THE SPIN OF THE PROTON IN A HYBRID CHIRAL BAG MODEL

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NOTE ADDED IN PROOF

The second equality in Eq. (18) neglects any mean orbital angular momentum of quarks inside the bag, and hence is only approximate. We have checked that including \( <L_3> \neq 0 \) effects does not alter our conclusion significantly. The first transition to cause a deviation from Eq. (20) is \( 1^+ \rightarrow 1^+ \), which is approximately a 20% effect for \( \theta = \pi/2 \) and serves as an upper bound on the sum of all such corrections. The net effect on our upper bound for \( R \) is to weaken it by at most 10%, which lies within the uncertainty expected for our calculation.

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ABSTRACT

We analyse the quark contributions to the proton spin in a hybrid chiral bag model which interpolates between the naive quark model and the Skyrme model. Care is taken to implement directly the anomalous axial U(1) symmetry. The total quark contribution to the proton spin varies rapidly with the bag radius R. If perturbative gluon corrections are neglected, recent EMC data on the quark spins constrain R to be less than 0.5 fm.
Until recently, the distribution of angular momentum among the constituents of a spinning proton has been very uncertain. The only pieces of information came from knowledge of axial current matrix elements, which determined flavour non-singlet combinations of quark contributions to the proton spin. Recently, two new pieces of information have become available about flavour-singlet contributions to the proton spin. These are derived from the European Muon Collaboration (EMC) measurement of polarized muon-proton deep inelastic scattering [1], and a precision measurement of neutrino-proton elastic scattering [2]. They indicate that (see for example ref. [3] and references therein)

\[ \sum_q (\Delta q - \frac{\alpha_s}{2\pi} \Delta g) \sim 0 \pm \frac{1}{4} \]  

(1)

where the \( \Delta q \) (\( \Delta g \)) are the integrated net quark (gluon) polarizations. The result (1) is in *prima facie* contradiction with the naïve quark model (NQM), which suggests that

\[ \Delta u + \Delta d = 1 \quad ; \quad \Delta q = 0 \quad \text{for} \quad q \neq u, d \quad ; \quad \Delta g = O(\alpha_s) \]  

(2)

However, the result (1) finds a natural interpretation within the Skyrme model, which suggests that [3, 4]

\[ \sum_q \Delta q \approx 0 \quad ; \quad \Delta g \approx 0 \]  

(3)

The Skyrme model can be derived from large-\( N_c \) QCD in the chirally symmetric limit of massless quarks [5]: \( m_q \to 0 \), which is likely to be a good approximation for the u and d quarks which only weigh a few MeV. Although the NQM gives a very good description of the systematics of hadron spectroscopy and good values for the static properties of baryons, it has not been derived from QCD, except in the limit of very massive quarks: \( m_q \gg 1 \) GeV. In view of the apparent failure (1,2) of the NQM to describe the spin content of the proton, it is desirable to
explore models that interpolate between the Skyrme model and the NQM. Such models could tell us whether the real world is closer to one or the other, and why the NQM works well for some quantities and not for others.

One such class of interpolating models is the hybrid chiral bag model (HCBM) [6]. The original bag model [7] gave a field-theoretical description of confined quarks in NQM-like wave functions. However, it did not respect chiral symmetry at the surface of the bag. This defect was remedied [8] in the HCBM, whose quarks were coupled to pions at a surface of arbitrary radius R in such a way that the SU(2) × SU(2) axial currents were continuous and chiral symmetry was restored. The HCBM looks like a NQM at small radii r < R, and like the Skyrme model at large radii r > R. The NQM is recovered in the limit R → ∞, and the Skyrme model in the limit R → 0. It was found that HCBM predictions for many physical quantities such as $g_A = A_0 - A_d$ were relatively insensitive to the bag radius R [9]. If one assumes a priori the correctness of the Skyrme model, this independence of R could be taken as an a posteriori reason why the NQM is successful in many predictions.

In this paper we analyse contributions to the proton spin in the HCBM. To treat the flavour-singlet sector correctly, we first improve previous versions of the HCBM by including an isosinglet η field and requiring continuity of the axial baryon number current at the bag boundary. We neglect O(αs) corrections inside the bag, and the axial anomaly is reflected in a non-zero η mass outside the bag. We show that the singlet axial current matrix element $g_A^0 : <p|A_0^0|p> \equiv g_A^0 S^0_\mu(p)$, where $S_\mu(p)$ is the proton spin, receives important contributions from the $\bar{q}q$ sea within the bag, as well as from the external η field. The total value of $g_A^0$ is very sensitive to the bag radius R, and the experimental result (1) suggests that

$$R \lesssim 0.5 \text{ fm}$$

(4)
if perturbative QCD corrections are neglected. For such small radii, there are no valence quarks inside the bag.

We begin our discussion with a description of the interpolating HCBM used in this paper. We restrict ourselves initially to two massless quark flavours (u,d), and will discuss the possible effects of strange quarks at the end. The Lagrangian is therefore given by [6]

\[ L_0 = L_{\text{quark}} + L_{\text{meson}} \]  

(5a)

where

\[ L_{\text{quark}} = \begin{cases} -\overline{\psi} \gamma^\mu \psi & \text{inside the bag} \\ 0 & \text{outside} \end{cases} \]  

(5b)

and

\[ L_{\text{meson}} = \begin{cases} \frac{e^2}{4} Tr \left( \partial_\mu U^\dagger \partial^\mu U \right) + \frac{1}{32\pi^2} Tr \left( \frac{1}{2} (\partial_i U^\dagger \partial_i U) \right) & \text{outside the bag} \\ 0 & \text{inside the bag} \end{cases} \]  

(5c)

We impose the boundary condition

\[ \psi(x) = \exp \left[ i \gamma_5 \tau \cdot \Theta(x) \right] \psi(x) \]  

(6)

at the bag surface, where \( n_\mu \) is the outward normal to the bag surface, the \( \tau \) are the isospin Pauli matrices, \( U = \exp \left[ i \tau \cdot \theta(x) \right] \), and the chiral angle \( \theta \) is related to the pion field by \( \pi = f_\pi \theta \sin \theta(x) \), with \( f_\pi = 93 \text{ MeV} \) the pion decay constant.

In the case of a spherically-symmetric bag, the ground state of this effective theory is given by the static hedgehog Ansatz

\[ \Theta(x,t) = \Theta(r) \hat{z} \]  

(7)

where \( r = |x| \). The chiral angle \( \theta = |\theta| \) at the bag radius \( R \) is determined by requiring continuity of the isovector axial current

\[ A_\mu = A_\mu^{\text{quarks}} + A_\mu^{\text{mesons}} \]

\[ A_\mu^{\text{quarks}} = \begin{cases} \frac{i}{2} \gamma_5 \sigma_{\mu \nu} \psi \gamma^\nu \psi & \text{inside the bag} \\ 0 & \text{outside} \end{cases} \]  

(8a)
\[ A^{\text{mesons}} = \left\{ \begin{array}{ll} 0 & \text{inside} \\ i f_R (1 + \frac{\xi^2}{R^2}) & \text{outside} \end{array} \right. \]  

so that \( \theta = \theta(R) \). In the limit \( R \to 0 \), \( \theta \to \pi \) and the model reduces to the Skyrme model, whilst for \( f_R \gg 1 \), \( \theta \to 0 \) and the model approaches the original MIT bag model*. In this latter limit, the isosinglet axial current matrix element at zero momentum transfer is given by [10]

\[ \langle p, \uparrow | A_3^\pi (0) | p, \uparrow \rangle = \frac{i}{2} g_A = \frac{1}{2} \frac{9}{2} \left( \frac{\omega_0}{3(\omega_0 - 1)} \right) \]

where \( g_A = \Delta u - \Delta d \) is the axial coupling measured in neutron decay to be \( 1.246 \pm 0.014 \), and \( \omega_0 \approx 2.04 \) determines the energy \( E_0 = \omega_0 / R \) of the valence quark orbital. The corresponding value of the isosinglet axial coupling \( g_A^0 = \Delta u + \Delta d \) is given by

\[ g_A^0 = \frac{3}{5} g_A \approx \frac{2}{3} \]

which is rather large.

In the Skyrme model, on the other hand, the isosinglet axial current vanishes locally [4], and therefore \( g_A^0 \approx 0 \) in the large-\( N_c \) limit. At intermediate bag radii, one might naively expect \( g_A^0 \) to receive no contribution from the meson sector and, if \( g_A^0 (\text{bag}) = 3/5 g_A (\text{bag}) \), to be relatively large. However, this turns out to be incorrect for two reasons. (a) The model defined by equations (5) and (6) breaks explicitly the classical axial \( U_A(1) \) global symmetry of QCD. This symmetry must be restored by introducing an isosinglet pseudoscalar meson \( \eta \), coupled in such a way that the isosinglet axial current is also continuous at the bag surface. We then find that

\[ g_A^0 (\eta) = \frac{1}{2} g_A^0 (\text{bag}) \]

*) Strictly speaking, this bag model corresponds to \( \theta \to 0 \) at fixed \( R \), but the difference is not essential.
(b) As \( \theta \to \pi/2 \), which happens when \( R \sim 0.5 \text{ fm} \), \( \Omega \to 0 \) and the valence orbital disappears into the Dirac sea. Vacuum polarization becomes very important and in particular

\[
q^0_A (l_{\text{bag}}) = \frac{3}{5} q^0_A (l_{\text{bag}})
\]

as well as being reduced from its valence-orbital value.

Before discussing these effects in more detail, we first review the computation of \( g_A \) in the HCBM. The boundary condition (6) implies that the quark orbitals inside the bag cannot have well-defined spin or isospin unless \( \theta = 0 \). They are instead eigenstates of the operator \( K = J + 1/2 \tau_z \), where \( J = L + S \) is the total angular momentum. The quark ground state is the \( K^F = 0^- \) orbital, and the hedgehog Ansatz (7) implies that only this orbital can be occupied. Its scaled energy \( \Omega_0 = E_0 R \) is fixed by the boundary condition (6):

\[
j_1(J_0) = j_0(J_0) \cos \theta \left/ (1 + \sin \theta) \right.
\]

(11)

where the \( j_1 \) are standard spherical Bessel functions. Thus \( \Omega_0(\theta = 0) = -\Omega_0(\theta = \pi) = 2.04 \) and \( \Omega_0(\pi/2) = 0 \).

The expectation value of the isovector axial current in the bag is

\[
q^0_A (l_{\text{bag}}; O^-) = -\frac{1 + \frac{y^2}{1 + y^2 - 2y_j J_0}}{1 + \frac{2y_j J_0}{J_0(J_0)}}
\]

(12)

for three quarks in \( O^- \) orbitals. However, this does not correspond immediately to the quark contribution to \( g_A \), since the nucleon state built out of \( K^F = 0^- \) orbitals is a mixture of nucleon and \( \Delta \) states without a well-defined angular momentum. In order to build eigenstates of \( J^2 \) and \( \tau^2 \), one notes that the ground state energy is invariant under a global isospin rotation of \( \theta \) accompanied by a spatial rotation, provided that the quark orbitals are simply rotated in \( K \)-space. Upon quantization of time-dependent such rotations, the Hamiltonian becomes \( H = E(O^-) + J^2/2I \), where \( I \) is the moment of
inertia, and \( J^2 = (1/2 \, \tau)^2 \). The eigenstates of \( H \) are then automatically eigenstates of \( J \) and \( 1/2 \, \tau \) with \( \langle J^2 \rangle = (1/2 \, \tau)^2 \). This so-called cranking of the quark system implies that its contribution to \( g_A \) is given by \([11]\)

\[
 g_A^{J\pi = 0+} = -\frac{1}{3} \left[ g_A^{J\pi = 0+} \right] = \frac{1}{3} \left[ 1 + \frac{y^2}{y^2 - 2y_x^2} \right] R^2.
\]  

(13)

However, this is not yet the full bag contribution to \( g_A \). Since the spectrum is not symmetric about \( E = 0 \) when \( \theta \neq 0, \pi \), in general fermion operators have non-zero vacuum expectation values. For example, the vacuum contribution to the baryon number inside the bag is \([12]\) \( N = \frac{\theta - \sin \theta \cos \theta}{\pi} \) for \( \theta \in [-\pi/2, \pi/2] \), with \( N(\theta + \pi) = N(\theta) \). Likewise, one finds a vacuum contribution to \( g_A \):

\[
 g_A^{(\text{vac})} = -\frac{2}{3} \left( \frac{N_c}{2} \right) \frac{d\rho_v}{d\Theta}.
\]  

(14)

where \( \rho_v \) is the Casimir energy per colour of the vacuum \([6, 13]\). Note that, although \( d\rho_v/d\theta \) is discontinuous at \( \theta = \pi/2 \), the total quark contribution to \( g_A \), namely \( g_A^{(\text{bag})} + g_A^{(\text{vac})} \), is continuous at \( \theta = \pi/2 \):

\[
 g_A^{(\text{quark})} \Big|_{\frac{\pi}{2} - \epsilon} = g_A^{(\text{vac})} \Big|_{\frac{\pi}{2} + \epsilon} = \frac{1}{2}
\]

\[
 g_A^{(\text{quark})} \Big|_{\frac{\pi}{2} + \epsilon} = \left[ g_A^{(\text{bag})} + g_A^{(\text{vac})} \right] \Big|_{\frac{\pi}{2} + \epsilon} = \frac{1}{2}
\]

(15)

as expected physically. The total \( g_A \) must, of course, include the contribution from the meson sector: \( g_A = g_A^{(\text{quarks})} + g_A^{(\text{mesons})} \). Using current conservation, the total \( g_A \) can be rewritten in terms of the long-distance behaviour of the external meson field:

\[
 g_A = -\frac{1}{3} \left( -8\pi \frac{\tau^2}{R^4} \right)
\]

(16)

where \( U(\xi) = 1 + \text{i}A \xi \cdot \frac{\xi}{r^2} + \ldots \) as \( r \to \infty \). Thus \( g_A \) reduces to the Skyrme model value in the limit \( R \to 0 \), and is not very different if \( R \) is chosen to be \( \lesssim 1 \) fm.
We now turn to the corresponding calculations of $g_\alpha^0$ in the HCBM. This involves the cranking procedure explicitly, since $g_\alpha^0 = \langle \tau^0 \sigma_3 \rangle = \langle \sigma_3 \rangle$, and $\langle 0^+ | \sigma_3 | 0^+ \rangle = \langle \text{vac} | \sigma_3 | \text{vac} \rangle = 0$. To see this, note that the vacuum $|\text{vac}\rangle$ is effectively a $K=0$ state since all single-particle levels contribute to $\langle \text{vac} | \sigma_3 | \text{vac} \rangle$. The vanishing of $\langle \text{vac} | \sigma_3 | \text{vac} \rangle$ also follows from the observation that, in the absence of rotation, nothing polarizes the medium to give $\langle \sigma_3 \rangle \neq 0$. Under a slow rotation $\omega$, the wave function of each level $k$ changes to become, in the rotating frame [11]:

$$|k_\omega\rangle = |k\rangle - \frac{1}{2} \sum_{p \neq k} \frac{\langle p | \omega \cdot \hat{z} | k \rangle}{E_p - E_k} |p\rangle + O(\omega^2)$$

(17)

where $|k\rangle$ denotes the valence and filled Dirac sea levels, and $|p\rangle$ denotes the unoccupied level. Thus to order $\omega$

$$\langle k_\omega | \sigma_3 | k_\omega \rangle = - \sum_{p \neq k} \frac{\langle p | \omega \cdot \hat{z} | k \rangle \langle k | \sigma_3 | p \rangle}{E_p - E_k} = \sum_{p \neq k} \frac{\langle p | \omega \cdot \hat{z} | k \rangle \langle k | \sigma_3 | p \rangle}{E_p - E_k}$$

(18)

and the quarks' contribution to $g_\alpha^0$ is therefore

$$g_\alpha^0(\text{quarks}) = \sum_{k_\omega} \langle k_\omega | \sigma_3 | k_\omega \rangle = \omega \sum_{p \neq k} \frac{\langle p | \sigma_3 | k \rangle \langle k | \sigma_3 | p \rangle}{E_p - E_k} = 2\omega I_q$$

(19)

where we have taken $\omega = (0,0,0)$, and $I_q$ is the quarks' contribution to the total moment of inertia $I$. Upon quantization of the rotation, in which $\omega \rightarrow J/I$, we find

$$g_\alpha^0(\text{quarks}) = \frac{I_q}{I}$$

(20)

where $I_q$ and $I$, including the vacuum contributions, are given in ref. [11].

The pion field does not contribute to the isosinglet axial current, and (20) would be the only contribution to $g_\alpha^0$ in the initial HCBM defined by equations (5) and (6). However, as already noted, one should modify the model to restore the $U_A(1)$ symmetry broken by the
continuity condition (6). This is done by introducing an isosinglet pseudoscalar meson field $\eta = (\bar{u}_5 u - \bar{d}_5 d)/\sqrt{2}$ in this 2-flavour case, and modifying the continuity condition (6) to become

$$\nabla \cdot J = \exp \left( i \gamma_5 \left( \frac{2}{f^2} + \frac{1}{2} \cdot \frac{\eta}{f} \right) \right) \cdot \nabla \cdot J \quad (6')$$

In the large-$N_c$ limit the $\eta$ decouples from the pions, and we modify (5) to become

$$L_0' = L_0 + \frac{1}{2} \left( \partial_\mu \eta \partial^{\mu} \eta - m_\eta^2 \eta^2 \right) \quad (5')$$

The isosinglet axial current is then given by $A^0_\mu = A^0_{quarks} + A^0_{mesons}$, where

$$A^0_{quarks} = \left\{ \begin{array}{ll} -\frac{i}{2} \gamma_5 \tau^a \nabla \cdot J & \text{inside the bag} \\ 0 & \text{outside} \end{array} \right. \quad (8'a)$$

$$A^0_{mesons} = \left\{ \begin{array}{ll} 0 & \text{inside the bag} \\ \frac{i}{2} \gamma_5 \tau^a \nabla \cdot J & \text{outside} \end{array} \right. \quad (8'b)$$

The isosinglet axial anomaly is $\partial_\mu A^0_\mu = \alpha_s/\pi \Gamma_{\mu\nu} F^\mu\nu$. In the spirit of the HCBM, we assume that gluonic effects may be treated perturbatively and neglected as a first approximation within the bag: $\partial_\mu A^0_{quarks} = 0$, whereas non-perturbative effects generate $\partial_\mu A^0_{mesons}$ outside the bag. This non-conservation of $A^0_{mesons}$ is represented by choosing $m_\eta \neq 0$. The continuity of $A^0_\mu$ at the bag surface imposes $A^0_{mesons} = A^0_{quarks} = 0$, so the $\eta$ field is a dipole: $\eta(r) = P \cdot x/r^2$. In the limit of small momentum transfer $q^2 \rightarrow 0$ we then find

$$A^0_{mesons} (q) = P(z) \left[ -\frac{4\pi f^2}{\gamma_5^2} \left( P - \frac{1}{q^2} \right)^2 - \frac{m_\eta^2}{q^2 + m_\eta^2} - \frac{P \cdot q}{q^2} \right] \quad (21)$$

where $P(z) \equiv e^{-z}(1 + z + 1/3 z^2)$; $z \equiv m_\eta R$, and the continuity of $A^0_{mesons}$ gives

$$P = -\frac{3}{8\pi f^2} \left[ \frac{e^{-z}}{1 + 2 z + z^2} \right] q^0_{(\eta)} \left( \text{quarks} \right) \quad (22)$$

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The $\eta$ contribution to $g_A^0$ is therefore

$$g_A^0(\text{mesons}) = \frac{1}{2} g_A^0(\text{quarks})$$

(23)

and hence the total

$$g_A^0 = g_A^0(\text{quarks}) + g_A^0(\text{mesons}) = \frac{3}{2} g_A^0(\text{quarks}) = \frac{3}{2} \frac{I_q}{I}$$

(24)

which is our main result.

The recent measurement of the polarized proton structure function $g_1^p$ by the EMC [1] allows one to extract the information that $g_1^p = 0.00 \pm 0.24$ [3]. This result applies to the real world with 3 light flavours, but we interpret it as corresponding to $g_A^0 < 1/4$ in our toy 2-flavour model. From the numerical results for $I_q/I$ in ref. [11], we infer that $R \leq 0.5$ fm in the limit $m_\eta \to 0$. For such small bag radii there is no valence quark orbital, since $\theta(R) \leq \pi/2$ for $R \leq 0.5$ fm and $E_0(\theta \leq \pi/2) \leq 0$.

We find that the contribution of the $\eta$ cloud to the total nucleon mass is small: $6E_\eta \leq 50$ MeV. Hence the parameters previously determined by fitting the total energy to the proton mass are probably not significantly altered.

Our results are probably not very sensitive to the various approximations made. For $m_\eta \neq 0$,

$$g_A^0(\text{mesons}) = \frac{1}{2} g_A^0(\text{quarks}) \left(1 - \frac{1}{6} (m_\eta R)^2 + \ldots \right)$$

(25)

where $m_\eta R \leq 1$ for $R \leq 0.5$ fm if $m_\eta = 540$ MeV is used, as appropriate for the 2-flavour case. A more realistic model would of course include strange quarks, in which case $m_\eta = m_\eta', = 958$ MeV and $(m_\eta, R)$ might be large. However, in the limit of large $m_\eta$

$$g_A^0(\text{mesons}) \to \frac{1}{2} g_A^0(\text{quarks}) \frac{2}{3}$$

(26)
Hence the total $g_A^0$ and our resulting bound on $R$ are insensitive to $m_\eta$. Higher-order corrections to $g_A^0$ (quarks) due to higher powers of $\omega$ in equation (19) are small as long as the nucleon-$\Delta$ mass splitting is attributed to rotational energy, since $\omega \propto m_\Delta - m_p \ll m_p$, and corrections to $g_A^0$ (quarks) are $O(\omega^2)$. Finally, we recall that we have neglected any contribution to $g_A^0$ that could come from gluon polarization $\Delta G$ via the perturbative anomaly in $A^0_\mu$ [14]. Clearly $\Delta G \neq 0$ only inside the bag where the colour magnetic field due to rotation can polarize the gluons. Hence $\Delta g \to 0$ as $R \to 0$, as has been shown explicitly in the Skyrme model [3]. For $R \neq 0$, the gluon condensate inside the bag would be quite small for $R \lesssim 0.5$ fm, so we would expect gluon polarization $\Delta G$ to make only a small contribution to $g_A^0$.

We have constructed an HCBM that interpolates between the Skyrme model and a quark bag model, and incorporates the approximate conservation of the isosinglet axial current in an appropriate way. In contrast to other baryon observables which are relatively insensitive to the bag radius $R$, the isosinglet-axial current matrix element $g_A^0$ varies rapidly with $R$. We interpret the EMC determination of $g_A^0$ as implying an upper bound $R \lesssim 0.5$ fm, in which case the bag contains no valence quark orbitals, and the proton just consists of a filled Dirac sea surrounded by a meson cloud. All our knowledge is consistent with $R$ being vanishingly small, in which case the HCBM becomes the Skyrme model. The relative insensitivity of other baryon observables to $R$ may explain many successes of naive quark models.

**Note added:**

After completing this paper, we received ref. [15] which covers similar ground. That paper neglects the vacuum contribution to the axial current, and saturates the isosinglet axial charge with the anomaly, using a high value of $\alpha_s$ that makes their perturbative analysis unreliable.
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