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Z-boson pair production via gluon fusion

E. W. N. Glover

J.J. van der Bij

CERN, CH-1211 Geneva 23, Switzerland

ABSTRACT

We study the coupling of two gluons with two Z -bosons via a quark loop keeping the full dependence on the quark mass. We discuss the structure of the $ggZZ$ polarisation tensor and give explicit formulas for the helicity amplitudes. The results are used to study the production of Z -boson pairs at hadron colliders both with and without the presence of a Higgs boson. Invariant mass, transverse momentum and rapidity distributions of the produced Z -bosons are shown. The Z -bosons from the gluon fusion process are produced somewhat more centrally than those from the quark-antiquark background. The continuum gluon fusion process, $gg \rightarrow ZZ$, is important, (60 – 70%), but never dominant over the $q\bar{q} \rightarrow ZZ$ process at the SSC. The Higgs boson signal and the continuum background interfere destructively at large $\hat{s}$ and this leads to a reduction in the cross section at large invariant mass and transverse momentum.

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I. Introduction.

Although the standard $SU(2)_L \times U(1)_Y$ electroweak gauge theory successfully explains all current data, certain parts of the theory have not been tested experimentally. The most important part of these is the mechanism of spontaneous symmetry breaking. Since in the standard model spontaneous symmetry breaking is due to the Higgs sector, the search for a Higgs particle is one of the main tasks for future colliders.

If the mass of the Higgs boson, m_H , is below 80 GeV then the process $e^+e^- \rightarrow ZH$ provides a signal at LEP200 [1]. For larger values of m_H one has to consider high energy hadron colliders such as the LHC (pp collisions at $\sqrt{s} = 16$ TeV) or the SSC (pp collisions at $\sqrt{s} = 40$ TeV). If the Higgs lies below the WW threshold then it will mainly decay into the heaviest quarks available, $H \rightarrow t\bar{t}$ if $m_H > 2m_t$ and $H \rightarrow b\bar{b}$ otherwise. In this case there is an overwhelming direct QCD background, which dominates the signal. In this mass region the Higgs is therefore very difficult to find, even though one could try to use some rare decays as a signal. Rare decays already considered in the literature include, for example, $H \rightarrow \tau^+\tau^-$ [2-5], $H \rightarrow \gamma\gamma$ [2-9], $H \rightarrow Z\gamma$ [4,5,10], $H \rightarrow ZZ^*$ [4,5,11,12] or $H \rightarrow WW^*$ [4,11,12,13]. All of these signals are at best marginal [5]. The situation is much better for heavy Higgs bosons ($m_H > 2m_W$). In this case the Higgs decays mainly into vector boson pairs, W^+W^- or ZZ [14]. A clear signal for the Higgs then consists of a peak in the invariant mass spectrum of the produced vector bosons. The double leptonic decay of the Z -boson, $H \rightarrow ZZ \rightarrow l^+l^-l'^+l'^-$, leads to a particularly clean signal [15]. Furthermore in principle by studying the polar angle of the decay leptons relative to the Z -boson, one can use the fact that the Higgs decays mainly into longitudinally polarised vector bosons to enhance the signal. In this paper we will only be concerned with the decay into Z -bosons and will ignore the subsequent weak decay.

The main background to detecting a heavy Higgs in the ZZ decay mode at hadron machines is the continuum production of Z -boson pairs. The most direct source of continuum vector boson pairs is via $q\bar{q}$ annihilation [16]. Two other sources for continuum vector boson pair production are weak boson fusion [17] and gluon fusion [18-21]. In weak boson fusion the incoming quarks radiate two vector bosons which subsequently scatter off each other. This process is mainly of interest as a source of Higgs bosons, the Higgs appearing as a s -channel resonance. Away from the Higgs peak weak boson fusion is only a small fraction of the basic $q\bar{q} \rightarrow ZZ$ process [17].

In the gluon fusion process, $gg \rightarrow ZZ$, the Z -bosons are produced via the box and triangle quark loops shown in fig. 1. Because of the loop this process is suppressed by a factor α_s^2 . Nonetheless it can be an important source of Z -bosons at hadron supercolliders because the gluon-gluon luminosity is much higher than the $q\bar{q}$ luminosity at small x . Gluon fusion is also a major production mechanism for the Higgs boson, because the Higgs couples to the massive top quark inside the loop. Except for the Higgs signal itself, the gluonic production of Z -bosons has not been widely studied. In ref. [18] a rough estimate for the background to longitudinal Z -boson production, $gg \rightarrow Z_L Z_L$, has been given. A nu-

merical estimate for the background process, $gg \rightarrow ZZ$, using Veltman's FORMF program has been given in ref. [19]. The results indicate that the gluon fusion process becomes dominant over the $q\bar{q}$ process at low invariant masses for the Z -boson pair. Subsequently, [20], the interference effects between the Higgs triangle graph and the box graphs were studied. A constructive interference between the Higgs graphs and box graphs was found at large invariant masses. In ref. [21] we presented some results of our analysis, which disagrees with ref. [20]. In particular, the interference between the Higgs graph and the box diagrams is constrained to be destructive by unitarity.

In this paper we give the full details of our calculation. We present analytic results for the parton level cross section $gg \rightarrow ZZ$. The form of the $ggZZ$ polarisation tensor is discussed in section 2. We used the reduction algorithm of ref. [22] to express the results in terms of the set of basic scalar loop integrals described in ref. [23]. These integrals, and their expression in terms of complex Spence functions, are given in appendix A. We use the full polarisation tensor to derive the eight independent helicity amplitudes in section 3. In order to avoid some of the numerical problems noted in refs. [11,12], we explicitly expand the integrals in the limit that the mass of the quark in the loop is smaller than all other quantities,

$$m_q^2 \ll p_{TZ}^2, m_Z^2, \hat{s}, |\hat{t}|, |\hat{u}|.$$

These expansions are given in appendix B.

In section 4 we make a numerical comparison of the $gg \rightarrow ZZ$ process with $q\bar{q} \rightarrow ZZ$ in the absence of the Higgs boson. We show the invariant mass, rapidity and transverse momentum distributions of the Z -bosons. At small invariant masses, where the cross section is largest, the gluon process is quite important (60-70%) at the SSC, but not dominant over the $q\bar{q}$ process. At the LHC the gluon process is less important (35-45%) due to the diminishing of the gluon structure function at larger x . The invariant mass and transverse momentum distributions for the two processes have a similar shape, however, the gluon fusion process produces the Z -bosons somewhat more centrally in rapidity than the $q\bar{q}$ process.

In section 5 we discuss the effects of the Higgs boson. The Higgs shows up as a peak in the invariant mass spectrum of the Z -bosons. At small invariant masses the Higgs interferes constructively with the $gg \rightarrow ZZ$ box diagrams while at large invariant mass the interference is destructive. The Z -bosons produced by the Higgs are longitudinally polarised, while those produced by either quark- antiquark annihilation or the non-Higgs gluon fusion graphs are mainly transversely polarised. There is a characteristic Jacobian peak in the transverse momentum of the Z -boson produced by Higgs boson decay which may be used to select Higgs boson events [15]. Finally, section 6 contains our conclusions.

II. The $ggZZ$ polarisation tensor.

The coupling of the Z -boson to the quarks is given by,

$$g_Z Z^\mu \bar{\psi}(v_q + a_q \gamma_5) \gamma_\mu \psi, \quad (2.1)$$

with,

$$v_u = \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W, \quad a_u = \frac{1}{4}, \quad (2.2)$$

$$v_d = -\frac{1}{4} + \frac{1}{3} \sin^2 \theta_W, \quad a_d = -\frac{1}{4}, \quad (2.3)$$

$$g_Z = \frac{e}{\sin \theta_W \cos \theta_W}, \quad (2.4)$$

θ_W being the Weinberg angle.

The $ggZZ$ coupling is generated by the box graphs shown in fig. 1. From charge conjugation invariance it is clear that there is no contribution proportional to the product $a_q v_q$. We may therefore write the contribution of a quark loop to the $ggZZ$ polarisation tensor as,

$$P^{\mu\nu\rho\sigma} = \frac{g_s^2 g_Z^2}{(2\pi)^4} \frac{\delta^{\alpha\beta}}{2} \{v_q^2 P_v^{\mu\nu\rho\sigma} + a_q^2 P_a^{\mu\nu\rho\sigma}\} \quad (2.5)$$

g_s is the strong coupling constant and α and β are the colour indices of the two gluons. We denote the momenta and Lorentz indices of the gluons by p_1^μ and p_2^ν while those of the Z -bosons are p_3^ρ and p_4^σ . All momenta are taken to be outgoing. We define,

$$\begin{aligned} s &= (p_1 + p_2)^2, \quad t = (p_2 + p_3)^2, \quad u = (p_1 + p_3)^2, \\ s_2 &= s - 2m_Z^2, \quad s_4 = s - 4m_Z^2, \quad t_1 = t - m_Z^2, \quad u_1 = u - m_Z^2, \\ Y &= ut - m_Z^4 = s \cdot p_{TZ}^2. \end{aligned} \quad (2.6)$$

The fact that the gluons are on shell yields the conditions,

$$p_1^\mu = p_2^\nu = p_1 \cdot p_1 = p_2 \cdot p_2 = 0. \quad (2.7)$$

Gauge invariance tells us that the $ggZZ$ polarisation tensor $P^{\mu\nu\rho\sigma}$ is transversal to the gluon momenta,

$$p_1^\mu P^{\mu\nu\rho\sigma} = p_2^\nu P^{\mu\nu\rho\sigma} = 0. \quad (2.8)$$

Similarly, the vector part of the polarisation tensor is also transversal to the Z -boson momenta,

$$p_3^\rho P^{\mu\nu\rho\sigma} = p_4^\sigma P^{\mu\nu\rho\sigma} = 0. \quad (2.9)$$

Finally we note that in the limit, $m_q = 0$, the axial vector current is also conserved and so,

$$p_3^\rho P_a^{\mu\nu\rho\sigma} = p_4^\sigma P_a^{\mu\nu\rho\sigma} = 0 \quad (m_q = 0). \quad (2.10)$$

The tensor $P_y^{\mu\nu\rho\sigma}$ is actually the same as the four photon tensor in QED and has been thoroughly studied in the literature. For an exhaustive review of this tensor we refer the reader to ref. [24], with which we are in complete agreement. We note that both here and in ref. [24] the factor -1 for the fermion loop has been left out in the definition of the tensor.

The axial part $P_a^{\mu\nu\rho\sigma}$ has not been discussed in the literature. The complete axial piece of the tensor is quite long, however the difference, $P_a - P_v$, is quite simple,

$$\begin{aligned}
P_a^{\mu\nu\rho\sigma} - P_v^{\mu\nu\rho\sigma} = & \\
& A_1(s, t, u) \delta^{\rho\sigma} (\delta^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}) \\
& + A_2(s, t, u) \delta^{\rho\sigma} \{ \delta^{\mu\nu} + \frac{2}{p_{TZ}^2} p_3^\mu p_3^\nu + \frac{m_Z^2}{p_{TZ}^2 p_1 \cdot p_2} p_1^\nu p_2^\mu - \frac{2p_1 \cdot p_3}{p_{TZ}^2 p_1 \cdot p_2} p_2^\mu p_3^\nu - \frac{2p_2 \cdot p_3}{p_{TZ}^2 p_1 \cdot p_2} p_1^\nu p_3^\mu \} \\
& + A_3(s, t, u) \{ \delta^{\mu\sigma} \delta^{\nu\rho} + \frac{\delta^{\mu\nu} p_1^\sigma p_2^\rho}{p_1 \cdot p_2} - \frac{\delta^{\nu\rho} p_1^\sigma p_2^\mu}{p_1 \cdot p_2} - \frac{\delta^{\mu\sigma} p_1^\nu p_2^\rho}{p_1 \cdot p_2} \} \\
& + A_4(s, t, u) \{ \delta^{\mu\rho} \delta^{\nu\sigma} + \frac{\delta^{\mu\nu} p_1^\rho p_2^\sigma}{p_1 \cdot p_2} - \frac{\delta^{\nu\sigma} p_1^\rho p_2^\mu}{p_1 \cdot p_2} - \frac{\delta^{\mu\rho} p_1^\nu p_2^\sigma}{p_1 \cdot p_2} \} \\
& + A_5(s, t, u) \frac{1}{p_1 \cdot p_2} \{ \delta^{\mu\sigma} p_1^\rho p_3^\nu - \delta^{\mu\rho} p_1^\sigma p_3^\nu + \delta^{\nu\sigma} p_2^\rho p_3^\mu - \delta^{\nu\rho} p_2^\sigma p_3^\mu \\
& \quad + \frac{p_2 \cdot p_3}{p_1 \cdot p_2} \delta^{\mu\rho} p_1^\nu p_1^\sigma - \frac{p_2 \cdot p_3}{p_1 \cdot p_2} \delta^{\mu\sigma} p_1^\nu p_1^\rho + \frac{p_1 \cdot p_3}{p_1 \cdot p_2} \delta^{\nu\rho} p_2^\mu p_2^\sigma - \frac{p_1 \cdot p_3}{p_1 \cdot p_2} \delta^{\nu\sigma} p_2^\mu p_2^\rho \}
\end{aligned} \tag{2.11}$$

The five formfactors, A_i , are given by,

$$\begin{aligned}
A_1(s, t, u) = & \\
\frac{8m_q^2}{s} \{ 4u_1 C_1(u) + 4t_1 C_1(t) - 2Y D(u, t) + s(s - 4m_q^2) [D(s, u) + D(s, t) + D(u, t)] \} & \tag{2.12}
\end{aligned}$$

$$\begin{aligned}
A_2(s, t, u) = & \frac{16m_q^2}{Y} \{ 2uu_1 C_1(u) + 2tt_1 C_1(t) - ss_2 C(s) + (2Y - ss_4) C_2(s) \\
& - su^2 D(s, u) - st^2 D(s, t) - 2m_q^2 Y [D(s, u) + D(s, t) + D(u, t)] \} & \tag{2.13}
\end{aligned}$$

$$A_3(s, t, u) = 8m_q^2 s [D(s, t) - D(s, u) - D(u, t)] \tag{2.14}$$

$$A_4(s, t, u) = A_3(s, u, t), \tag{2.15}$$

$$\begin{aligned}
A_5(s, t, u) = & \\
\frac{8m_q^2 s}{Y} \{ 2u_1 C_1(u) + 2t_1 C_1(t) + 2s C(s) - st D(s, t) - su D(s, u) - Y D(u, t) \} & \tag{2.16}
\end{aligned}$$

The C and D functions are the basic scalar three- and four-point integrals described in appendix A.

III. The $gg \rightarrow ZZ$ helicity amplitudes.

Using the $gg \rightarrow ZZ$ polarisation tensor we can construct the helicity amplitudes. We work in the rest frame of the p_1 and p_2 system. The momenta are given in this frame by,

$$\begin{aligned} p_1^\mu &= (0, 0, -p; -p), \\ p_2^\mu &= (0, 0, p; -p), \\ p_3^\mu &= (q \sin \theta, 0, q \cos \theta; p), \\ p_4^\mu &= (-q \sin \theta, 0, -q \cos \theta; p). \end{aligned} \tag{3.1}$$

The helicity vectors are,

$$\begin{aligned} e_1^+ &= e_2^- = \frac{1}{\sqrt{2}} (-i, 1, 0; 0), \\ e_1^- &= e_2^+ = \frac{1}{\sqrt{2}} (i, 1, 0; 0), \\ e_3^+ &= e_4^- = \frac{1}{\sqrt{2}} (i \cos \theta, 1, -i \sin \theta; 0), \\ e_3^- &= e_4^+ = \frac{1}{\sqrt{2}} (-i \cos \theta, 1, i \sin \theta; 0), \\ e_3^0 &= \frac{1}{m_Z} (p \sin \theta, 0, p \cos \theta; q), \\ e_4^0 &= \frac{1}{m_Z} (-p \sin \theta, 0, -p \cos \theta; q). \end{aligned} \tag{3.2}$$

$e_i^\pm$ is the $\pm$ polarisation vector of particle i and e_i^0 is the longitudinal polarisation of the Z -bosons.

There are twenty four helicity amplitudes which are related by parity

$$\mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \mathcal{M}_{-\lambda_1 -\lambda_2 -\lambda_3 -\lambda_4}^* \tag{3.3}$$

where λ_i denotes the polarisation of particle i . The remaining twelve amplitudes are further related,

$$\begin{aligned} \mathcal{M}_{++--}(\beta) &= \mathcal{M}_{++++}(-\beta), \\ \mathcal{M}_{+++-}(\beta) &= \mathcal{M}_{++-+}(\beta), \\ \mathcal{M}_{+---}(\beta) &= \mathcal{M}_{-++-}(\beta), \\ \mathcal{M}_{+--+}(\beta) &= \mathcal{M}_{-+-+}(-\beta), \\ \mathcal{M}_{++00}(\beta) &= \mathcal{M}_{++0+}(\beta) = \mathcal{M}_{++-0}(-\beta) = \mathcal{M}_{++0-}(-\beta), \\ \mathcal{M}_{+-00}(\beta) &= -\mathcal{M}_{+-0+}(-\beta) = \mathcal{M}_{+-0-}(-\beta) = -\mathcal{M}_{+-0+}(\beta), \end{aligned} \tag{3.4}$$

leaving eight independent amplitudes. To make the structure of the helicity amplitudes manifest, we write the helicity amplitude for a single quark of mass m_q as a sum over the different coupling combinations,

$$\mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = (v_q^2 + a_q^2) \mathcal{A}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} + (v_q^2 - a_q^2) \mathcal{B}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} + \frac{a_q^2 s}{m_Z^2} \mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}. \quad (3.5)$$

Both the $\mathcal{B}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ and $\mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ subamplitudes are proportional to the quark mass, and so vanish for massless quarks. The factor s/m_Z^2 multiplying the $\mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ subamplitude arises from the fact that the polarisation vector for longitudinally polarised Z -bosons is given by,

$$\epsilon_0^\mu = \frac{p_Z^\mu}{m_Z} + O\left(\frac{m_Z}{p_Z^0}\right). \quad (3.6)$$

Since the axial current is not conserved for massive quarks, the first term in eq. (3.6) leads to an enhancement of $\sqrt{s}/m_Z$ in the amplitude. Consequently, if neither Z -boson is longitudinally polarised the subamplitude $\mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ is zero.

We use the notation of eq. (2.6), and define the quantities,

$$\beta = \sqrt{1 - \frac{4m_Z^2}{s}}, \quad (3.7)$$

$$\Delta = \sqrt{\frac{-2m_Z^2}{sY}}. \quad (3.8)$$

As in eq. (2.5) we extract an overall factor,

$$\frac{g_s^2 g_Z^2}{(2\pi)^4} \frac{\delta^{\alpha\beta}}{2}, \quad (3.9)$$

in the definition of the helicity amplitudes.

We find that the non-zero subamplitudes, $\mathcal{A}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$, $\mathcal{B}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ and $\mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ are,

$$\begin{aligned} \mathcal{A}_{++++}(s, t, u, \beta) = & \frac{8[sY\beta + m_Z^2(2Y - ss_4)]}{s_4 u_1 t_1} \\ & + \frac{4(s_2 + \beta s)}{s_4} \left(\left(\frac{2m_Z^4}{t_1^2} - 1 - \frac{2t}{s} \right) B_1(t) + \left(\frac{2m_Z^4}{u_1^2} - 1 - \frac{2u}{s} \right) B_1(u) \right) \\ & - \frac{2(2Y - ss_4)(s_2 + \beta s)}{s^2 s_4} E(u, t) \\ & + \frac{4m_q^2}{s_4} \left(4(s\beta + 2m_Z^2) C(s) + \frac{(2Y - ss_4)(s_2 + \beta s)}{s} D(u, t) \right. \\ & \quad \left. + 2m_Z^2(s_2 + \beta s) \left[\frac{2}{u_1} C_1(u) + \frac{2}{t_1} C_1(t) + D(s, t) + D(s, u) \right] \right) \end{aligned} \quad (3.10)$$

$$\begin{aligned}
\mathcal{A}_{+-+-}(s, t, u, \beta) = & \\
& \frac{8}{s_4 u_1 t_1} \left(s_2 Y + m_Z^2 s \beta (t - u) \right) - \frac{8 s_2}{s_4} B_1(s) \\
& + \frac{4}{s_4} \left\{ \left([t(t s_2 + 2 m_Z^2 s_4) + m_Z^4 s_2] + \beta s t_1 (3t + m_Z^2) \right) \frac{B_1(t)}{t_1^2} + (t \leftrightarrow u, \beta \leftrightarrow -\beta) \right\} \\
& + \frac{4s}{s_4} \left(s_2 C(s) + s_4 C_2(s) \right) + \frac{64 \beta m_Z^6}{s_4^2} \left(2C_1(t) - 2C_1(u) + t_1 D(s, t) - u_1 D(s, u) \right) \\
& - \frac{4}{s_4} \left\{ \left([(t^2 + m_Z^4)(3s + 2m_Z^2) - ts(4s - 10m_Z^2)] \right. \right. \\
& \quad \left. \left. - \beta [3t^2 s + t(4s^2 + 2sm_Z^2 + 8m_Z^4) - m_Z^4(5s + 8m_Z^2)] \right) \frac{C_1(t)}{t_1} + (t \leftrightarrow u, \beta \leftrightarrow -\beta) \right\} \\
& + \frac{1}{s_4} \left\{ \left(-[8t^2 s - 6t s s_2 + 8s m_Z^4] + \beta [16t_1 m_Z^4 + 10t s^2 - 8m_Z^4 s] \right) D(s, t) + (t \leftrightarrow u, \beta \leftrightarrow -\beta) \right\} \\
& + \frac{4s}{s_4^2 Y} \left(s_4 + \beta(t - u) \right) \left(2m_Z^4 Q_2(s, t, u) + s_4 Q_3(s, t, u) \right) \\
& + \frac{4s}{s_4 Y^2} \left[s \left(s_4 + \beta(t - u) \right) - 2Y \right] Q_4(s, t, u) \\
& + \frac{4m_q^2}{s_4} \left\{ \left[\left(\frac{8m_Z^2}{s} (s s_4 - Y) + 4t_1 (s_4 + \beta s) \right) \frac{C_1(t)}{t_1} + (t \leftrightarrow u, \beta \leftrightarrow -\beta) \right] \right. \\
& \quad \left. + \frac{2s_2 Y}{s} D(u, t) \right. \\
& \quad \left. + \left([s s_2 (1 - 4\beta) - 2m_Z^2 (s + 2m_Z^2) + 3s(t - u)\beta] D(s, t) + (t \leftrightarrow u, \beta \leftrightarrow -\beta) \right) \right\} \\
& + \frac{8m_q^2 s}{s_4 Y} \left(s_4 + \beta(t - u) \right) \left(Q(s, t, u) + t^2 D(s, t) + u^2 D(s, u) \right) \\
& + 16m_q^4 \left(D(s, t) + D(u, t) + D(s, u) \right)
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\mathcal{A}_{+++-}(s, t, u, \beta) = & \\
& -8 \frac{s_2 Y}{s_4 u_1 t_1} + \frac{8m_Z^2}{s_4} \left(\left(1 + \frac{2t}{s} - \frac{s_4}{t_1} - \frac{m_Z^2 s_2}{t_1^2}\right) B_1(t) + \left(1 + \frac{2u}{s} - \frac{s_4}{u_1} - \frac{m_Z^2 s_2}{u_1^2}\right) B_1(u) \right) \\
& + \frac{8m_Z^2 Y}{s^2 s_4} E(u, t) \\
& - m_q^2 \left(\frac{16m_Z^4}{s_4} \left[\frac{2}{t_1} C_1(t) + \frac{2}{u_1} C_1(u) + D(s, u) + D(s, t) \right] + \frac{16s_2}{s_4} C(s) + \frac{16m_Z^2 Y}{s s_4} D(u, t) \right. \\
& \quad \left. + \frac{8}{s} E(u, t) - \frac{8s Q(s, t, u)}{Y} \right) \\
& + 16m_q^4 (D(s, u) + D(s, t) + D(u, t))
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\mathcal{A}_{+-++}(s, t, u, \beta) = & \\
& -\frac{8s_2 Y}{s_4 u_1 t_1} + \frac{8m_Z^2}{s_4} \left(\frac{2m_Z^2 t + Y}{t_1^2} B_1(t) + \frac{2m_Z^2 u + Y}{u_1^2} B_1(u) \right) \\
& + \frac{8s_2 m_Z^2}{s_4} \left[\frac{t}{t_1} C_1(t) + \frac{u}{u_1} C_1(u) \right] - \frac{4m_Z^2}{s_4} (E(s, t) + E(s, u)) + \frac{4s s_2 m_Z^2 Q(s, t, u)}{s_4 Y} \\
& + m_q^2 \left(\frac{16s_2 Y}{s s_4} \left[\frac{C_1(t)}{t_1} + \frac{C_1(u)}{u_1} \right] \right. \\
& \quad \left. + \frac{8m_Z^2}{s s_4} \{ s s_2 [D(s, u) + D(s, t) + D(u, t)] - 2u_1 t_1 D(u, t) \} \right)
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\mathcal{A}_{+++0}(s, t, u, \beta)/\Delta = & \\
(1 + \beta) \{ & + \frac{4(t-u)Y s}{s_4 u_1 t_1} \frac{4Y}{s_4} \left(\frac{2t t_1 - m_Z^2 s}{t_1^2} B_1(t) - \frac{2u u_1 - m_Z^2 s}{u_1^2} B_1(u) \right) \\
& - \frac{2(t-u)Y}{s s_4} E(u, t) \\
& + \frac{m_q^2}{s_4} \left(+8s \left(\frac{2m_Z^2 u_1 - s u}{u_1} C_1(u) - \frac{2m_Z^2 t_1 - s t}{t_1} C_1(t) \right) \right. \\
& \quad \left. - \frac{4\beta(t-u)}{(1+\beta)} E(u, t) + 4Y(t-u) D(u, t) \right) \}
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\mathcal{A}_{+-+0}(s, t, u, \beta)/\Delta = & \\
& \frac{4sY}{u_1 t_1 s_4} (s\beta + u - t) + \frac{4}{s_4} (s\beta + u - t) \left(\frac{m_Z^2 s u}{u_1^2} B_1(u) + \frac{m_Z^2 s t}{t_1^2} B_1(t) - s B_1(s) \right) \\
& + \frac{4s}{s_4} \left((1 + \beta)(u + m_Z^2) B_1(u) - (1 - \beta)(t + m_Z^2) B_1(t) + 2s\beta B_1(s) \right) \\
& - \frac{4s}{s_4} (s\beta + t - u) \left(\frac{t^2}{t_1} C_1(t) + \frac{u^2}{u_1} C_1(u) + (s - 3m_Z^2) C_2(s) \right) - \frac{8m_Z^2 s s_2}{s_4} \beta C_2(s) \\
& + \frac{2s}{s_4} \left((t_1(1 + \beta) + 2m_Z^2 \beta) E(s, t) - (u_1(1 - \beta) - 2m_Z^2 \beta) E(s, u) \right) \\
& - \frac{2s^2}{s_4 Y} (s\beta + t - u) \left((u + t) Q(s, t, u) + ut Q_1(s, t, u) \right) \\
& + m_q^2 \left(\frac{8s}{s_4} \{ s\beta C(s) + (s\beta + u - t) \left[\frac{t}{t_1} C_1(t) + \frac{u}{u_1} C_1(u) \right] \} \right. \\
& \quad + \frac{4}{s_4} \{ s((st - 2m_Z^2 t_1) + 2t_1^2 \beta) D(s, t) - s((su - 2m_Z^2 u_1) - 2u_1^2 \beta) D(s, u) \\
& \quad \left. + (2s\beta + u - t) E(u, t) \} \right)
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\mathcal{A}_{++00}(s, t, u, \beta) = & \\
& - \frac{32m_Z^2 Y}{s_4 u_1 t_1} + \frac{16m_Z^2}{s_4} \left(\left(1 + \frac{2t}{s} - \frac{2m_Z^4}{t_1^2}\right) B_1(t) + \left(1 + \frac{2u}{s} - \frac{2m_Z^4}{u_1^2}\right) B_1(u) \right) \\
& + \frac{16m_Z^2 Y}{s^2 s_4} E(u, t) \\
& - \frac{16m_Z^2 m_q^2}{s s_4} \left(4s C(s) + 4s m_Z^2 \left[\frac{C_1(t)}{t_1} + \frac{C_1(u)}{u_1} \right] + s s_2 [D(s, u) + D(s, t)] + 2Y D(u, t) \right)
\end{aligned} \tag{3.16}$$

$$\begin{aligned}
\mathcal{A}_{+-00}(s, t, u, \beta) = & \\
& + \frac{32m_Z^2 Y}{s_4 u_1 t_1} + \frac{16m_Z^2}{s_4} \left(\left(1 + \frac{2t m_Z^2}{t_1^2}\right) B_1(t) + \left(1 + \frac{2u m_Z^2}{u_1^2}\right) B_1(u) \right) \\
& + \frac{8m_Z^2}{s_4} \left(E(s, u) + E(s, t) - 2s_2 \left[\frac{u}{u_1} C_1(u) + \frac{t}{t_1} C_1(t) \right] - \frac{s s_2 Q(s, t, u)}{Y} \right) \\
& - \frac{16m_q^2 m_Z^2}{s s_4} \left(2Y \left[\frac{2}{u_1} C_1(u) + \frac{2}{t_1} C_1(t) + D(u, t) \right] + s_2 s [D(s, u) + D(s, t)] \right)
\end{aligned} \tag{3.17}$$

$$\begin{aligned}
B_{++++}(s, t, u, \beta) = & \\
& -\frac{4m_q^2}{ss_4} \left(4\beta s^2 C(s) + s^2(s_4 + \beta s_2) [D(s, u) + D(s, t)] + 2(s\beta + s_4) E(u, t) \right) \\
& + 16m_q^4 \left(D(s, u) + D(s, t) + D(u, t) \right)
\end{aligned} \tag{3.18}$$

$$\begin{aligned}
B_{+-++}(s, t, u, \beta) = & \\
& 8m_q^2 \left(\frac{2t}{t_1} C_1(t) + \frac{2u}{u_1} C_1(u) - 2 C_2(s) + \frac{Y}{s_4} D(u, t) + \frac{s Q(s, t, u)}{Y} \right) \\
& + 16m_q^4 \left(D(u, t) + D(s, u) + D(s, t) \right)
\end{aligned} \tag{3.19}$$

$$B_{+--+}(s, t, u, \beta) = \frac{4m_q^2}{s_4} \left(s[s_4 - \beta(t - u)] - 2Y \right) D(u, t) \tag{3.20}$$

$$\begin{aligned}
B_{+++0}(s, t, u, \beta)/\Delta = & \\
& \frac{4m_q^2 s}{s_4} \left(2(t - u) C(s) - (su - 2m_Z^2 u_1) D(s, u) + (st - 2m_Z^2 t_1) D(s, t) \right)
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
C_{+++0}(s, t, u, \beta)/\Delta = & \\
& \frac{2m_q^2 (1 + \beta)}{s_4} \left(2s(t - u) C(s) + s(2m_Z^2 u_1 - su) D(s, u) - s(2m_Z^2 t_1 - st) D(s, t) \right. \\
& \left. + \beta(t - u) E(u, t) \right)
\end{aligned} \tag{3.22}$$

$$\begin{aligned}
C_{+-+0}(s, t, u, \beta)/\Delta = & \\
& \frac{2m_q^2}{s} (s\beta + u - t) \left(E(s, u) + E(s, t) - 2s_2 C_2(s) + \frac{(s + 4m_Z^2)Y}{s_4} D(u, t) \right)
\end{aligned} \tag{3.23}$$

$$C_{++00}(s, t, u, \beta) = -16m_q^2 \left(C(s) + \frac{m_Z^2}{s^2} E(u, t) \right) - 16m_q^4 \frac{s_2}{s} \left(D(s, u) + D(s, t) + D(u, t) \right) \tag{3.24}$$

$$\begin{aligned}
C_{+-00}(s, t, u, \beta) = & \\
& -\frac{8m_q^2}{s} \left(\frac{2s_2 u}{u_1} C_1(u) + \frac{2s_2 t}{t_1} C_1(t) - \frac{4m_Z^2 Y}{s_4} D(u, t) - E(s, u) - E(s, t) \right. \\
& \left. + \frac{ss_2 Q(s, t, u)}{Y} \right) \\
& -\frac{16m_q^4 s_2}{s} \left(D(u, t) + D(s, t) + D(s, u) \right)
\end{aligned} \tag{3.25}$$

The form of the above helicity amplitudes is rather complicated. In order to get some insight in the dependence on the top quark mass fig. 2 shows the squared matrix elements for right angle scattering for a fixed invariant mass as a function of m_t . In this graph we keep only the contribution of the top quark. We show curves where both of the outgoing Z -bosons are transversely polarised (TT), one transversely and one longitudinally polarised (TL) and both longitudinally polarised (LL). We sum over all contributing helicity amplitudes.

One notices that for very large top mass ($m_t \gg \sqrt{s}$) all amplitudes go to zero. This shows the decoupling of the top quark. At large enough values of m_t the only effects due to the top quark are pointlike interactions such as $F_{\mu\nu} F^{\mu\nu} Z_\alpha Z^\alpha$ which are suppressed by powers of $1/m_t^2$ or worse.

The TT contribution falls monotonically with increasing top quark mass. There is a change of behaviour at $m_t = \sqrt{s}/2$ and at $m_t = m_Z/2$ corresponding to the possibility of producing real top quarks in the loop in either the s -channel or at one of the $Zt\bar{t}$ vertices.

For small m_t the LL piece is very small compared to the TT part, since massless quarks couple mainly to transversely polarised gauge bosons. The cross section rises steeply as one enters the region $\sqrt{s} \gg m_t$ where the $\mathcal{M}_{++00}$ amplitude behaves as,

$$\frac{a_q^2 m_t^2}{m_Z^2} \log^2\left(\frac{s}{m_t^2}\right). \quad (3.26)$$

This growth is due entirely to the fact that the axial vector current is not conserved for massive quarks and the s/m_Z^2 enhancement of the $\mathcal{C}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ subamplitude which cancels the $1/s$ behaviour of the form factor. Finally, when m_t becomes larger than typical mass scales of the problem like m_Z the top quark decouples and the cross section goes to zero for very heavy top masses.

The TL piece behaves somewhat like the LL piece, but is always a small fraction of the total, being much smaller than the TT piece for light quarks and smaller than the LL piece for heavier quarks.

We also note that several of the subamplitudes, eqs (3.10)-(3.25), contain factors of $1/Y \sim 1/p_{TZ}^2$. This problem is particularly apparent in the $\mathcal{A}_{+-+}$ subamplitude which has terms $O(1/Y^2)$. These factors are artefacts of the method [22] employed to reduce the full diagram to a sum of scalar integrals. In principle there are precise cancellations between the various integrals which render the amplitudes finite as $p_{TZ} \rightarrow 0$. We have checked explicitly that this is so in the $m_q \rightarrow \infty$ limit. In practice there are numerical problems which may be avoided by either making a small p_{TZ} cut on the Z -bosons or by evaluating the scalar integrals to sufficient precision. We employ the former approach as is fully explained in the next section.

IV. Continuum Z-boson pair production.

The physical production rate of Z-bosons in proton collisions is given by integration over the gluon structure functions,

$$d\sigma(pp \rightarrow gg \rightarrow ZZ) = dx_1 dx_2 f_g(x_1, Q^2) f_g(x_2, Q^2) d\hat{\sigma}(\hat{s}) \quad (4.1)$$

with $\hat{s} = x_1 x_2 s$ and,

$$d\hat{\sigma}(\hat{s}) = \frac{\alpha_s^2(Q^2)\alpha_Z^2}{32\hat{s}} \sum_{\lambda_1 \dots \lambda_4 = \pm} \left(2 \sum_q |\mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \right) \times \frac{1}{2} \frac{d^3 p_Z}{2E_Z(2\pi)^3} \frac{d^3 p_{Z'}}{2E_{Z'}(2\pi)^3} \delta_4(p_Z + p_{Z'} - p_1 - p_2) \quad (4.2)$$

The factor $1/2$ for identical particle phase space has been written explicitly, as has a factor 2 for the color sum. The subprocess variables are denoted with a hat i.e., $\hat{s}$, $\hat{t}$ and $\hat{u}$. We use Duke and Owens structure functions [25] with $\Lambda = 0.2$ GeV evolved to a scale $Q^2 = \hat{s}/4$ and $\alpha_s = 12\pi/(23 \log(Q^2/\Lambda^2))$. For the electroweak parameters we choose $m_Z = 94.6$ GeV and $\sin^2 \theta_W = 0.23$. The various cross sections are now obtained by numerically integrating over phase space and structure functions.

As mentioned in section 3, there are severe numerical problems at small p_{TZ} especially when $m_q^2/\hat{s}$ is also small. To cure this problem, the contribution from the light quarks, u , d , s , c , and b , is evaluated using the explicit $m_q \rightarrow 0$ expansion of the scalar integrals given in appendix B. The matrix elements for light quarks are then well behaved over all phase space provided $p_{TZ} > 2$ GeV. The contribution from $p_{TZ} < 2$ GeV is negligible since the cross section vanishes as $p_{TZ} \rightarrow 0$. For massive quarks, $m_t \sim 40$ GeV, the numerical problems persist at large $\hat{s}$ and small p_{TZ} . To avoid these problems we make an invariant mass cut, $\sqrt{\hat{s}} < 2$ TeV. The contribution from the region with invariant mass, $\sqrt{\hat{s}} > 2$ TeV, is small since the gluon structure functions are small in this region. Furthermore, in the region, $2 \text{ GeV} < p_{TZ} < 10 \text{ GeV}$, we keep only the contribution of the light quarks. At small p_{TZ} the cross section is dominated by the light quarks and the error made by ignoring the top quark in this region is less than a few %. For $m_t \geq m_Z$ the numerical problems are much less severe.

To give some idea of the size of observable cross sections at the SSC($\sqrt{s} = 40$ TeV) and LHC($\sqrt{s} = 16$ TeV) we give the total cross section for Z-boson pair production with loose ($|y_Z| < 2.5$) and strict ($|y_Z| < 1.5$) rapidity cuts for some masses of the top quark in Table 1. We see that the cross sections become smaller as the top quark becomes heavier. This can be understood by considering the extreme cases $m_t = 0$ and $m_t = \infty$. In the last case the top quark decouples from the amplitudes and one is left with only the contribution of five light quarks. For light quarks the $\mathcal{B}$ and $\mathcal{C}$ subamplitudes vanish and the amplitudes have the following structure,

$$\mathcal{M} \sim \sum_q (v_q^2 + a_q^2). \quad (4.3)$$

The ratio of the cross sections for six and five light quarks is therefore,

$$\frac{\sigma(m_t = 0)}{\sigma(m_t = \infty)} = \frac{\left(\sum_{q=1}^6 (v_q^2 + a_q^2)\right)^2}{\left(\sum_{q=1}^5 (v_q^2 + a_q^2)\right)^2} \sim 1.37 \quad (4.4)$$

This ratio is insensitive to the precise value of $\sin^2 \theta_W$.

It is also clear that the gluon fusion process produces the Z -bosons somewhat more centrally than the $q\bar{q}$ process. This is made more explicit in fig. 3. where we show the rapidity distribution of the Z -bosons. At the SSC the point where $d\sigma/dy_Z$ has dropped to half its maximum at $y_Z = 0$ is at $y_Z = 3.8$ for the $q\bar{q}$ process and $y_Z = 3.0$ for the gluon process. At the LHC the corresponding numbers are $y_Z = 2.8$ and $y_Z = 2.2$. The shape of the rapidity distribution is essentially unaffected by the mass of the top quark.

We also see that the gluon fusion process is relatively more important at the SSC than at the LHC. At the SSC $\sigma(gg \rightarrow ZZ)$ is about 60-70% of $\sigma(q\bar{q} \rightarrow ZZ)$, while at the LHC it is only about 35-45%. This is a reflection of the fact that the gluon structure functions fall faster with increasing x than the quark structure functions. Going from the SSC to the LHC one loses about a factor two in the $q\bar{q}$ luminosity and about a factor four in the gg luminosity.

The precise ratio of the $q\bar{q}$ and gg process is uncertain because the structure functions are not well known at low values of x and because higher order corrections have not been calculated. Although the uncertainties in the quark and gluon structure functions are large, the uncertainties in Q^2 and Λ cancel to some extent in the ratio of the quark and gluon luminosities. Some higher order corrections are known for $q\bar{q} \rightarrow ZZ$ [26,27]. In particular the K -factor is given by [27],

$$K = 1 + \frac{8\pi}{9}\alpha_s, \quad (4.5)$$

indicating effects of $O(30\%)$. Since the corresponding calculation for $gg \rightarrow ZZ$ does not exist, we do not include this factor in our results. One might expect the radiative corrections to $gg \rightarrow ZZ$ to be quite important, since this process is of $O(\alpha_s^2)$ and therefore rather sensitive to the value of Q^2 .

In fig. 4 we show the invariant mass distribution of Z -boson pairs at the SSC and the LHC. To illustrate the effect of the top quark mass we show curves for two extreme cases for the top quark mass, $m_t = 40$ GeV and $m_t = 200$ GeV. One notices that at low invariant masses a smaller top quark mass leads to a larger cross section. This is in agreement with the argument given previously for the decoupling of a very heavy top quark for the total cross section. However, at large invariant masses the cross section becomes bigger if one increases the top mass. This is due to the growth of the longitudinal amplitudes, (see eq. (3.26)), caused by the $\hat{s}/m_Z^2$ enhancement of the longitudinal amplitudes due to the non-conservation of the axial vector current for massive

quarks, and which cancels the $1/\hat{s}$ behaviour of the form factor. Therefore increasing the top mass will increase the cross section. This argument applies obviously only when $\sqrt{\hat{s}} > 2m_t$, since for lower masses the top quark should be considered as heavy and should satisfy the decoupling theorem. Indeed we see from the graphs that the point where $d\sigma/dM_{ZZ}(m_t = 200 \text{ GeV}) \approx d\sigma/dM_{ZZ}(m_t = 40 \text{ GeV})$ is $M_{ZZ} \sim 550 \text{ GeV}$, which is fairly close to $2m_t = 400 \text{ GeV}$. Although the gluon fusion process is important at low invariant mass, we find that even at threshold the $q\bar{q}$ process is still the dominant one. In this respect we disagree with ref. [20], which finds $gg \rightarrow ZZ$ to be more important in this region.

The transverse momentum distribution shown in fig. 5 exhibits a similar behaviour to the invariant mass distribution. At low values of p_{TZ} lowering m_t increases the cross section while at higher p_{TZ} the enhancement of the longitudinal amplitudes leads to the opposite behaviour. The turn over point where $d\sigma/dp_T(m_t = 200 \text{ GeV}) \approx d\sigma/dp_T(m_t = 40 \text{ GeV})$ is at about $p_{TZ} \approx m_t = 200 \text{ GeV}$. The overall behaviour of the p_{TZ} distribution is very similar for $q\bar{q} \rightarrow ZZ$ and $gg \rightarrow ZZ$. Both distributions peak around $p_{TZ} = 30 \text{ GeV}$ and fall off steeply for larger p_{TZ} . We notice that the region $p_{TZ} < 10 \text{ GeV}$ lies to the left of the peak of the transverse momentum distribution and therefore contains only a very small part of the cross section and justifies our approximate treatment of this region.

V. The Higgs signal.

The Higgs boson can be made from two gluons through its Yukawa coupling to a quark inside a triangle loop graph. Since the Higgs couples to quarks with a strength proportional to their mass only massive quarks are important. The Higgs boson may then decay into a Z -boson pair and thus contributes to the process $gg \rightarrow ZZ$. Only two of the eight independent helicity amplitudes receive a contribution from the Higgs particle,

$$\mathcal{M}_{++++}^{Higgs} = \frac{2m_q^2}{\hat{s} - m_H^2 + i\Gamma_H m_H} \left(-2 + (\hat{s} - 4m_q^2)C(\hat{s}) \right), \quad (5.1)$$

$$\mathcal{M}_{++00}^{Higgs} = \frac{2m_q^2}{\hat{s} - m_H^2 + i\Gamma_H m_H} \frac{s_2}{2m_Z^2} \left(-2 + (\hat{s} - 4m_q^2)C(\hat{s}) \right), \quad (5.2)$$

where Γ_H is the total Higgs boson width.

We notice that in the Higgs case, heavy quarks ($m_q^2 \gg \hat{s}$) do not decouple but give a contribution,

$$\mathcal{M}_{++++}^{Higgs}(m_q \rightarrow \infty) = -\frac{\hat{s}}{\hat{s} - m_H^2 + i\Gamma_H m_H}, \quad (5.3)$$

$$\mathcal{M}_{++00}^{Higgs}(m_q \rightarrow \infty) = -\frac{\hat{s}}{\hat{s} - m_H^2 + i\Gamma_H m_H} \frac{s_2}{2m_Z^2}, \quad (5.4)$$

These contributions are due to the effective Higgs-gluon coupling, generated by a very heavy quark,

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} \frac{H}{v} G_{\mu\nu}^a G_a^{\mu\nu}, \quad (5.5)$$

where $v = 2m_W/g_w$ is the vacuum expectation value of the Higgs field. Since heavy quarks do not decouple the quarks of a hypothetical fourth generation would contribute coherently to the Lagrangian of eq. (5.5) and would therefore greatly enhance the Higgs production cross section. In the following we ignore this possibility and keep only the contribution of the top quark, for which we take $m_t = 120$ GeV.

In fig. 6 we compare the invariant mass distributions for $gg \rightarrow ZZ$ for $m_t = 120$ GeV both with and without a Higgs of mass $m_H = 500$ GeV at SSC energies. The contributions for $gg \rightarrow Z_L Z_L$ are shown separately. We see that the cross section for longitudinally polarised Z -bosons is always small except at the Higgs peak itself, where practically all Z -bosons are produced with a longitudinal polarization. In fig. 7 we show the transverse momentum distributions for the same parameters. There is a Jacobian peak in the Higgs boson signal at,

$$p_{TZ} = p_{TZ}^{Jac} = \frac{m_H}{2} \sqrt{1 - \frac{4m_Z^2}{m_H^2}}, \quad (5.6)$$

which is visible above the monotonically falling continuum background. In both cases, we see that at large invariant mass, $M_{ZZ} > m_H$, and large transverse momentum, $p_{TZ} > p_{TZ}^{Jac}$, the $gg \rightarrow ZZ$ cross section is reduced relative to the non-Higgs case. Similarly, the cross section is reduced relative to the case when only the graph containing the Higgs boson is calculated. It is clear that the large p_{TZ} tail of the Higgs boson signal [15] has been almost completely eliminated. At smaller values of M_{ZZ} and p_{TZ} the cross section is increased. Since the Higgs decays almost completely into longitudinal Z -bosons the interference may be understood by examining the $\mathcal{M}_{++00}$ amplitude at large $\hat{s}$.

$$\begin{aligned} \mathcal{M}_{++00}^{box}(\hat{s} \rightarrow \infty) &= -\frac{8a_q^2 m_q^2}{M_Z^2} \log^2\left(\frac{\hat{s}}{m_q^2}\right), \\ \mathcal{M}_{++00}^{Higgs}(\hat{s} \rightarrow \infty) &= \frac{\hat{s} m_q^2}{2M_Z^2(\hat{s} - m_H^2 + i\Gamma_H m_H)} \log^2\left(\frac{\hat{s}}{m_q^2}\right), \end{aligned} \quad (5.7)$$

where $a_q = \pm 1/4$. We see that the two contributions cancel.

The nature of the interference can be understood as follows. At large $\hat{s}$, the crossed box in fig. 1 does not contribute as is obvious from eq. (5.7). If we now make an s -channel cut on the other three graphs we obtain the process $t\bar{t} \rightarrow Z_L Z_L$. For this process, unitarity requires a cancellation between the Higgs graph and the other two graphs at large $\hat{s}$. The integration over loop momenta does not change this cancellation. We therefore disagree with the results of ref. [20], which finds a constructive interference at large $\hat{s}$.

In fig. 8 we show the invariant mass distribution for Z -boson pairs at the LHC and SSC. To get an idea of an observable cross section, we restrict the Z -boson rapidity, $|y_Z| < 2.5$. We show curves for $m_t = 120$ GeV and $m_H = 250, 500$ and 750 GeV. The increasing width of the Higgs boson (3.7, 57 and 197 GeV) respectively is clearly visible as the signal is spread over a larger range of M_{ZZ} . Consequently, the signal for $m_H = 750$ GeV never

becomes bigger than the $q\bar{q} \rightarrow ZZ$ background at the LHC. The situation is considerably worse at the LHC compared to the SSC, because of the reduced ratio of the gg and $q\bar{q}$ luminosities.

One way to improve the signal might be to look at the p_{TZ} distribution, shown in fig. 9. Since the background is peaked at low transverse momentum, while the Higgs boson decays isotropically in its centre of mass frame, one expects the Jacobian peak to stand out from the background. The effect is more pronounced for heavy Higgs bosons. Indeed we see in fig. 9 that for a Higgs boson mass $m_H = 750$ GeV, the signal is bigger than the background at the LHC. For low mass Higgs the invariant mass distribution seems to be a better signal.

We may use the large transverse momentum behaviour to improve the Higgs boson signal in the invariant mass distribution by requiring that the Z -bosons have large transverse momentum [15]. We illustrate this in fig. 10 for the case,

$$p_{TZ} > 0.25 \times M_{ZZ}, \quad (5.8)$$

for a Higgs mass $m_H = 750$ GeV at the LHC. The enhancement of the signal over the background may be clearly seen by comparing with the $m_H = 750$ GeV curve in fig. 8(a). The background is reduced by $\sim 50\%$ while the signal is reduced by only 15% at both the LHC and SSC.

Since the Z -bosons from Higgs decay are produced relatively centrally, we might expect that reducing the allowed rapidity range of the Z -bosons might also lead to a relative enhancement of the signal. In fig. 10 we see that this is the case. Reducing the allowed rapidity range from $|y_Z| < 2.5$ to $|y_Z| < 1.5$ reduces the signal by 37% (47%) and the background by 75% (74%) at the LHC (SSC). Decreasing the allowed rapidity range therefore leads to an improved signal-to-background ratio, but at the cost of reducing the number of signal events. Whether such a loss can be tolerated depends on a more detailed analysis of the final state, including the Z -boson decays and the detector acceptance, and on the available luminosity.

Finally we note that as an additional check on potential $H \rightarrow ZZ$ events, one may examine the angular distribution of the Z -boson decay products (see Duncan, ref. [17]). Since the Z -bosons from Higgs decay are predominantly produced with longitudinal polarisation (see figs. 6 and 7), the angular distribution of the lepton in the Z rest frame is of the form,

$$\frac{d\sigma}{d\cos\theta} \sim \sin^2\theta, \quad (5.9)$$

where θ is the angle of the lepton in the Z -boson rest frame relative to the Z direction of motion. The background Z -bosons are mainly transversely polarised and so exhibit behaviour of the form,

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \cos^2\theta. \quad (5.10)$$

Such a study may be used to confirm the Higgs signal.

VI. Conclusions.

In conclusion we have studied the coupling of two gluons with two Z -bosons via a quark loop. We kept the full dependence on the quark mass inside the loop. The $ggZZ$ polarisation tensor depends on both the vector and axial vector coupling of the quark to the Z -boson. While the vector piece of the tensor is related to the four-photon polarisation tensor, which is known from the QED literature, the axial vector piece had not been previously studied. The difference between the two pieces of the tensor is remarkably simple and we have presented it here.

The results of the full tensor were used to construct explicit formulae for all helicity amplitudes. We then applied these results to both continuum and Higgs boson resonance production of Z -boson pairs in high energy hadron colliders. The total continuum production of Z -boson pairs is reduced when the top quark mass is increased due to the decoupling of heavy quarks. The total cross section is, however, fairly insensitive to m_t since the light quarks dominate in the loop. At large invariant mass or large transverse momentum the cross section grows with increasing the top quark mass. This effect is due to the production of longitudinal Z -bosons for which the coupling grows with the quark mass. The gluon fusion process produces the Z -bosons somewhat more centrally than the basic $q\bar{q} \rightarrow ZZ$ process.

We have also studied the effects of the Higgs boson in the gluonic production of Z -boson pairs. At large $\hat{s}$, there is a cancellation between the triangle and box graphs which is not explicitly required by unitarity. The process $t\bar{t} \rightarrow Z_L Z_L$ is directly related to $gg \rightarrow Z_L Z_L$ and in this case a cancellation is indeed required at large $\hat{s}$ to preserve unitarity. Hence, we found a negative interference at large $\hat{s}$ which is at variance with the literature [20]. The net result is that the interference between the box and triangle graphs is destructive at large invariant mass and large transverse momentum and constructive at small M_{ZZ} and small p_{TZ} . Consequently, neglecting this interference may lead to an over-optimistic estimate of the $H \rightarrow ZZ$ signal at large M_{ZZ} .

To illustrate the signal from Higgs boson decay into a Z -boson pair we examined the case $m_t = 120$ GeV for a range of Higgs boson masses. The invariant mass distribution clearly shows the Breit-Wigner resonance structure for small m_H . For large m_H , the large width of the Higgs boson smears the signal and makes the Higgs hard to detect. We note, however, that the Jacobian peak in the transverse momentum distribution is more pronounced than the invariant mass peak and so leads to a cleaner signal. This fact may then be used to improve the invariant mass signal by requiring that the Z -bosons have large transverse momentum. Alternatively one may make use of the relatively central production of the Z -bosons in Higgs decay to enhance the signal by reducing the allowed rapidity range.

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Appendix A.

In this appendix we define the integrals appearing in the calculation. All momenta are outgoing. In the following we use the conventions p_1 and p_2 are the gluon momenta and p_3 and p_4 are the Z -boson momenta,

$$p_1^2 = p_2^2 = 0, \quad (A.1)$$

$$p_3^2 = p_4^2 = m_Z^2. \quad (A.2)$$

As in section 2 we use the notation,

$$(p_1 + p_2)^2 = s, \quad (p_1 + p_3)^2 = u, \quad (p_2 + p_3)^2 = t, \quad (A.3)$$

$$s_2 = s - 2m_Z^2, \quad s_4 = s - 4m_Z^2, \quad t_1 = t - m_Z^2, \quad u_1 = u - m_Z^2, \quad (A.4)$$

$$Y = ut - m_Z^4 = s \cdot p_T^2, \quad (A.5)$$

$$\beta = \sqrt{1 - \frac{4m_Z^2}{s}}. \quad (A.6)$$

The two-point function $B(s)$ is given by,

$$\begin{aligned} B(s) &= -i\pi^2 \int_0^1 dx \log(m_q^2 - i\epsilon - sx(1-x)) \\ &= -i\pi^2 \left\{ \log(m_q^2) - 2 + \sqrt{1 - \frac{4(m_q^2 - i\epsilon)}{s}} \log\left(\frac{-z}{1-z}\right) \right\} \end{aligned} \quad (A.7)$$

with,

$$z = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4(m_q^2 - i\epsilon)}{s}} \right). \quad (A.8)$$

In the amplitudes only the following combination is present,

$$B_1(s) = B(s) - B(m_Z^2). \quad (A.9)$$

The next integral that appears is the three-point function $C(p_1, p_2)$ which is given by,

$$C(p_1, p_2) = \int \frac{d^4 q}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)}. \quad (A.10)$$

The three-point function with two massless external lines is,

$$\begin{aligned} C(p_1, p_2) &= C(s) = i\pi^2 \int_0^1 \frac{dx}{sx} \log\left(1 - i\epsilon - \frac{s}{m_q^2} x(1-x)\right) \\ &= \frac{i\pi^2}{2s} \left[\log\left(\frac{-z}{1-z}\right) \right]^2, \end{aligned} \quad (A.11)$$

with z given by (A.8).

The three-point function with one massless external line is given by,

$$C(p_1, p_3) = C_1(u) = \frac{1}{u_1} (uC(u) - m_Z^2 C(m_Z^2)). \quad (\text{A.12})$$

$C(p_2, p_3)$ is obtained by exchanging u and t in eq. (A.12).

In order to describe the three-point function with no massless external lines and the four-point functions with only two massless external lines we introduce two auxiliary functions,

$$I_1(a, b, y_0) = \int_0^1 \frac{dy}{y - y_0} (\log(ay + b - i\epsilon) - \log(ay_0 + b - i\epsilon)), \quad (\text{A.13})$$

$$I_2(a, b, y_0) = \int_0^1 \frac{dy}{y - y_0} (\log(a - i\epsilon - by(1 - y)) - \log(a - i\epsilon - by_0(1 - y_0))). \quad (\text{A.14})$$

In terms of Spence functions one has,

$$I_1(a, b, y_0) = Sp\left(\frac{y_0}{y_0 - z}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z}\right), \quad (\text{A.15})$$

with,

$$z = -\frac{(b - i\epsilon)}{a}, \quad (\text{A.16})$$

and,

$$I_2(a, b, y_0) = Sp\left(\frac{y_0}{y_0 - z_1}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_1}\right) + Sp\left(\frac{y_0}{y_0 - z_2}\right) - Sp\left(\frac{y_0 - 1}{y_0 - z_2}\right), \quad (\text{A.17})$$

with,

$$z_1 = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4(a - i\epsilon)}{b}}\right), \quad (\text{A.18})$$

and,

$$z_2 = 1 - z_1. \quad (\text{A.19})$$

The three-point function with no massless external lines,

$$C(p_3, p_4) = C_2(s) = \frac{1}{s\beta} \left(2I_2(m_q^2, m_Z^2, \frac{\alpha}{1 + \alpha}) - I_2(m_q^2, s, \frac{\alpha^2}{\alpha^2 - 1})\right), \quad (\text{A.20})$$

with,

$$\alpha = -\frac{1 + \beta}{1 - \beta}. \quad (\text{A.21})$$

Finally there is the four-point function,

$$D(p_1, p_2, p_3) = \quad (\text{A.22})$$

$$\int \frac{d^4 q}{i\pi^2} \frac{1}{(q^2 - m_q^2)((q + p_1)^2 - m_q^2)((q + p_1 + p_2)^2 - m_q^2)((q + p_1 + p_2 + p_3)^2 - m_q^2)}.$$

There are two basic four point functions. The cross box, $D(p_1, p_3, p_2)$, is given by,

$$D(p_1, p_3, p_2) = D(u, t) = \frac{2}{Y R_1} \left\{ 2 I_2(m_q^2, m_Z^2, x_+) - I_2(m_q^2, u, x_+) - I_2(m_q^2, t, x_+) \right. \\ \left. + \log\left(-\frac{x_-}{x_+}\right) \left[2 \log(m_q^2 - i\epsilon - m_Z^2 x_+ x_-) - \log(m_q^2 - i\epsilon - u x_+ x_-) - \log(m_q^2 - i\epsilon - t x_+ x_-) \right] \right\}, \quad (A.23)$$

with,

$$x_{\pm} = \frac{1}{2}(1 \pm R_1), \quad (A.24)$$

and,

$$R_1 = \sqrt{1 + \frac{4sm_q^2}{Y}}. \quad (A.25)$$

The other basic function is the straight box, $D(p_1, p_2, p_3)$ given by,

$$D(p_1, p_2, p_3) = D(s, t) = -\frac{1}{st R_2} \left\{ -I_1(s, 0, \beta_1 + y_{1+}) + I_1(s, 0, \beta_1 + y_{1-}) \right. \\ + I_1(-u_1, -t_1, y_{1+}/(1 - \beta_1)) - I_1(-u_1, -t_1, y_{1-}/(1 - \beta_1)) \\ - I_1(t_1, -t_1, -y_{1+}/\beta_1) + I_1(t_1, -t_1, -y_{1-}/\beta_1) \\ + I_1(s, 0, \beta_2 + y_{2+}) - I_1(s, 0, \beta_2 + y_{2-}) \\ - I_1(-u_1, -t_1, y_{2+}/(1 - \beta_2)) + I_1(-u_1, -t_1, y_{2-}/(1 - \beta_2)) \\ + I_1(t_1, -t_1, -y_{2+}/\beta_2) - I_1(t_1, -t_1, -y_{2-}/\beta_2) \\ - I_2(m_q^2, M_Z^2, y_{1+}/(1 - \beta_1)) + I_2(m_q^2, M_Z^2, y_{1-}/(1 - \beta_1)) \\ + I_2(m_q^2, t, -y_{1+}/\beta_1) - I_2(m_q^2, t, -y_{1-}/\beta_1) \\ - I_2(m_q^2, s, y_{2+} + \beta_2) + I_2(m_q^2, s, y_{2-} + \beta_2) \\ + I_2(m_q^2, M_Z^2, y_{2+}/(1 - \beta_2)) - I_2(m_q^2, M_Z^2, y_{2-}/(1 - \beta_2)) \\ \left. - I_2(m_q^2, M_Z^2, -y_{2+}/\beta_2) + I_2(m_q^2, M_Z^2, -y_{2-}/\beta_2) \right\}, \quad (A.26)$$

with,

$$\beta_1 = \frac{t}{t_1}, \quad (A.27)$$

$$\beta_2 = \frac{1}{2} (1 - \beta), \quad (A.28)$$

$$y_{1\pm} = -\frac{t}{2t_1}(1 \pm R_2), \quad (A.29)$$

$$y_{2\pm} = \frac{-M_Z^2 + t_1\beta \mp tR_2}{2t_1 + s(1 + \beta)}, \quad (A.21)$$

$$R_2 = \sqrt{1 + \frac{4m_q^2 Y}{st^2}}. \quad (\text{A.30})$$

$D(p_3, p_1, p_2) = D(s, u)$ is obtained by exchanging t and u in eqs. (A.26) - (A.30).

Next, we define the following combinations of scalar integrals as auxiliary functions,

$$E(u, t) = 2u_1 C_1(u) + 2t_1 C_1(t) - Y D(u, t), \quad (\text{A.31})$$

$$E(s, t) = sC(s) + s_2 C_2(s) + 2t_1 C_1(t) - ts D(s, t), \quad (\text{A.32})$$

$$E(s, u) = sC(s) + s_2 C_2(s) + 2u_1 C_1(u) - us D(s, u). \quad (\text{A.33})$$

Finally, the following auxiliary functions are always associated with a pole in p_T ,

$$Q(s, t, u) = s_2 C(s) + s_4 C_2(s) + \frac{2u^2}{u_1} C_1(u) + \frac{2t^2}{t_1} C_1(t) + t^2 D(s, t) + u^2 D(s, u), \quad (\text{A.34})$$

$$Q_1(s, t, u) = 2C(s) - \frac{2u}{u_1} C_1(u) - \frac{2t}{t_1} C_1(t) - u D(s, u) - t D(s, t), \quad (\text{A.35})$$

$$Q_2(s, t, u) = t_1(t + m_Z^2) \left(\frac{2}{t_1} C_1(t) + D(s, t) \right) + u_1(u + m_Z^2) \left(\frac{2}{u_1} C_1(u) + D(s, u) \right), \quad (\text{A.36})$$

$$\begin{aligned} Q_3(s, t, u) = & 2s_2 B_1(s) + \frac{2t^3}{t_1^2} B_1(t) + \frac{2u^3}{u_1^2} B_1(u) - ss_2 C(s) - (ss_4 + 2m_Z^4) C_2(s) \\ & + \frac{2u^2(2s + u)}{u_1} C_1(u) + \frac{2t^2(2s + t)}{t_1} C_1(t) + 2t^2 s D(s, t) + 2u^2 s D(s, u), \end{aligned} \quad (\text{A.37})$$

$$\begin{aligned} Q_4(s, t, u) = & s_2(ss_4 + m_Z^4) C(s) + s_4(ss_4 + 3m_Z^4) C_2(s) + \frac{2t^4}{t_1} C_1(t) + \frac{2u^4}{u_1} C_1(u) \\ & + t^4 D(s, t) + u^4 D(s, u). \end{aligned} \quad (\text{A.38})$$

Appendix B.

In this Appendix we make an explicit expansion of the integrals defined in Appendix A in the limit that the quark mass is smaller than all other quantities,

$$m_q^2 = \mu^2 \ll p_T^2, m_Z^2, s, |t|, |u| \text{ etc..} \quad (B.1)$$

In this limit the two point function, $B_1(s)$, is given by,

$$B_1(s) = -\log\left(\frac{|s|}{m_Z^2}\right) + i\pi(\theta(s) - 1), \quad (B.2)$$

where,

$$\theta(s) = \begin{cases} 1, & s > 0, \\ 0, & s < 0. \end{cases} \quad (B.3).$$

The three point functions, $C(s)$, $C_1(t)$, and $C_2(s)$ are,

$$C(s) = \frac{1}{2s} \left(\log^2\left(\frac{\mu^2}{s}\right) - \pi^2 + 2i\pi \log\left(\frac{\mu^2}{s}\right) \right), \quad (B.4)$$

$$C_1(t) = \frac{1}{2t_1} \left(\log\left(-\frac{m_Z^2}{t}\right) \left\{ \log\left(-\frac{\mu^2}{t}\right) + \log\left(\frac{\mu^2}{m_Z^2}\right) \right\} + \pi^2 - 2i\pi \log\left(\frac{\mu^2}{m_Z^2}\right) \right), \quad (B.5)$$

$$C_2(s) = \frac{2}{s\beta} \left(Sp\left(-\frac{1-\beta}{1+\beta}\right) - Sp\left(-\frac{1+\beta}{1-\beta}\right) \right). \quad (B.6)$$

$C_1(u)$ is obtained by exchanging u and t in eq. (B.5).

The four point functions, $D(u, t)$ and $D(s, t)$, are given by,

$$\begin{aligned} D(u, t) = & \frac{2}{sp_T^2} \left(Sp\left(-\frac{p_T^2}{u}\right) + Sp\left(-\frac{p_T^2}{t}\right) - 2Sp\left(-\frac{p_T^2}{m_Z^2}\right) - 2i\pi \left\{ \log\left(\frac{\mu^2}{p_T^2}\right) + \log\left(\frac{p_T^2 + m_Z^2}{m_Z^2}\right) \right\} \right. \\ & \left. + \log\left(-\frac{\mu^2}{u}\right) \log\left(\frac{p_T^2 + u}{u}\right) + \log\left(-\frac{\mu^2}{t}\right) \log\left(\frac{p_T^2 + t}{t}\right) - 2 \log\left(\frac{\mu^2}{m_Z^2}\right) \log\left(\frac{p_T^2 + m_Z^2}{m_Z^2}\right) \right), \end{aligned} \quad (B.7)$$

and,

$$\begin{aligned} D(s, t) = & \frac{1}{st} \left(4Sp(x) + \frac{1}{2} \log^2\left(\frac{\mu^2}{s}\right) + 2 \log\left(\frac{\mu^2}{s}\right) \log\left(\frac{1-x}{x}\right) + 2 \log^2(x) + \frac{\pi^2}{6} \right. \\ & \left. + i\pi \{ 2 \log(x(1-x)) - \log\left(\frac{\mu^2}{s}\right) \} \right), \end{aligned} \quad (B.8)$$

where,

$$x = \frac{t}{t_1}. \quad (B.9)$$

$D(s, u)$ is obtained by exchanging u and t in eq. (B.8).

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Table 1

Cross sections for ZZ production, $q\bar{q} \rightarrow ZZ$ and $gg \rightarrow ZZ$ in pp collisions at $\sqrt{s} = 40$ TeV ($\sqrt{s} = 16$ TeV).

$\sigma(pb)$	$\sigma(q\bar{q} \rightarrow ZZ)$	$\sigma(gg \rightarrow ZZ)$ $m_t = 40$ GeV	$\sigma(gg \rightarrow ZZ)$ $m_t = 120$ GeV	$\sigma(gg \rightarrow ZZ)$ $m_t = 200$ GeV
$ y_Z < 2.5$	15.3 (7.0)	10.6 (2.7)	9.3 (2.3)	9.1 (2.3)
$ y_Z < 1.5$	7.0 (3.4)	5.4 (1.5)	4.6 (1.3)	4.6 (1.3)

Figure Captions

- Fig. 1: Feynman diagrams contributing to the process $gg \rightarrow ZZ$.
- Fig. 2: The exact $gg \rightarrow ZZ$ matrix elements squared for a single quark of mass m_t at $s = 9m_Z^2$ and $u = t$. The contributions when both Z -bosons are transversely polarised (TT), one transverse and one longitudinal (TL) and both longitudinal (LL) are shown. The vertical scale is in arbitrary units.
- Fig. 3: The rapidity, y_Z , distribution of Z -bosons produced via $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at (a) $\sqrt{s} = 16$ TeV and (b) $\sqrt{s} = 40$ TeV. To illustrate the variation with the top quark mass we show curves for $m_t = 40$ GeV and $m_t = 200$ GeV. The normalisation is such that the area under the curve is half the total ZZ cross section.
- Fig. 4: The invariant mass, M_{ZZ} , distribution of Z -bosons produced via $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at (a) $\sqrt{s} = 16$ TeV and (b) $\sqrt{s} = 40$ TeV. To illustrate the variation with the top quark mass we show curves for $m_t = 40$ GeV and $m_t = 200$ GeV. Both Z -bosons have rapidity $|y_Z| < 2.5$.
- Fig. 5: The transverse momentum, p_{TZ} , distribution of Z -bosons produced via $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at (a) $\sqrt{s} = 16$ TeV and (b) $\sqrt{s} = 40$ TeV. To illustrate the variation with the top quark mass we show curves for $m_t = 40$ GeV and $m_t = 200$ GeV. Both Z -bosons have rapidity $|y_Z| < 2.5$.
- Fig. 6: The invariant mass, M_{ZZ} , distribution of Z -bosons for $gg \rightarrow ZZ$ in pp collisions at $\sqrt{s} = 40$ TeV. We choose $m_t = 120$ GeV and show curves both with a Higgs boson of mass $m_H = 500$ GeV and without the Higgs boson. We show curves for longitudinally polarised Z -boson pair production in addition to the sum over all polarisations of the Z -boson. Both Z -bosons have rapidity $|y_Z| < 2.5$.
- Fig. 7: The transverse momentum, p_{TZ} , distribution of Z -bosons for $gg \rightarrow ZZ$ in pp collisions at $\sqrt{s} = 40$ TeV. We choose $m_t = 120$ GeV and show curves both with a Higgs boson of mass $m_H = 500$ GeV and without the Higgs boson. We show curves for longitudinally polarised Z -boson pair production in addition to the sum over all polarisations of the Z -boson. Both Z -bosons have rapidity $|y_Z| < 2.5$.
- Fig. 8: The invariant mass, M_{ZZ} , distribution of Z -bosons for $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at (a) $\sqrt{s} = 16$ TeV and (b) $\sqrt{s} = 40$ TeV. We choose $m_t = 120$ GeV and show curves with a Higgs boson of mass $m_H = 250, 500$ and 750 GeV. Both Z -bosons have rapidity $|y_Z| < 2.5$.
- Fig. 9: The transverse momentum, p_{TZ} , distribution of Z -bosons for $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at (a) $\sqrt{s} = 16$ TeV and (b) $\sqrt{s} = 40$ TeV. We choose $m_t = 120$ GeV and show curves with a Higgs boson of mass $m_H = 250, 500$ and 750 GeV. Both Z -bosons have rapidity $|y_Z| < 2.5$.

Fig. 10: The invariant mass, M_{ZZ} , distribution of Z -bosons for $gg \rightarrow ZZ$ and $q\bar{q} \rightarrow ZZ$ in pp collisions at $\sqrt{s} = 16$ TeV. We choose $m_t = 120$ GeV and $m_H = 750$ GeV. We show curves for (i) rapidity $|y_Z| < 2.5$ and transverse momentum $p_{TZ} > 0.25 \times M_{ZZ}$ and (ii) rapidity $|y_Z| < 1.5$.

Fig. 1.

Fig. 2.

Fig. 3(a).

Fig. 3(b).

Fig. 4(a).

Fig. 4(b).

Fig. 5(a).

Fig. 5(b).

Fig. 6.

Fig. 7.

Fig. 8(a).

Fig. 8(b).

Fig. 9(a).

Fig. 9(b).

Fig. 10.