CERN-TH.5101/88
Univ. of Roma preprint 633
Erratum

MATRIX ELEMENTS OF LEFT-RIGHT FOUR-FERMION OPERATORS AND THE
ELECTROPENGUIN CONTRIBUTION TO ϵ'/ϵ IN LATTICE QCD WITH WILSON FERMIONS

E. Franco
INFN - Sezione di Roma, Italy

L. Maiani
Dipartimento di Fisica, Università di Roma
INFN - Sezione di Roma, Italy

G. Martinelli
CERN - Geneva

and

A. Morelli
Dipartimento di Fisica, Università di Roma
INFN - Sezione di Roma

Due to the overlooking of a factor of two in the operator basis and to other minor errors, the numerical values of the electropenguin contribution to ϵ'/ϵ are about a factor two larger than given in the original version of the paper. We also take the occasion to correct some trivial misprint.

The correct results and formulae are as follows.

Abstract - 4th line: $(\epsilon'/\epsilon)_{\text{EMP}} = (0.74 \pm 0.27) \times 10^{-3} \sin \varphi$.

Abstract - 6th line: $\Omega_{\text{EMP}} \cong 0.40$

$$\left| \frac{\epsilon'}{\epsilon} \right|_{\text{EMP}} = (0.74 \pm 0.27) 10^{-3} \left(\frac{A}{1.05} \right)^2 \left(\frac{\lambda}{0.221} \right)^4 \left(\frac{\rho}{0.6} \right) \sin \varphi \quad (6)$$

$$\Omega_{\text{EMP}} \simeq +0.40 \quad (8)$$

CERN-TH.5101/88
Univ. of Roma preprint 633
Erratum
November 1988

$$\langle \pi^+ \pi^- | O_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} = -\frac{i}{f_\pi} \langle \pi^+ | O_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} = i(6.9 \pm 2.2) \times 10^{-3} \text{ a}^{-3} \quad (34)$$

$$\langle \pi^+ \pi^- | (O_c)_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} = -\frac{i}{f_\pi} \langle \pi^+ | (O_c)_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} = i(7.2 \pm 2.6) \times 10^{-3} \text{ a}^{-3} \quad (35)$$

$$\left(\frac{\text{Im } A_2}{A_2} \right)_{\text{EMP}} = \left(\frac{-\lambda_t}{\sin \theta_c} \right) \frac{3}{c_4} \frac{(\tilde{y}_7 + \frac{1}{3} \tilde{y}_8) \langle \pi^+ \pi^- | \hat{O}_{LR} | \bar{K}^0 \rangle + 2 \tilde{y}_8 \langle \pi^+ \pi^- | \hat{O}_c |_{LR} | K^0 \rangle}{\langle \pi^+ \pi^- | \hat{O}_{LL}^{(4)} | K^0 \rangle} \quad (37)$$

$$\left(\frac{\text{Im } A_2}{A_2} \right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c} \right) \frac{3}{c_4} \frac{(\tilde{y}_7 + \frac{1}{3} \tilde{y}_8) A_{LR} + 2 \tilde{y}_8 (A_c)_{LR}}{(m_k^2 - m_\pi^2) \gamma_{LL}^{(4)}} \quad (38)$$

$$\gamma_{LL}^{(4)} = \frac{\langle \pi^+ | \hat{O}_{LL}^{(4)} | K^+ \rangle}{M_p^2} \Big|_{\text{chiral limit}} \quad (39)$$

$$\frac{\tilde{c}_7}{\alpha_{\text{em}}} (2 \text{ GeV}) \equiv_2 \frac{\tilde{y}_7}{\alpha_{\text{em}}} (2 \text{ GeV}) \equiv -0.18 \quad (40)$$

$$\frac{\tilde{c}_8}{\alpha_{\text{e.m.}}} (2 \text{ GeV}) \equiv_2 \frac{\tilde{y}_8}{\alpha_{\text{e.m.}}} (2 \text{ GeV}) \simeq -0.03 \quad (41)$$

$$\left(\frac{\text{Im } A_2}{A_2} \right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c} \right) \alpha_{\text{e.m.}} \frac{a^{-2}}{m_k^2 - m_\pi^2} \cdot (0.24 \pm 0.09) \quad (42)$$

$$\left(\frac{\text{Im } A_2}{A_2} \right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c} \right) (3.4 \pm 1.4) \times 10^{-2} \quad (43)$$

The unnumbered equation following eq.(43): corrected as Eq.(6).

$$\frac{\text{Im } A_0}{A_0} \simeq \left(\frac{\lambda_t}{\sin \theta_c} \right) (-0.146) \quad (44)$$

$$\Omega_{\text{EMP}} = 0.39 \quad (45)$$

Sentence following eq.(47): which is close to the previous estimate if $B_6 \sim 0.5$, as found in ref.(13).

MATRIX ELEMENTS OF LEFT-RIGHT FOUR-FERMION OPERATORS AND THE
ELECTROPENGUIN CONTRIBUTION TO ϵ'/ϵ IN LATTICE QCD WITH WILSON FERMIONS^{*)}

E. Franco
INFN - Sezione di Roma, Italy

L. Maiani
Dipartimento di Fisica, Università di Roma
INFN - Sezione di Roma, Italy

G. Martinelli
CERN - Geneva

and

A. Morelli
Dipartimento di Fisica, Università di Roma
INFN - Sezione di Roma

A B S T R A C T

We compute the $K\pi$ and $K\pi\pi$ matrix elements of left-right four-fermion operators with $\Delta I = 3/2$ in quenched lattice QCD and $\beta = 6.0$, with Wilson fermions. This enables us to compute the electropenguin contributions to ϵ'/ϵ . We estimate: $(\epsilon'/\epsilon)_{\text{EMP}} = (0.43 \pm 0.16) \times 10^{-3} \sin\phi$, with ϕ the CP violating C-K-M phase, assuming standard values for the other C-K-M angles. Alternatively, we quote $\Omega_{\text{EMP}} \approx 0.20$ for the ratio of $(\text{Im } A_2/A_2)_{\text{EMP}}$ to $(-\text{Im } A_0/A_0)$, the latter quantity being estimated from vacuum saturation in the continuum theory.

By comparing $K\pi$ and $K\pi\pi$ matrix elements, we check, within 50% errors, the soft-pion relations derived from the chiral structure of the operators. We observe a large enhancement of the matrix element $\langle \pi^-\bar{K}^0 | 0 | \pi^- \rangle$, which we interpret as the effect of a scalar octet pole in the $\pi^-\bar{K}^0$ channel. This observation has interesting implications for the related calculation of $\Delta I = \frac{1}{2}$, $K \rightarrow \pi\pi$ weak matrix elements.

^{*)} Partially supported by Ministero della Pubblica Istruzione, Italy.

1. - Introduction and Summary of Results

The electromagnetic contribution to the CP-violating parameter ϵ' induced by the electropenguin diagrams has been widely discussed in recent literature⁽¹⁻⁴⁾. After integration of t and c-quark virtual loops, the electropenguin contribution to the $I = 2$, $K \rightarrow \pi\pi$ amplitude, A_2 , is determined by the matrix elements of the following four fermion operators:

$$\hat{O}_{LR} = \frac{1}{3} \{ (\bar{s}_L \gamma_\mu d_L) [\bar{u}_R \gamma^\mu u_R - \bar{d}_R \gamma^\mu d_R] + (\bar{s}_L \gamma_\mu u_L) (\bar{u}_R \gamma^\mu d_R) \} \quad (1)$$

$$\begin{aligned} \hat{O}_{LR} = \frac{1}{3} \{ (\bar{s}_L \gamma_\mu t^A d_L) [\bar{u}_R \gamma^\mu t^A u_R - \bar{d}_R \gamma^\mu t^A d_R] \\ + (\bar{s}_L \gamma_\mu t^A u_L) (\bar{u}_R \gamma^\mu t^A d_R) \} \end{aligned} \quad (2)$$

The operators in eqs.(1-2) have pure $\Delta I=3/2$. From the point of view of flavour SU(3), they are a superposition of 27-plet and decuplet, unlike the fully left-handed operator⁽⁵⁾, $\hat{O}_{LL}^{(4)}$:

$$\hat{O}_{LL}^{(4)} = (\bar{s}_L \gamma_\mu d_L) [(\bar{u}_L \gamma^\mu u_L) - (\bar{d}_L \gamma^\mu d_L)] + (\bar{s}_L \gamma_\mu \bar{u}_L) (\bar{u}_L \gamma^\mu d_L) \quad (3)$$

which is a pure **27**.

Besides the interest for the theory of CP-violation, a fully non-perturbative calculation of the $K \rightarrow \pi$ and $K \rightarrow \pi\pi$ matrix elements of the LR operators (1-2) is worthwhile being done for a number of other reasons.

- i) The $K \rightarrow \pi$ matrix elements of the operators (1-2) may be compared to their values in the vacuum saturation approximation (VSA). In the chiral limit, the VSA values of the amplitudes are dominated by the matrix elements of the pseudoscalar density between the vacuum and one pseudoscalar meson. This is to be contrasted with the matrix elements of the LL operator which vanish in the chiral limit.
- ii) By comparing $K \rightarrow \pi\pi$ to $K \rightarrow \pi$ correlation functions and matrix elements, one may test the predictions of the soft-pion theorems on the lattice. The chiral $SU(3)_L \times SU(3)_R$ behavior of the operators (1-2) is that of a **(8,8)**, while the LL operator in eq.(3) behaves as a **(27,1)**. Thus the soft-pion theorems tested with the LR operators are independent from and complementary to those obeyed by the LL operator in (3), which have been analysed in ref.(6).

- iii) Due to the non-vanishing component over the SU(3) decuplet, the operators (1-2) may have a non vanishing matrix element between one pseudoscalar and one scalar octet meson. If the mass of the scalar meson M_s is not much larger than the pseudoscalar meson mass M_p in the range of quark masses one is working with, the matrix elements $\langle \pi^- \bar{K}^0 | \hat{O}_{LR} | \pi^- \rangle$ and $\langle \pi^- \bar{K}^0 | (\hat{O}_c)_{LR} | \pi^- \rangle$ may be enhanced by the effects of the scalar meson propagator which contains a factor $(M_s^2 - 4M_p^2)^{-1}$. It has been suggested⁽⁷⁾ that an effect of this kind is a possible source of contamination to the calculation of the $\Delta I=1/2$, $K \rightarrow \pi\pi$ weak amplitude on the lattice, with the method proposed in ref.(8). It is important to see whether the enhancement due to the scalar meson is present for $\Delta I=3/2$, LR operators, where we control at the same time matrix elements with and without the scalar meson intermediate states.

In this paper, we present a lattice calculation, in the quenched approximation with Wilson fermions, of the $K-\pi$ and $K-\pi-\pi$ matrix elements of operators (1-2). The quark propagators are the same as those used in ref.(6). They have been computed on a lattice of dimension $20 \times 10^2 \times 40$, at $\beta=6.0$, corresponding to an inverse lattice spacing $a^{-1} \simeq 2.2$ GeV. Three values of quark masses have been considered, corresponding to the hopping parameter $K=0.1515, 0.1530, 0.1545$. The overall statistics for $(K-\pi-\hat{O})$ correlation functions corresponds to 30 configurations for each K . 15 configurations have been generated with the CRAY XMP of CINECA (Bologna) and the other 15 with the CRAY XMP at the Rutherford Laboratory. The quark propagators with one pseudoscalar source insertion, needed for the $(K-\pi-\pi-\hat{O})$ correlation functions, are available only for the 15 CINECA configurations. Thus, in this case we have only one half of the previous statistics. Further details about the calculation of the gauge configurations and of quark propagators can be found in ref.(6), which is also the source of the values of meson masses, a^{-1} , f_π and other parameters used in the present paper.

Our results can be summarized as follows.

- i) In the chiral limit, the $K-\pi$ matrix elements of $\hat{O}_{LR}$ and $(\hat{O}_c)_{LR}$ are both very close to the their VSA value, the latter being also computed on the lattice. More precisely, we find:

$$B_{LR}(a^{-1} = 2.2 \text{ GeV}) = 1.0 \pm 0.1 \quad (4)$$

$$(B_c)_{LR}(a^{-1} = 2.2 \text{ GeV}) = 0.95 \pm 0.1 \quad (5)$$

- ii) Chiral relations are obeyed within about 50% accuracy, similarly to what is found in ref.(6) for $\hat{O}_{LL}^{(4)}$.
- iii) For both $\hat{O}_{LR}$ and $(\hat{O}_c)_{LR}$, the matrix elements, $\langle \pi^- \bar{K}^0 | \hat{O} | \pi^- \rangle$ which allow for a scalar octet intermediate state, are between 5 to 10 times larger than the matrix elements

$\langle \pi^+ \pi^- | \hat{O} | K^0 \rangle$ and $\langle \pi^- | \hat{O} | \pi^- K^0 \rangle$, which do not. This is strongly suggestive of an important role of the scalar octet also in the $K \rightarrow \pi\pi$, $\Delta I = 1/2$ amplitude. It suggests that a reliable calculation of $\Delta I = 1/2$ CP-conserving and CP-violating $K \rightarrow \pi\pi$ amplitudes requires quark masses much closer to the chiral limit^(f).

Finally, we may quote our results on the electropenguin contribution to ϵ'/ϵ in two different ways.

- a) From a full lattice calculation of $(\text{Im } A_2/A_2)_{\text{EMP}}$ we find:

$$\left| \frac{\epsilon'}{\epsilon} \right|_{\text{EMP}} = (0.43 \pm 0.16) 10^{-3} \left(\frac{A}{1.05} \right)^2 \left(\frac{\lambda}{0.221} \right)^4 \left(\frac{\rho}{0.6} \right) \sin \varphi \quad (6)$$

where the error is purely statistical; A , λ , ρ and φ are the parameters of the C-K-M matrix⁽¹⁰⁻¹¹⁾ in the Wolfenstein parametrization⁽¹²⁾, and the Wilson coefficients have been taken from ref.(4) at a subtraction point $\mu \simeq a^{-1}$.

- b) Writing:

$$\epsilon' = -1 e^{i(\delta_2 - \delta_0)} \frac{\omega}{\sqrt{2}} \frac{\text{Im } A_0}{A_0} (1 + \Omega_{\text{EMP}} + \Omega_{\eta, \eta'}) \quad (7)$$

with $\Omega_{\eta, \eta'}$ arising from π - η and π - η' mixing,⁽¹³⁾ we find:

$$\Omega_{\text{EMP}} \simeq +0.20 \quad (8)$$

As discussed in Sect.5, we prefer not to quote an error for the result in eq.(8). Our value of Ω_{EMP} is of the same order of the estimated value of $\Omega_{\eta, \eta'}$ ^[1], but with the opposite sign. This is quite different from the result obtained with the $1/N_C$ expansion⁽³⁾, which gives quite a smaller value of Ω_{EMP} , while it essentially agrees with ref.[13].

The analysis of K - π - π matrix elements reported in this paper is new. Previous lattice calculations of the K - π matrix elements have given B-parameters close to unity for the LR operators (1-2), both with Wilson⁽⁸⁾ and staggered⁽¹³⁾ fermions at comparable values of a^{-1} . In ref.(13), a full calculation of ϵ'/ϵ , from the K - π matrix elements with staggered fermions, is also reported.

The paper is organized as follows. We discuss the results for K - π matrix elements and B-parameters in Sect.2. K - $\pi\pi$ matrix elements are given and discussed in Sect.3. Sect.4 contains an illustration of the chiral relations between K - π and K - $\pi\pi$ amplitudes. The electropenguin contribution to ϵ' is considered in Sect.5.

^(f) Of course the calculation of the $\Delta I = 1/2$, CP-conserving, amplitude from K - π matrix elements and soft-pion theorems⁽⁹⁾ is unaffected by the scalar pole⁽⁷⁾. Unfortunately this latter method cannot be used, with Wilson fermions, for the $\Delta I = 1/2$, part of ϵ' , due to the need of one further subtraction, see ref.(9).

2. - K- π Matrix elements

We have computed the following three point functions:

$$G_O(t, \vec{t}) = \sum_{\vec{x} \vec{y}} \langle (\bar{u} \gamma_5 s)(t, \vec{x}) O(o) (\bar{d} \gamma_5 u)(\vec{t}, \vec{y}) \rangle. \quad (9)$$

with $O = O_{LR}, (O_C)_{LR}$, for all values of t , fixed $\vec{t}=6$, in the exact SU(3) limit, and for the three values of the hopping parameter K . Only eight-shaped diagrams contribute, where quark lines from the external sources are all contracted with the four-fermion operators. O is the lattice transcription of the corresponding continuum operator, i.e. the appropriate linear combination of dimension six operators and the naive operator, with coefficients computed in perturbation theory, ref.(14). We shall denote by O_{LR} and $(O_C)_{LR}$ the lattice transcriptions of the continuum operators in eqs.(1-2).

With the methods extensively described in ref.(6), we have extracted, from each correlation function, the two matrix elements:

$$\langle \pi^+ | O | K^+ \rangle, \langle K^- \pi^+ | O | o \rangle \quad (10)$$

The results (in lattice units) are given in Table 1. To extrapolate the matrix elements to the chiral limit, $M_p = 0$, we have assumed the behaviour:

$$\begin{aligned} \langle \pi^+ | O | K^+ \rangle &= A + B M_p^2 \\ \langle K^- \pi^+ | O | o \rangle &= A + D M_p^2 \end{aligned} \quad (11)$$

as function of the pseudoscalar mass M_p .

The values of the chiral limit for each operator are also given in Table 1. The parameter $A = A_{LR}, (A_C)_{LR}$ in eqs.(11) can be obtained with the following methods:

- (a) extrapolating to the chiral limit the matrix elements $\langle \pi^+ | O | K^+ \rangle$ or the matrix elements $\langle K^- \pi^+ | O | o \rangle$, separately;
- (b) fitting simultaneously all matrix elements with the linear behaviour given in eq.(11), i.e. by imposing $\langle \pi^+ | O | K^+ \rangle$ and $\langle K^- \pi^+ | O | o \rangle$ to have a common intercept at $M_p = 0$.

As seen from Table 1, methods (a) and (b) give compatible results. The errors quoted in the Table are statistical only, and obtained from the dispersion of the results over six clusters of five configurations each.

The values of the matrix elements versus $\pm M_p^2$ (for $K \rightarrow \pi$ and $\bar{K} \pi \rightarrow 0$ respectively) are reported in Fig.1, where the Bologna and RAL results are shown separately, together

with the overall averages. The results from the two different sets of configurations are quite consistent with each other.

In conclusion, we find:

$$\langle \pi^+ | O_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} \equiv A_{LR} = (-0.52 \pm 0.12) \times 10^{-3} a^{-4} \quad (12)$$

$$\langle \pi^+ | (O_C)_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} \equiv (A_C)_{LR} = (-0.54 \pm 0.16) \times 10^{-3} a^{-4} \quad (13)$$

It is common practice in the literature to express the matrix elements of four-fermion operators in terms of the so-called B-parameter times their value in the vacuum-saturation approximation (VSA) which reads^(f):

$$\begin{aligned} \langle \pi^+ | \hat{O}_{LR} | K^+ \rangle_{\text{VSA}} &= \frac{1}{18} \langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 u | K^+ \rangle \equiv -\frac{C^2}{18} \\ \langle \pi^+ | \hat{O}_C | K^+ \rangle_{\text{VSA}} &= \frac{2}{27} \langle \pi^+ | \bar{u} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 u | K^+ \rangle \equiv -\frac{2}{27} C^2 \end{aligned} \quad (14)$$

where, in the chiral limit:

$$C \simeq f_\pi \frac{m_K^2}{m_s} \quad (15)$$

In the continuum, with $m_s \simeq 160$ MeV, the value of C is:

$$C \simeq 0.20 \text{ GeV}^2 (f_\pi \simeq 130 \text{ MeV}). \quad (16)$$

In the scheme of ref.(14), the lattice operators O_{LR} and $(O_C)_{LR}$ are so defined as to correspond to the continuum operators normalized at a momentum scale $\mu \simeq a^{-1}$. In order to keep the same convention for the pseudoscalar densities, the $(\bar{q} \gamma_5 q)$ operators on the lattice have to be multiplied by a finite factor, Z_P , which we take from perturbation theory⁽¹⁵⁾:

$$Z_P \simeq 0.81 (\beta=6.0) \quad (17)$$

^(f) The VSA as given by eq.(14) is valid only at the chiral limit. For $M_p^2 \neq 0$, the contribution arising by contracting the currents in eqs.(1-2) with the external mesons should also be included. We omit the latter contribution in the present discussion, aimed to obtain the B-parameters in the chiral limit.

Accordingly, we define the B-parameters as follows:

$$B_{LR}(a^{-1}) = \frac{18\langle\pi^+|O_{LR}|K^+\rangle}{Z_p^2\langle\pi^+|\bar{u}\gamma_5 d|o\rangle\langle o|\bar{s}\gamma_5 u|K^+\rangle} \quad (18)$$

$$(B_C)_{LR}(a^{-1}) = \frac{27}{2} \frac{\langle\pi^+|O_C|K^+\rangle}{Z_p^2\langle\pi^+|\bar{u}\gamma_5 d|o\rangle\langle o|\bar{s}\gamma_5 u|K^+\rangle}$$

with both numerator and denominator computed on the lattice. The pseudoscalar density matrix elements are computed from the two-point functions in the standard way.

To obtain the B-parameters extrapolated to the chiral limit, we have proceeded in two ways. We have computed the ratios in eq.(18) for the three values of the hopping parameter K , both for the $K^+ \rightarrow \pi^+$ and $K^+ \rightarrow 0$ matrix elements (see Table 2). We have then extrapolated to the chiral limit independently each set of ratios (a), or both sets together, requiring a common value for the chiral limit (b). The results are also reported in Table 2. Alternatively, we have extrapolated separately to K_C the matrix element of the scalar densities, with the result:

$$a^4 Z_p^2 \langle\pi^+|\bar{u}\gamma_5 d|o\rangle\langle o|\bar{s}\gamma_5 u|K^+\rangle = -(0.8 \pm 0.3) 10^{-2} \quad (19)$$

Combining eqs.(12), (13) and (19), for the central values we obtain:

$$B_{LR}(a^{-1}) = 1.2 \quad (20)$$

$$(B_C)_{LR}(a^{-1}) = 0.92$$

These numbers differ from those obtained by extrapolating the B-factors themselves by a somewhat larger amount than the statistical error quoted in Table 2 case (b). The discrepancy gives an idea of the systematic errors involved in the extrapolation. A further source of error more difficult to quantify is the use of perturbation theory in the renormalization of all operators.

As a conclusion, we quote the following values as our estimate of the B-parameters at the chiral limit:

$$B_{LR}(a^{-1}) = 1.0 \pm 0.10 \quad (21)$$

$$(B_C)_{LR}(a^{-1}) = 0.95 \pm 0.10$$

Values of the B-parameters for LR operators have been computed independently in lattice QCD by the authors of ref.(13) with staggered fermions, and ref.(8) with Wilson fermions. Our results are consistent with these calculations.

Keeping into account the QCD anomalous dimension matrix of the operators in eqs.(1-2), as well as the anomalous dimension of the pseudoscalar density, one obtains the following formulae, which give the B-factors for any renormalization point μ , (provided $\Lambda_{\text{QCD}} < \mu < a^{-1}$):

$$\begin{aligned} B_{\text{LR}}(\mu) - (B_{\text{C}})_{\text{LR}}(\mu) &= \frac{y_1(\mu)}{[y_p(\mu)]^2} (B_{\text{LR}}(a^{-1}) - (B_{\text{C}})_{\text{LR}}(a^{-1})) \\ B_{\text{LR}}(\mu) + 8 (B_{\text{C}})_{\text{LR}}(\mu) &= \frac{y_2(\mu)}{[y_p(\mu)]^2} (B_{\text{LR}}(a^{-1}) + 8 (B_{\text{C}})_{\text{LR}}(a^{-1})) \\ y_i(\mu) &= \left[\frac{\alpha_s(\mu)}{\alpha_s(a^{-1})} \right]^{d_i} \end{aligned} \quad (22)$$

with $d_1 = -1/b_f$, $d_2 = 8/b_f$, $d_p = -2/b_f$ ($b_f = 11 - 2/3 f$, for f flavours).

From the matrix elements of the pseudoscalar densities computed on the lattice one obtains an estimate of the quantity C , defined in the continuum by eqs.(15-16). Using $a^{-1} \simeq 2 \text{ GeV}$, we get:

$$C(a^{-1}) \simeq 0.36 \text{ GeV}^2 \quad (23)$$

a value about twice the result^(f) in eq.(16). This shows that a value of $B_{\text{LR}} \sim 1$ on the lattice corresponds in fact to a matrix element of the four fermion operator which is about four times larger than one would estimate in the continuum with $B=1$ and C given by eq.(16) (with an uncertainty about a factor 2 due to the value of a^{-1}). This is at variance with the situation for purely lefthanded operators. Since the experimental value of f_π is well reproduced by the lattice calculation⁽⁶⁾, $B_{\text{LL}} = 1$ corresponds to a matrix element of the four fermion operator which coincides with the vacuum saturation approximation in the continuum.

^(f) The value $m_\pi \simeq 160 \text{ MeV}$ used in eq.(16) is obtained from the baryon mass differences, i.e. at a mass scale $\mu \simeq 1 \text{ GeV}$. The variation of C , in going from $\mu \simeq a^{-1} \simeq 2.2 \text{ GeV}$ to $\mu \simeq 1 \text{ GeV}$ is essentially negligible.

3. - $K\pi\pi$ matrix elements and the role of the scalar octet

We present in this Section the results obtained from a calculation of the four-point functions:

$$G_O(t_K, t_\pi, \bar{t}) = \sum_{\vec{x} \vec{y} \vec{z}} \langle (\bar{d} \gamma_5 s)(t_K, \vec{x}) O(o) (\bar{d} \gamma_5 u)(t_\pi, \vec{y}) (\bar{u} \gamma_5 d)(\bar{t}, \vec{z}) \rangle . \quad (24)$$

with $O = O_{LR}, (O_C)_{LR}$. We have computed these functions for all values of t_K and t_π , and $\bar{t}=6$. As discussed in ref.(6), the quark propagator with the insertion of a pseudoscalar source at $\bar{t} = 6$ is computed separately, and combined with the usual quark propagator to obtain the relevant diagrams, given in Fig.2.

Quark propagators with a pseudoscalar insertion are available only for the 15 configurations generated at CINECA, on which all results presented in this Section are based.

The plateau method⁽⁶⁾ allows us to obtain the matrix elements:

$$\langle \pi^- \pi^+ | O | K^0 \rangle ; \langle \pi^- | O | \pi^- K^0 \rangle \quad (25)$$

from the correlation functions (24) with fixed $t_K = 34$ and letting t_π to be variable. In addition, the matrix elements:

$$\langle \pi^- | O | \pi^- K^0 \rangle ; \langle \pi^- \bar{K}^0 | O | \pi^- \rangle \quad (26)$$

are obtained by fixing $t_\pi = 34$, and letting t_K to vary. (In all cases, one π^- source is fixed at the time $\bar{t}=6$). The results are listed in Table 3, where it is seen that the two ways which lead to the same matrix element, $\langle \pi^- | O | \pi^- K^0 \rangle$, indeed give quite close results. If we define the $K\pi$ matrix elements to be real, the matrix elements (25) and (26) are purely imaginary, and we give in Table 3 their imaginary parts.

There are two conspicuous features of the results which apply to both O_{LR} and $(O_C)_{LR}$:

- i) The matrix elements $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$ are much larger than the others, and show a quite non-smooth extrapolation to chiral limit;
- ii) The matrix elements $\langle \pi^- \pi^+ | O | K^0 \rangle$ and $\langle \pi^- | O | \pi^- K^0 \rangle$ are similar and smoothly varying (except for the point at $K=0.1545$ where all our results are of worse quality).

What makes the matrix elements $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$ so special can be simply understood by noticing that the two mesons on the same side are allowed to be in an octet state, with

$J=0$ and even charge conjugation^(f). If the $q\bar{q}$ scalar octet has a mass M_s close to $2M_p$, (as indicated by the APE data at $\beta=6.0$ ⁽¹⁶⁾) propagator effects due to S-exchange will enhance the matrix elements. On the contrary, simple isospin arguments show that the two meson states in the other two matrix elements have pure isospin 2 or 3/2, so that no scalar octet intermediate state is allowed. The observed enhancement indicates a quite sizeable matrix element $\langle S|O|\pi \rangle$ and supports the considerations made in ref.(7), about possible large effects induced in the $\Delta I=1/2$ non leptonic $K \rightarrow \pi\pi$ matrix element, at the present values of β and quark masses.

The ratio of the two other matrix elements is uniquely fixed by SU(3) symmetry. This is because the parity violating part of the operators (1-2) is a pure decuplet, and there is only one independent coupling of the operators to give rise to 27-two meson states, such as the $\pi^-\pi^+$ or π^-K^0 states in eq.(25). A simple calculation shows that:

$$\langle \pi^-\pi^+ | O | K^0 \rangle = \langle \pi^- | O | \pi^- K^0 \rangle \quad (27)$$

in the SU(3) limit, for any value of M_p . The SU(3) relation is well verified by the results of Table 3, except perhaps at $K = 0.1545$.

To conclude this Section, we report the values of the matrix elements extrapolated to the chiral limit. In Table 3 column (a) we list the chiral limit value of the different matrix elements, each extrapolated linearly in M_p^2 and independently of the others. We have quoted also the linearly extrapolated values of $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$, although the smoothness hypothesis is dubious here. Column (b) reports chiral limits obtained with a joint fit to $\langle \pi^+\pi^- | O | K^0 \rangle$ and $\langle \pi^- | O | \pi^- K^0 \rangle$ or a joint fit to $\langle \pi^- | O | \pi^- K^0 \rangle$ and $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$.

Finally, the values in column (c) are obtained with the same procedure as (b), omitting all points at 0.1545, where correlation functions are of rather poor quality. Summing up, we quote as final result the values in Table 3 (c):

$$\langle \pi^-\pi^+ | O_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} = \langle \pi^- | O_{LR} | \pi^- K^0 \rangle \Big|_{\text{chiral limit}} = 1(4 \pm 1) \times 10^{-3} a^{-3} \quad (28)$$

$$\langle \pi^-\pi^+ | (O_C)_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} = \langle \pi^- | (O_C)_{LR} | \pi^- K^0 \rangle \Big|_{\text{chiral limit}} = 1(4 \pm 2) \times 10^{-3} a^{-3} \quad (29)$$

errors quoted are statistical only.

^(f) The purely lefthanded operator considered in ref.(6) is a pure SU(3) 27-plet, no scalar octet is allowed as intermediate state, and indeed a smooth behaviour was observed for all its matrix elements.

4. - Testing the soft pion relations

As a first test of the chiral behaviour of four versus three-point functions, we have computed the sum:

$$\bar{G}_O(t_\pi, \bar{t}) \equiv 2m \sum_{t_k} G_O(t_k, t_\pi, \bar{t}) \quad (30)$$

m is the bare quark mass, defined by the relation⁽¹⁷⁾:

$$\frac{2m}{Z_A} \equiv \frac{M_p \langle 0 | (\bar{u} \gamma_0 \gamma_5 d) | \pi^+ \rangle}{\langle 0 | (\bar{u} \gamma_5 d) | \pi^+ \rangle} \quad (31)$$

All quantities in the r.h.s. of eq.(31) are computed directly on the lattice, ref.(6).

We shall take Z_A from perturbation theory, which gives⁽¹⁵⁾:

$$Z_A = 0.87 \quad (32)$$

In the continuum limit, one derives the chiral Ward-identity⁽¹⁷⁾:

$$\bar{G}_O(t_\pi, \bar{t}) = G_O(t_\pi, \bar{t}) + \dots \quad (33)$$

where G_O is defined in eq.(9), and dots represent three-point functions with one scalar and one pseudoscalar density, which may be safely neglected when time distances are large.

After computing $\bar{G}_O$, we have applied to it the same procedure that leads to the matrix elements (10) from G_O . The $K \rightarrow \pi$ and $\bar{K} \pi \rightarrow 0$ matrix elements obtained from $\bar{G}_O$ are reported in Fig.3 (a,b) versus $\pm M_p^2$, and compared to those already given in Fig.1. The agreement is quite remarkable. In particular it gives us confidence that lattice artifacts are not very important at $\beta=6.0$.

Another way to test the chiral behaviour, is to compare the $K\text{-}\pi\text{-}\pi$ matrix elements at the chiral point, with the soft pion predictions:

$$\langle \pi^+ \pi^- | O_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} \equiv -\frac{1}{f_\pi} \langle \pi^+ | O_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} = 1(6.9 \pm 0.3) \times 10^{-3} \text{ a}^{-3} \quad (34)$$

$$\langle \pi^+ \pi^- | (O_C)_{LR} | K^0 \rangle \Big|_{\text{chiral limit}} \equiv -\frac{1}{f_\pi} \langle \pi^+ | (O_C)_{LR} | K^+ \rangle \Big|_{\text{chiral limit}} = 1(7.2 \pm 0.4) \times 10^{-3} a^{-3} \quad (35)$$

Numerical values correspond to the matrix elements given in eqs.(12-13) and to f_π taken from ref.(6). Within 50% these values agree with the corresponding ones quoted in the previous Section, eqs.(28) and (29).

5. - The Electropenguin contribution to ϵ'/ϵ

The phase convention independent parameter ϵ' , measured in neutral K decays, is given by:

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \omega \left[-\frac{\text{Im } A_0}{A_0} + \frac{\text{Im } A_2}{A_2} \right] \quad (36)$$

where δ_I are the S-wave pion-pion phase shifts, A_I the transition matrix elements for $K_0 \rightarrow (\pi\pi)_{S,I}$, and $\omega^{-1} = A_0/A_2 \simeq 22$. The contribution to $\text{Im } A_2/A_2$ arising from the electropenguin diagrams can be completely evaluated on the lattice, if we combine the results of Sect.2 with the calculation of the K- π matrix element of $\hat{O}_{LL}^{(4)}$ given in ref.(6) and use the soft-pion theorems^(f). We write:

$$\left(\frac{\text{Im } A_2}{A_2} \right)_{\text{EMP}} = \left(\frac{-\lambda_t}{\sin \theta_c} \right) \frac{3}{2} \frac{\tilde{y}_7 + \frac{1}{3} \tilde{y}_8 \langle \pi^+ \pi^- | \hat{O}_{LR} | \bar{K}^0 \rangle + 3 \tilde{y}_8 \langle \pi^+ \pi^- | (\hat{O}_C)_{LR} | K^0 \rangle}{c_4 \langle \pi^+ \pi^- | \hat{O}_{LL}^{(4)} | K^0 \rangle} \quad (37)$$

where $\sin \theta_c$ is the Cabibbo angle, λ_t is a combination of the parameters in the C-K-M matrix⁽¹⁰⁻¹²⁾:

$$\lambda_t = S_\gamma S_\beta S_\phi = A^2 \lambda^5 \rho \sin \phi \quad (38)$$

and $\hat{O}_{LL}^{(4)}$ is given in eq.(3).

The Wilson coefficients $\tilde{y}_{7,8}$ and c_4 are defined as in ref.(3) and ref.(5), respectively. Reducing one pion, we obtain, in the chiral limit:

^(f) Since we have checked the soft pion theorems both here and in ref.(6), there is not much point in using directly the $K \rightarrow \pi\pi$ matrix elements, which have considerably larger errors.

$$\left(\frac{\text{Im } A_2}{A_2}\right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c}\right) \frac{3}{2} \frac{\tilde{y}_7 + \frac{1}{3} \tilde{y}_8}{c_4 (m_K^2 - m_\pi^2)} \frac{A_{LR} + 3 \tilde{y}_8 (A_C)_{LR}}{\gamma_{LL}^{(4)}} \quad (39)$$

$$\gamma_{LL}^{(4)} = \frac{\langle \pi^+ | \hat{O}_{LL}^{(4)} | K^+ \rangle}{M_p^2} \Big|_{\text{chiral limit}}$$

We use the values of the Wilson coefficients computed in ref.(4), where:

$$\frac{\tilde{c}_7}{\alpha_{\text{em}}} (2 \text{ GeV}) \equiv 2 \frac{\tilde{y}_7}{\alpha_{\text{em}}} (2 \text{ GeV}) \equiv 0.18 \quad (40)$$

$$\frac{\tilde{c}_8}{\alpha_{\text{e.m.}}} (2 \text{ GeV}) \equiv 2 \frac{\tilde{y}_8}{\alpha_{\text{e.m.}}} (2 \text{ GeV}) \simeq -0.03$$

and $c_4 (2 \text{ GeV}) \simeq 0.55$. Furthermore, from ref.(6), we obtain^(f):

$$\gamma_{LL}^{(4)} = (1.5 \pm 0.5) 10^{-3} \text{ a}^{-2} \quad (41)$$

Combining with our results for A_{LR} and $(A_C)_{LR}$, we get:

$$\left(\frac{\text{Im } A_2}{A_2}\right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c}\right) \alpha_{\text{e.m.}} \frac{\text{a}^{-2}}{m_K^2 - m_\pi^2} \cdot (0.14 \pm 0.05) \quad (42)$$

The structure of eq.(42) reflects directly the difference in the chiral behaviour of L-R versus L-L operators.

Inserting $\text{a}^{-1} = 2.2 \text{ GeV}$, we find:

$$\left(\frac{\text{Im } A_2}{A_2}\right)_{\text{EMP}} = \left(\frac{\lambda_t}{\sin \theta_c}\right) (2.0 \pm 0.8) \times 10^{-2} \quad (43)$$

or, equivalently, the result already quoted in Sect.1:

$$\left| \frac{\epsilon'}{\epsilon} \right|_{\text{EMP}} = (0.43 \pm 0.16) \times 10^{-3} \sin \varphi \left(\frac{A}{1.05}\right)^2 \left(\frac{\lambda}{0.221}\right)^4 \left(\frac{\rho}{0.6}\right).$$

^(f) We have used the relation: $(\gamma_{LL}^{3/2}) \text{ a}^2 = \left[\frac{8}{3} (f_\pi \text{ a})^2 B_{K^0 \bar{K}^0}\right] \frac{1}{8}$, with $B_{K^0 \bar{K}^0} = 0.81 \pm 0.16$ and $(f_\pi \text{ a}) = 0.075 \pm 0.016$.

where we have used the experimental value for $|ε|$.

A full lattice calculation of $\text{Im } A_0/A_0$ with Wilson fermions is not available, yet^(7,9). We may however compare the result in eq.(43) with the conventional calculation of $\text{Im } A_0/A_0$, done in the continuum with the vacuum insertion hypothesis. The result quoted in ref.(18) is:

$$\frac{\text{Im } A_0}{A_0} \simeq \left(\frac{\lambda_t}{\sin \theta_c} \right) (-0.085) \quad (44)$$

Defining Ω_{EMP} from eq.(7) of Sect.1, we find, from eqs.(43) and (44):

$$\Omega_{\text{EMP}} = 0.23 \quad (45)$$

An alternative way to compute Ω_{EMP} , is to follow ref.(4) and write:

$$\Omega_{\text{EMP}} \simeq 0.23 \frac{3\tilde{y}_7 B_{\text{LR}} + \tilde{y}_8 [B_{\text{LR}} + 8 (B_c)_{\text{LR}}]}{9 \alpha_{\text{e.m.}} \tilde{y}_6 B_6} \quad (46)$$

where B_6 is the B-parameter of the penguin operator, O_6 ⁽⁵⁾. Using the values of B_{LR} and $(B_c)_{\text{LR}}$ found in Sect.2, we obtain ($\tilde{c}_6 \equiv 2\tilde{y}_6 = -0.12$):

$$\Omega_{\text{EMP}} \simeq \frac{0.17}{B_6} \quad (47)$$

which is close to the previous estimate if $B_6 \sim 1$. We have remarked in Sect.2 that B_{LR} and $(B_c)_{\text{LR}}$ close to unity correspond in fact to matrix elements of $\hat{O}_{\text{LR}}$ and $(\hat{O}_c)_{\text{LR}}$ about four times larger than what obtained in the continuum, using $m_s \equiv 160$ MeV. In the computation of $(\text{Im } A_2/A_2)_{\text{EMP}}$ which leads to the result in eq.(43), this factor four is partly compensated because A_2 on the lattice is (about twice) larger than the experimental amplitude. These two facts essentially explain the difference between the two results (45) and (47).

The statistical errors in (45) and (47), estimated from those of the B-parameters are rather small, of the order of 10%. The result is however affected by systematic errors which are quite difficult to evaluate. In particular the errors involved in the use of the soft-pion theorems can be of the order of 50%, see Sect.4. We prefer not to quote a specific value of the error, but 50% could be a good reference value.

A full lattice calculation of ϵ'/ϵ with staggered fermions has been presented in ref.(13). These authors find B-parameters close to one for the L-R operators (and a B-parameter of $\hat{O}_6$ close to 0.5). We agree with these authors so that we get the same result

for the magnitude of Ω_{EMP} . On the other hand our results do not support the suppression of the matrix element of $\hat{O}_{\text{LR}}$ (which would vanish in the $N_c \rightarrow \infty$ limit, N_c being the number of colours) and the very small value of Ω_{EMP} found in ref.(3).

Conclusions

We have presented a lattice calculation of the matrix elements of LR, $\Delta I=3/2$, four-fermion operators between $K-\pi$ and $K-\pi-\pi$ states. This calculation has been used to estimate the electropenguin contribution to ϵ'/ϵ and to test the soft-pion theorems on the lattice.

A relevant related result is the enhancement of the $\langle \pi^- \bar{K}^0 | O | \pi^- \rangle$ matrix elements over the others. A natural interpretation is afforded by the propagator effect, due to a nearby scalar octet pole in the $\pi^- \bar{K}^0$ channel. This result has interesting consequences for the program of evaluating the $\Delta I=1/2$ CP-conserving and CP-violating amplitudes in K decay. To obtain these amplitudes from the $K-\pi\pi$ matrix elements, which need no subtraction of divergent operators⁽⁹⁾ in the $SU(3)$ limit⁽⁸⁾, one has to check whether the scalar pole is not contaminating the result. Since most of the calculations are done for rather heavy quark mass, a calculation of the $K-\pi\pi$ matrix elements of $\hat{O}_{\text{LR}}$, together with those of the relevant terms of the weak hamiltonian $\hat{O}^{(-)}$ and $\hat{O}^{(6)}$, could be very useful. The LR operators can be used to monitor the relevance of the scalar pole, in a channel in which no dynamical enhancement is otherwise expected, as we let the quark mass to approach the chiral limit.

Acknowledgements

We are grateful to C.Sachrajda for his very useful collaboration in the use of the Rutherford Laboratory gauge fields configurations and propagators. We acknowledge many interesting discussions with M.B.Gavela, M.Lusignoli, O.Pène and S.Petrarca. We also thank the members of the APE collaboration for communicating to us their results on hadron spectrum prior to publication. Financial support from INFN is gratefully acknowledged.

References

- 1) J.F.Donoghue, E.Golowich, B.R.Holstein and J.Trampetic, Phys. Lett. B179 (1986) 361 and B188 (1987) 511.
- 2) J.Bijnens and M.B.Wise, Phys. Lett. B137 (1984) 245.
- 3) A.Buras and J.M.Gerard, Phys. Lett. B192 (1987) 156.
- 4) S.R.Sharpe, Phys. Lett. B194 (1987) 551.
- 5) M.Shifman, A.Vainshtein and V.Zakharov, Nucl.Phys. B120(1977) 31.
- 6) M.B.Gavela et al. CERN-TH 4905/87, to appear in Nuclear Physics B.
- 7) M.B.Gavela et al. Roma preprint 599/1988 and CERN TH 5032/88. To appear in Physics Letters B.
- 8) C.Bernard et al. Nucl.Phys. B (Proc.Suppl.) 4 (1988) 483.
- 9) L.Maiani et al. Nucl. Phys. B289 (1987) 505.
- 10) M.Kobayashi and T.Maskawa, Progr. Theor. Phys. 49 (1973) 652.
- 11) L.Maiani, Proc. of the Int. Symp. on Lepton and Photon Interactions at High Energies, Hamburg 1977.
- 12) L.Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
- 13) S.R.Sharpe et al. Phys. Lett. B192 (1987) 149.
- 14) G.Martinelli, Phys.Lett. 141B (1984) 395;
C.Bernard , A.Soni and T.Draper, Phys. Rev. D36 (1987) 363.
- 15) G.Martinelli and Y.C.Zhang, Phys. Lett. B123 (1983) 433.
- 16) APE Collaboration, private communication.
- 17) M.Bochicchio et al. Nucl.Phys. B262 (1985) 331;
L.Maiani and G.Martinelli, Phys.Lett. B178 (1986) 265.
- 18) G.Altarelli and P.J.Franzini, CERN TH 4914/87 to appear in Present Trends, Concepts and Instruments of Particle Physics, Symp. in Honour of M.Conversi, ed. by G.Baroni, L.Maiani, G.Salvini, Roma 1987.

K aM _P	.1515 .524	.1530 .443	.1545 .346	.1569 .0	(a)	(b)
$\langle \pi O_{LR} K \rangle a^4 10^3$	- 1.63(17)	- 1.28(15)	- 0.95(11)	- 0.43(06)		- 0.52(12)
$\langle \bar{K} \pi O_{LR} O \rangle a^4 10^3$	- 1.08(09)	- 0.96(12)	- 0.82(17)	- 0.63(22)		
$\langle \pi (O_c)_{LR} K \rangle a^4 10^3$	- 1.87(17)	- 1.53(17)	- 1.16(16)	- 0.61(13)		- 0.54(16)
$\langle \bar{K} \pi (O_c)_{LR} O \rangle a^4 10^3$	- 1.63(13)	- 1.31(15)	- 0.98(19)	- 0.48(19)		

Table 1

Matrix elements of O_{LR} and $(O_c)_{LR}$ between K - π and $\bar{K}\pi$ -0 states, versus the pseudoscalar mass M_P (or the hopping parameter, K). The last two column give two determinations of the values extrapolated to the chiral limit ($K=K_c$, $M_P=0$), see text. External particles have vanishing space momentum. $\beta=6.0$, 30 confs. Matrix elements are given in units of $10^{-3} a^{-4}$, $a \approx (2.2 \text{ GeV})^{-1}$ is the lattice spacing.

K aM _P	.1515 .524	.1530 .443	.1545 .346	.1569 .0	(a)	(b)
$B_{LR}(K \rightarrow \pi)$	1.10(4)	1.10(4)	1.10(2)	1.0(1)		1.0(1)
$B_{LR}(\bar{K}\pi \rightarrow 0)$	0.73(1)	0.79(2)	0.91(6)	1.0(1)		
$(B_c)_{LR}(K \rightarrow \pi)$	0.95(1)	0.95(2)	0.96(1)	0.98(4)		0.95(2)
$(B_c)_{LR}(\bar{K}\pi \rightarrow 0)$	0.83(2)	0.82(3)	0.82(5)	0.80(6)		

Table 2

B-parameters for O_{LR} and $(O_c)_{LR}$, from K - π and $\bar{K}\pi$ -0 matrix elements, versus the pseudoscalar mass (or the hopping parameter, K). The last two columns give two different determinations of the values extrapolated to the chiral limit ($K=K_c$, $M_P=0$), see text. External particles have vanishing space momentum. $\beta=6.0$, 30 confs, $a^{-1} \simeq 2.2 \text{ GeV}$.

K aM _p	.1515 .523	.1530 .436	.1545 .327		.1565 .0	
				(a)	(b)	(c)
$\langle \pi^- \pi^+ O_{LR} K^0 \rangle a^3 10^3$	4(1)	3.7(4)	2(1)	0(2)	1(1)	4(1)
$\langle \pi^- O_{LR} \pi^- K^0 \rangle a^3 10^3$	4(1)	4(1)	5(1)	5(2)		
	6(1)	6(2)	4(2)	3(2)	3(2)	5(5)
$\langle \pi^- \bar{K}^0 O_{LR} \pi^- \rangle a^3 10^3$	62(9)	46(9)	30(8)	9(5)		
$\langle \pi^- \pi^+ (O_c)_{LR} K^0 \rangle a^3 10^3$	11(2)	8(2)	4(1)	0(1)	1(1)	4(2)
$\langle \pi^- (O_c)_{LR} \pi^- K^0 \rangle a^3 10^3$	12(3)	11(3)	9(3)	8(1)		
	14(3)	13(4)	9(2)	6(2)	6(2)	9(5)
$\langle \pi^- \bar{K}^0 (O_c)_{LR} \pi^- \rangle a^3 10^3$	66(9)	49(10)	33(8)	11(6)		

Table 3

Matrix elements of O_{LR} and $(O_c)_{LR}$ from four point correlation functions, versus the pseudoscalar mass M_p (or the hopping parameter, K). Lines 2 and 3 (6 and 7) give different determinations of the same matrix element, obtained from the dependence of the correlation function with respect to the time of the kaon or of the pion source, see text. The last three columns list the values extrapolated to the chiral limit ($K=K_c$, $M_p=0$). In (c) the points at $K=0.1545$ have been omitted, see text. External particles have vanishing space momentum. $\beta=6.0$, 15 confs. Matrix elements are purely imaginary; we give here the imaginary part in units of $10^{-3} a^{-3}$, $a \simeq (2.2 \text{ GeV})^{-1}$ is the lattice spacing.

Figure Captions

Fig.1 Matrix element of LR operators for $K \rightarrow \pi$ and $\bar{K} \pi \rightarrow 0$ transitions vs $\pm M_p^2$, respectively (M_p is the pseudoscalar meson mass) in lattice units. (a) refers to $\hat{O}_{LR}$ and (b) to $(\hat{O}_C)_{LR}$. (x) results from the 15 CINECA configurations; ($\blacklozenge$) results for the 15 Rutherford configurations; ($\blacksquare$) combined average. $(k \bullet p)$ is the scalar product of the kaon and pion momenta. $(k \bullet p) = -M_p^2$ in the time-like region and $(k \bullet p) = +M_p^2$ in the space-like region.

Fig.2 Quark diagrams for the correlation function of $\hat{O}_{LR}$, see eq.(1) of text, with three pseudoscalar sources. The diagrams for $(\hat{O}_C)_{LR}$, see text, are the same except for colour matrices in the vertices.

Fig.3 Comparison of the $K \rightarrow \pi$ and $\bar{K} \pi \rightarrow 0$ matrix elements of $\hat{O}_{LR}$ (a) and $(\hat{O}_C)_{LR}$ (b) with the analogous matrix elements obtained from the four point correlation functions, after integration over the time of the kaon source. The agreement of the two sets of points is a test of the Ward-identity eq.(33) of text. ($\blacksquare$) Matrix elements of $\hat{O}_{LR}$ or $(\hat{O}_C)_{LR}$; (x) prediction from eq.(33). $(k \bullet p)$ is the scalar product of the kaon and pion momenta. $(k \bullet p) = -M_p^2$ in the time-like region and $(k \bullet p) = +M_p^2$ in the space-like region.

Fig. 1a

Fig.1b

Fig. 2

Fig. 3a

Fig. 3b