broken phase where the pseudoparticles are in a liquid phase, we expect this assumption to provide a reasonable description of the real (unquenched) system. Note that we *do* take into account the feed-back of the fermion determinant on the distribution of the sizes of the pseudoparticles. However, the fermion determinant is evaluated by a mean field approximation. We will refer to this approach as a semiquenched approximation. The distribution of the sizes will be discussed later.

In the thermodynamic limit and for large N_c , the eigenvalues of the overlap matrix D are circularly distributed [17]

$$\bar{\nu}(\lambda) = \frac{N}{\pi\kappa} \left(1 - \frac{\lambda^2}{4\kappa^2}\right)^{\frac{1}{2}}, \quad (2-9)$$

where κ^2 is the variance of the eigenvalues. From standard ergodicity arguments, it follows that the fluctuations in eigenvalues about (2-9) vanish in the thermodynamic limit [21]. With the help of this distribution function, the low-momentum factor of the fermion determinant reads

$$\det(m^2 + T^\dagger T) = \prod_f H(m_f), \quad (2-10a)$$

$$H(m) = \left(\frac{m}{2} + \frac{1}{2}(m^2 + 4\kappa^2)^{\frac{1}{2}}\right) \exp\left(\frac{1}{2} \frac{m - (m^2 + 4\kappa^2)^{\frac{1}{2}}}{m + (m^2 + 4\kappa^2)^{\frac{1}{2}}}\right). \quad (2-10b)$$

Under the assumption that $1/\bar{\rho} \gg M_1 \gg \kappa$ the M_1 -dependence in eq. (2-5) cancels the M_1 -dependence² of eq. (2-4). The variance κ^2 is proportional to the

²Even when the double inequality is only marginally satisfied, the M_1 dependence is weak. For $T = 0$ this fact has been demonstrated by Dyakonov and Petrov [17].

average of the trace of $T^\dagger T$. The averaging over the positions and the orientations can be performed analytically. The averaging over the sizes is performed by a leading-order cumulant expansion which is accurate when the distribution function is peaked. The result is

$$\kappa^2(T) = \frac{NT}{V_3 N_c} \alpha(T) \bar{\rho}^2, \quad (2-11)$$

with

$$\alpha(T) = \frac{1}{\bar{\rho}^2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\beta} \sum_n (k^2 + \omega_n^2) (k^2 A_n^2 + \omega_n^2 B_n^2)^2, \quad (2-12)$$

and the bar denotes the averaging over the distribution of the collective coordinates. The ω_n represent the Matsubara frequencies given by $2\pi(n + \frac{1}{2})/\beta$. The coefficients A_n and B_n enter in the evaluation of the density matrix of instantons and anti-instantons. They are given by

$$k A_n = \frac{1}{2\pi\bar{\rho}} \int_0^\infty 4\pi r^2 dr \int_0^\beta dt \left(\frac{\cos kr}{kr} - \frac{\sin kr}{k^2 r^2} \right) \cos \omega_n t \Pi^{\frac{1}{2}} \partial_r \frac{\phi}{\Pi}, \quad (2-13a)$$

$$\omega_n B_n = \frac{1}{2\pi\bar{\rho}} \int_0^\infty 4\pi r^2 dr \int_0^\beta dt \frac{\sin kr}{kr} \sin \omega_n t \Pi^{\frac{1}{2}} \partial_t \frac{\phi}{\Pi}. \quad (2-13b)$$

In the limit $\bar{\rho}/\beta \gg 1$, the dependence of A_n and B_n on $\bar{\rho}$ and β simplifies considerably

$$k A_n = \beta^2 F_n(k\beta), \quad (2-14a)$$

$$\omega_n B_n = \beta^2 G_n(k\beta), \quad (2-14b)$$

where F_n and G_n are $\bar{\rho}$ -independent functions of $k\beta$ accessible numerically. In this limit the terms with $n = 0$ or $n = -1$ are dominant. Dimensional arguments show that the function $\alpha(T)$ depends only on the dimensionless combination $\bar{\rho}^2 T^2$. The

shape of this function can be determined numerically. In Fig. 1 we show the result for $\bar{\rho}^2 = 1$. For small values of $\bar{\rho}^2 T^2$ it can be well approximated by

$$\alpha(T) = \frac{6.6209}{1 + \frac{2}{3}\pi^2 \bar{\rho}^2 T^2}. \quad (2-15)$$

The asymptotic tail of $\alpha(T)$ can be obtained by using the functional dependence of A_n and B_n in the limit $\bar{\rho}^2 T^2 \gg 1$ (see eqs. (2-14ab)). The result is

$$\alpha(T) = \frac{1.078}{\bar{\rho}^2 T^2}, \quad \bar{\rho}^2 T^2 \gg 1. \quad (2-16)$$

The constant 1.078 follows from a numerical integration. Note that at low temperatures the size of the instantons is *not* renormalized by $1/(1 + \frac{\pi^2}{3}\bar{\rho}^2 T^2)$ as argued by Gross, Pisarski and Yaffe [19]. The reason is that the variance is obtained by integration over the whole range of r and t whereas the renormalization factor of Gross, Pisarski and Yaffe applies only to distances $(r^2 + t^2)^{\frac{1}{2}} \ll \beta$. In the chiral limit we reproduce the functional dependence of the fermion determinant given by Ilgenfritz and Shuryak [16]. However, they ignored the finite temperature size renormalization given by $\alpha(T)/\alpha(0)$ in eq. (2-11).

To evaluate the resulting partition function (2-1), we use the Feynman variational principle (see ref. [11] for details). As a result only the average interaction $\bar{u}_{int}$ enters in the final expressions. By a straightforward calculation Dyakonov and Merlin [15] obtained ³

$$\bar{u}_{int} = \gamma^2 \bar{\rho}_1^2 \bar{\rho}_2^2, \quad \gamma^2 = \frac{27\pi^2}{4} \frac{N_c}{N_c^2 - 1}. \quad (2-17)$$

Note that the average interaction does not depend on the chirality of the instantons, and that the coefficient γ^2 is temperature independent. Using $\mu(\rho, T)$ as a

³It is assumed that the many-body forces do not play an important role at the instanton densities relevant for the description of the QCD vacuum. Again, this assumption will be justified *a posteriori* by the result that the instanton medium is dilute.

variational one-body density to account for the two-body interaction, the partition function is bounded from below by

$$\sum_{N_+, N_-} \frac{1}{N_+! N_-!} \int \prod_I d\Omega_I \mu(\rho_I, T) \exp(-\beta(\bar{\rho}) \sum_{I < J} \bar{u}_{int}(I, J)) - \sum_I \log \mu(\rho_I, T) / d(\rho_I, T)), \quad (2-18)$$

where the density $d(\rho, T)$ is equal to the product of $d_0(\rho, T)$ and a factor originating from the fermion determinant. The function μ is determined variationally. We find that the instantons and the anti-instantons are distributed according to the same distribution function μ ,

$$\mu(\rho, T) = d(\rho, T) \exp \left[\overline{\log \mu / d} + \frac{1}{2} \sum_f \frac{1}{H} \frac{\partial H}{\partial \bar{\rho}} (\rho - \bar{\rho}) - \frac{\beta(\bar{\rho}) \gamma^2 T N}{V_3} \bar{\rho}^2 (\rho^2 - \bar{\rho}^2) \right] \quad (2-19)$$

The term involving H is due to the μ -dependence of the average size. The argument of the running coupling constant is fixed after minimization with respect to μ , and is given by $\bar{\rho}^2 \Lambda^2$. The average squared size is determined selfconsistently from

$$\bar{\rho}^2 = \int d\rho \mu(\rho) \rho^2 / \int d\rho \mu(\rho), \quad (2-20)$$

and $\bar{\rho}$ is defined by $(\bar{\rho}^2)^{\frac{1}{2}}$.

The free energy $F \equiv \log Z$ is given by

$$F(T) = N \left[-\log \frac{n(T)}{2\Lambda^4} - \frac{1}{2} \sum_f \frac{1}{H} \frac{\partial H}{\partial \bar{\rho}} \bar{\rho} + \frac{1}{2} \beta(\bar{\rho}) \gamma^2 n(T) \bar{\rho}^2 + \log \int d\rho d(\rho, T) \Lambda^4 \exp \left(\frac{1}{2} \sum_f \frac{1}{H} \frac{\partial H}{\partial \bar{\rho}} \rho - \beta(\bar{\rho}) \gamma^2 n(T) \bar{\rho}^2 \rho^2 \right) \right], \quad (2-21)$$

where the density of pseudoparticles NT/V_3 is denoted by $n(T)$. In the thermodynamic limit $n(T)$ is determined by the value of N where F has its maximum. In the dilute limit the extremum can be determined analytically. In this limit the density dependence of the free energy density $\mathcal{F}$ is given by

$$\mathcal{F} = n(T)(c(T) + (\frac{N_f}{2} - 1) \log \frac{n(T)}{\Lambda^4}), \quad \frac{n(T)}{\Lambda^4} \ll 1, \quad (2-22)$$

where $c(T)$ is a temperature dependent constant. The free energy density at the extremum is $(N_f/2 - 1)n(T)/\Lambda^4$. In the three-flavor case, the free energy becomes negative in the dilute limit leading to a first order phase transition to the trivial solution $n(T) = 0$. In the two-flavor case, the free energy is zero in the dilute limit which results in a second order phase transition. In the one-flavor case, the free energy for the nontrivial solution is always positive and no phase transition takes place. This behavior is illustrated in Fig. 2 where we show the ratio $F(T)/F(0)$ as a function of the temperature T for one, two and three massless flavors. The derivative $\partial_T F(T)$ is discontinuous at the critical point for three flavors only. The free energy per particle $F(0)/N$ is 2.17, 1.21 and 0.672 for $N_f = 1, 2$ and 3 respectively. The curves shown in Fig. 3 represent the ratio $n(T)/n(0)$, where the density $n(T)$ behaves as expected from the T -dependence of the free energy. Below the critical temperature, the T -dependence of the size of the pseudoparticles is very weak. This is shown in Fig. 4, where the curves represent results for one, two and three flavors. In the chirally symmetric phase, the size of the instantons arises as a consequence of the finite temperature (see eq. (2-2)).

To reproduce the phenomenological value of the gluon condensate, we have chosen the coupling constant in the pre-exponential factor to be $8\pi^2/g^2 = -b \log 0.1$. The value of Λ_{PV} is fixed by requiring that the gluon condensate at zero temperature ($\langle F_{\mu\nu}^2 \rangle = (190 \text{ MeV})^4$) is saturated by NT/V_3 . In the three flavor-case, the latter quantity is equal to $0.2257\Lambda^4$ which yields a value of $\Lambda_{PV} = 275 \text{ MeV}$. The average size of the pseudoparticles at $T = 0$ is equal to 0.36 fm , whereas the distance between the pseudoparticles defined by $(NT/V_3)^{-\frac{1}{4}}$, is equal to 1.45 fm . This shows that our system is dilute. The validity of the semiclassical limit is guaranteed by the value of the running coupling constant, $8\pi/g^2 = -b \log \bar{\rho}\Lambda = 6.4$. Similar numerical values have been obtained for one and two flavors.

In this approach, massive flavors decouple from the light ones. This fact has been verified both analytically and numerically. This implies that the chiral phase transition in the case of 3 flavors with a massive strange quark is of second order. This is shown in Fig. 5 where we display the ratio $F(T)/F(0)$ as a function of T/Λ for quark masses equal to 0.024Λ , 0.024Λ and 0.6Λ . We find that the free energy behaves very much like the two-flavor massless case. In fact, because of the nonzero masses of the light quarks, the free energy always remains positive, and in the strict sense we do not have a phase transition. Also the density of pseudoparticles is a smooth function of the temperature.

3. Chiral order parameters.

In the mean field approximation, Dyakonov and Petrov [17] were able to obtain the chiral condensate and the constituent quark mass as a function of $N/V_4\bar{\rho}^4$. In Ref. [13] we simplified their derivation by the introduction of a generating function for the chiral condensate. This approach can be generalized to nonzero temperatures by replacing the zero temperature zero modes by zero modes corresponding to the caloron solution, and replacing the integral over the four-momentum by its finite temperature counterpart. As gap equation we obtain

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\beta} \sum_n \frac{M^2(\mathbf{k}, \omega_n)}{\mathbf{k}^2 + \omega_n^2 + M^2(\mathbf{k}, \omega_n)} = \frac{NT}{4V_3N_c}(1 - m\epsilon), \quad (3-1)$$

where the momentum-dependent effective mass is defined by

$$M(\mathbf{k}, \omega_n) = \frac{\epsilon NT}{2V_3N_c}(\mathbf{k}^2 + \omega_n^2)(\mathbf{k}^2 A_n^2 + \omega_n^2 B_n^2). \quad (3-2)$$

Also the result for the chiral condensate is a straightforward generalization of the zero temperature case

$$\langle \bar{q}q \rangle = -4N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\beta} \sum_n \frac{M(\mathbf{k}, \omega_n)}{\mathbf{k}^2 + \omega_n^2 + M^2(\mathbf{k}, \omega_n)}. \quad (3-3)$$

One can show that the chiral condensate and the constituent quark mass depend only on the dimensionless combinations $n(T)\rho^4$ and $\bar{\rho}^2 T^2$.

The value of ϵ is determined by solving eq. (3-1) iteratively. This solution enables us to calculate the constituent mass $M(\mathbf{k} = 0, \omega_{n=0})$ and the chiral condensate $\langle \bar{q}q \rangle$. Their temperature dependence for one, two and three flavors is displayed in Figs. 6 and 7. We clearly see that the order of the phase transition depends on the number of flavors. For low temperatures the chiral condensate is almost independent of the temperature. This is the result of two opposing effects. The chiral condensate obtained from eqs. (3-1), (3-2) and (3-3) increases both with temperature and with density. However, the actual density as obtained from the free energy (eq. (2-21)) decreases with increasing temperature. With the numerical parameters quoted above the $N_f = 3$ values at zero temperature of the constituent quark mass and the chiral condensate are equal to $370MeV$ and $(237MeV)^3$, respectively. The critical temperature in this case is $180MeV$. These values are in rough agreement with values obtained from a phenomenological analysis.

4. Conclusion

In conclusion, the order of the chiral phase transition in an instanton model for the QCD-vacuum depends sensitively on the number of massless flavors. Contrary to lattice QCD calculations [22] we find a second order phase transition in the two-flavor case. In the three-flavor case we agree with the lattice calculations on the order of the transitions. The numerical values of the observables are in reasonable agreement with lattice calculations and with values obtained by a phenomenological analysis [10].

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Figure Captions

Fig. [1]. Temperature dependence of the eigenvalues of the overlap matrix D as given by the function $\alpha(T)$ defined in eq. (2-12). The curve was evaluated for $\bar{\rho} = 1$.

Fig. [2]. Ratio of the free energy $F(T)$ to $F(0)$ as a function of the dimensionless temperature T/Λ . Here the quarks are massless. Going from left to right, the curves represent results for $N_f = 3, N_f = 2$ and $N_f = 1$.

Fig. [3]. Ratio of the density of pseudoparticles $n(T)$ to the vacuum density $n(0)$. Going from left to right, the curves show results for $N_f = 3, N_f = 2$ and $N_f = 1$. All quarks are massless.

Fig. [4]. Average size $\bar{\rho}$ of the pseudoparticles in units of the cutoff $1/\Lambda$ as a function of the temperature in units of Λ . The size at zero temperature is equal to 0.335, 0.419 and 0.525 for $N_f = 1, N_f = 2$ and $N_f = 3$, respectively.

Fig. [5]. Ratio $F(T)/F(0)$ as a function of T/Λ in the case of three flavors with quark masses equal to $0.024\Lambda, 0.024\Lambda$ and 0.6Λ .

Fig. [6]. Chiral condensate $\langle \bar{q}q \rangle \Lambda^3$ as a function of the temperature in units of Λ . From top to bottom the curves represent results for one, two and three massless flavors, respectively.

Fig. [7]. Constituent mass $M(0) \equiv M(\mathbf{k} = 0, \omega_{n=0})$ in units of Λ as a function of T/Λ . From top to bottom, the curves represent results for one, two and three massless flavors, respectively.

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