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PAIR PRODUCTION IN BUNCH CROSSING

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A B S T R A C T

Electron-positron colliders operating in the multi-TeV range, at the required very high luminosity, will have to contend with an intense beamstrahlung. Serious problems will originate from the energetic electrons and positrons which the radiated photons will produce before they leave the bunch. We calculate the pair production rate by a photon traversing a bunch, focusing here on the contribution proportional to the bunch length, in the deep quantum regime. The probability for pair production is of the same order as that for beamstrahlung. It is considerable.

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Electron-positron linear colliders in the TeV range are presently considered with much attention as potentially very valuable future machines [1]. With a power limitation set around 200 MW, the very high luminosity which such colliders should achieve imposes a strong bremsstrahlung when bunches collide. It is referred to as beamstrahlung [2]. The radiation losses are likely to occur in the deep quantum regime. They are considerable.

In a recent series of papers [3] we have studied the properties of such a beamstrahlung. Our approach focuses on the deep quantum regime and is based on the calculation of Feynman graphs. It complemented in a useful new way earlier calculations [4] and more recent ones [5].

One of the leading terms in the fractional energy loss δ is proportional to the bunch length L_b . We found a very simple expression for it, in terms of a new coherent radiation length ℓ_c , proper to the deep quantum regime, namely

$$\delta = K \frac{\alpha}{\pi} \frac{L_b}{\ell_c} \quad (1)$$

where K is a numerical factor of order 1 [3]. Here:

$$\ell_c = (L_e L_c^2)^{1/3} \quad (2)$$

where L_e and L_c are respectively the radiative length and the classical coherent radiation length [3].

The numerical value of δ is found to be typically of the order of 20% or larger for a uniform cylindrical bunch. The probability for photon radiation is rather large. Indeed $L_b/\ell_c = \Lambda$ is a large parameter, of order 10 to 100 with currently considered designs [1,3].

The intensity of this radiation is a serious matter which such colliders will have to contend with. It can be considered as a bonus when looking at such machines as being also powerful photon-photon colliders [6]. If a large number of energetic photons are radiated within a bunch, it is likely that they will in turn produce a large number of electron-positron pairs (and background muons!) in the intense field of the bunch [7]. This is a serious hazard. These very energetic electrons and positrons will be shot forwards with a sizeable angular spread, thus spraying the focusing device.

In this letter we estimate the magnitude of the expected effect in the deep quantum regime, considering the simpler problem of pair production by a photon traversing a bunch. We thus calculate the rate associated with the Feynman graph of figure 1-a,

while a proper treatment would require the calculation of the rate due to the process of Fig. 1-b. Here the crosses correspond to the interaction of the electron (positron) with the bunch. We leave the second calculation to a more detailed paper. Here we focus on the first process which already provides much insight into the problem.

We indeed find that the production rate per unit area has a leading term, proportional to the bunch length, which is

$$\frac{I(X)}{\pi R^2} = K_p \frac{\alpha}{\pi} \frac{L_b}{\ell_c} \frac{(1-X)^2 + X^2}{[X(1-X)]^{1/3}}, \quad (3)$$

where K_p is a numerical factor of order 1. Here X is the fraction of the photon energy taken by the electron, or the positron. The relevant coherent production length ℓ_c is the same as the one previously found for beamstrahlung [3]. Since L_b/ℓ_c is large, pair production, with a rather flat spectrum, is important.

We work in the low D approximation, where D is the disruption parameter, and also in the deep quantum regime ($\ell_c \gg L_c$), as we previously did in the case of beamstrahlung. We consider a cylindrical bunch with uniform density for this first approach. It is known [6] that both photon radiation (and pair production) would be reduced with ribbon bunches since the field will be lower for the same luminosity. The problems of ground vibration and of bringing the two bunches into good overlap are of course increased.

The method followed is similar in spirit to one which we developed in the case of beamstrahlung [8]. It shortcuts the use of Airy functions, normally associated with synchrotron radiation problems. We refer the reader to this paper for many details and focus here on these technical and important differences which are encountered in the case of pair production.

In order to simplify the presentation we consider the spinless (Klein-Gordon) case. In the deep quantum regime the rate obtained in the Dirac case is merely proportional to that calculated in the spinless case, since one helicity state is negligible [3]. The overall proportionality factor is now:

$$S(X) = \frac{(1-X)^2 + X^2}{X(1-X)}. \quad (4)$$

As in [3] we solve the wave equation of the electron (positron) in the presence of the bunch to obtain the phase of the production amplitude. The first difference encountered with the previous case of photon radiation is that the first-order contributions to the phase from the electron and the positron cancel each other out. As a result, the condition for a stationary phase in the radial direction is qualitatively different. The radial point of the stationary phase is now given by

$$\frac{\vec{r}}{R} = - \frac{12 \chi(1-\chi)}{(1-\beta)^3} \frac{\vec{K}_\perp}{\epsilon} - 3 \frac{\vec{k}_{f\perp}}{1-\beta} . \quad (5)$$

Here z is the longitudinal variable over which we integrate the radiation amplitude, and $\vec{k}_f$ is the momentum of the electron ($k_f = \chi k$) and $\vec{k}_{f\perp}$ is its component normal to the bunch axis. The normal component of the positron momentum is not quite opposite to it. One has $\vec{k}_{f',\perp} = -\vec{k}_{f\perp} + \vec{K}_\perp$ with $\vec{K}_\perp$ of order $\epsilon \vec{k}_\perp$, where $\epsilon = (N\alpha L_b)/(R^2 k)$ is equal to the disruption parameter, a small quantity. Here N is the number of particles in the bunch. One finds again [3]

$$\frac{L_b}{l_c} = \Lambda = (\epsilon N \alpha)^{1/3} . \quad (6)$$

In (5) and in the following all quantities are normalized. The actual position along the bunch is $z L_b/2$ ($-1 < z < 1$). All momenta are normalized to maximum bending $2N\alpha/R$, with R the radius of the bunch.

As in the case of beamstrahlung, there is in general no real actual stationary point in z . The derivative of the phase with respect to z , obtained after imposing condition (5) is

$$\frac{\partial \phi}{\partial \beta} = \frac{\epsilon N \alpha}{4 \chi(1-\chi)} \left| \frac{12 \chi(1-\chi)}{(1-\beta)^2} \frac{\vec{K}_\perp}{\epsilon} + \vec{k}_{f\perp} \right|^2 . \quad (7)$$

One notes again, though they now have a different form, the proportionality relation between this derivative and the production matrix element squared. This matrix element is indeed found to be

$$\vec{e}_\perp \cdot \vec{m}_\perp = \frac{2N\alpha}{R} \vec{e}_\perp \cdot \left[\frac{12X(1-X)}{(1-z)^2} \frac{\vec{K}_\perp}{\varepsilon} + \vec{k}_{f\perp} \right]. \quad (8)$$

It can be written as a two-component vector, contracted with the unit vector normal to the bunch axis corresponding to the photon polarization, $\vec{e}_\perp$.

We make use of this proportionality relation. It was indeed at the core of our new method in the case of beamstrahlung [8].

The production rate per unit area is written as [3]

$$\frac{I(X)}{\pi R^2} = \frac{\alpha}{\pi} \left(\frac{N\alpha}{\pi} \right)^2 18 X(1-X) \int dz dz' F(z, z'), \quad (9)$$

with

$$F(z, z') = \int d^2 \vec{k}_{f\perp} d^2 \vec{K}_\perp \frac{1}{(1-z)^3} \frac{1}{(1-z')^3} e^{i[\phi(z) - \phi(z')]} \left[\frac{12X(1-X)}{(1-z)^2} \frac{\vec{K}_\perp}{\varepsilon} + \vec{k}_{f\perp} \right] \cdot \left[\frac{12X(1-X)}{(1-z')^2} \frac{\vec{K}_\perp}{\varepsilon} + \vec{k}_{f\perp} \right]. \quad (10)$$

While the structure of F is now different, the steps followed are identical to those already presented [8].

We also introduce similarly a quantity $F_0(z, z')$

$$\begin{aligned} F_0(z, z') &= \int d^2 \vec{k}_{f\perp} d^2 \vec{K}_\perp e^{i[\phi(z) - \phi(z')]} \\ &= \int d^2 \vec{k}_{f\perp} d^2 \vec{K}_\perp e^{i \left(A \frac{\vec{K}_\perp^2}{\varepsilon^2} + B \frac{\vec{K}_\perp \cdot \vec{k}_{f\perp}}{\varepsilon} + C \vec{k}_{f\perp}^2 \right)} \end{aligned} \quad (11)$$

with:

$$\begin{aligned}
 A &= \frac{\epsilon N \alpha}{4 X(1-X)} [12 X(1-X)]^2 \frac{1}{3} \left[\frac{1}{(1-\beta)^3} - \frac{1}{(1-\beta')^3} \right], \\
 B &= \frac{\epsilon N \alpha}{4 X(1-X)} 24 X(1-X) \left(\frac{1}{1-\beta} - \frac{1}{1-\beta'} \right), \\
 C &= \frac{\epsilon N \alpha}{4 X(1-X)} (\beta - \beta').
 \end{aligned} \tag{12}$$

We note that the special z dependence which is now met has a formal similarity with that found in the case of a varying bunch density, which was considered in details in the last three papers of Ref.[3]. We indeed find that many complications can be eliminated as contributing at order Λ^{-1} only. Contributions at that order were calculated in detail for the case of varying densities [3]. They can be altogether neglected here, as we focus on the leading term of order Λ .

It is now a technical matter to use the proportionality between the derivative of the phase and the matrix element squared, in a way similar to that followed previously [8]. The leading term (order Λ) is given by

$$\begin{aligned}
 &F(\beta, \beta') \\
 &= - \frac{[12 X(1-X)]^2}{2} \left[\frac{1}{(1-\beta)^2} - \frac{1}{(1-\beta')^2} \right]^2 \frac{1}{(1-\beta)^3} \frac{1}{(1-\beta')^3} \frac{\partial}{\partial \Lambda} F_0(\beta, \beta').
 \end{aligned} \tag{13}$$

An analysis of the present phase $\phi(z)$ as compared with the previous phase of the beamstrahlung amplitude [3] shows that the coherent radiation length λ_c is identical in those two cases, namely $\lambda_c = L_b(\epsilon N \alpha)^{-1/3}$.

In practice, the coherence condition imposes that the dominant contribution in (9) originates from values of z and z' which do not differ by much more than λ_c , with $\lambda_c \ll L_b$. The term neglected in (13) is then found to be of order Λ^{-2} or $(\lambda_c/L_b)^2$ as compared with the term kept.

What this means physically is that, because of the large factor $\epsilon N \alpha$ in (7), the phase varies very much faster than the other z -dependent factors which are introduced by the specific features of the production process.

We now make a change of variables in (11)

$$\frac{\bar{K}_\perp}{\varepsilon} = \frac{K_\perp}{\varepsilon} + \frac{B}{2A} k_{f\perp} \quad (14)$$

to write F_0 as

$$F_0(z, z') = \int d^2 k_{f\perp} d^2 \bar{K}_\perp e^{i \left(A \frac{\bar{K}_\perp^2}{\varepsilon^2} + \frac{4AC - B^2}{4A} k_{f\perp}^2 \right)} \quad (15)$$

The integration limits on $k_{f\perp}$ correspond to

$$k_{f\perp}^2 \leq \frac{1}{4} (1-z)^2. \quad (16)$$

The integration limits on $K_\perp$ would be more tricky to handle. One can check however that, since z and z' differ within λ_c only, the coefficient of the first term is of order Λ^2 higher than that of the second one. The integral rapidly damps itself out at large $\vec{K}_\perp^2$ and one may thus freely integrate it to infinity. This simplifies (15) to

$$F_0(z, z') = \frac{\pi \varepsilon^2}{A} \int_0^{\frac{1}{4}(1-Z)^2} d^2 \vec{k}_{f\perp} e^{i \frac{4AC - B^2}{4A} k_{f\perp}^2} \quad (17)$$

We can now calculate $F(z, z')$ using (13). There are two terms. The latter one, corresponding to the derivation of the exponential, is however of order Λ^2 as compared with the former one, corresponding to the derivation of the factor in front. It is therefore legitimate to keep only the latter one.

In writing (17) we have again used the fact that z and z' do not in practice differ by much more than λ_c and introduced their average Z in the calculation of the phase space limit. With the same approximation, we combine (9) (13) and (17) to write, with $z' - z = \zeta$,

$$\frac{I(X)}{\pi R^2} = \frac{\alpha}{\pi} 4X(1-X) \int dZ d\zeta \frac{1}{(1-Z)^4} \left[i \frac{\Lambda^3}{4X(1-X)} \zeta \int_0^{\frac{1}{4}(1-Z)^2} u du e^{-i \frac{\Lambda^3 \zeta^3}{12X(1-X)(1-Z)^2} u} \right]. \quad (18)$$

With further obvious simplifications, it lead to

$$\frac{I(X)}{\pi R^2} = \frac{\alpha}{\pi} \frac{\Lambda^3}{8} \int i \zeta d\zeta \int_0^1 v dv e^{-i \frac{\Lambda^3 \zeta^3 v}{3X(1-X)}} \quad (19)$$

We remark that while our integration over Z is now trivial, it would not be the case for the terms which have been neglected. In those cases, the divergences at $z = z' = 1$ are however spurious and call for a detailed treatment of varying densities, as presented in the last paper of Ref.[3]. To order Λ^{-2} we can safely keep only the term arrived at in (19).

The integral over ζ is so quickly damped at large ζ that we can integrate to infinity. We have therefore a beta function removing also the i present in (19), namely

$$i \int d\zeta e^{-i \frac{\Lambda^3 \zeta^3 v}{3X(1-X)}} = \frac{1}{\sqrt{3}} \Gamma\left(\frac{2}{3}\right) \Lambda^{-2} \left[\frac{3X(1-X)}{v} \right]^{2/3} \quad (20)$$

The integral over v now gives a simple factor $3/4$. One therefore finds

$$\frac{I(X)}{\pi R^2} = \frac{\alpha}{\pi} \frac{\sqrt{3} \Gamma(2/3)}{32} \Lambda [3X(1-X)]^{2/3} \quad (21)$$

This is the pair production rate in the Klein-Gordon case with an electron (positron) of fractional energy X . There is the extra factor $S(X)$ (4) in the Dirac case.

One finds that while the rate for pair production is proportional to α/π , it is also proportional to the large quantity Λ . The expression arrived at is particularly simple and it is accurate in the deep quantum regime. Pair production is therefore a serious problem.

There is also a term of order Λ^0 which includes a $\ln L_e$ term [3]. This contribution is easily calculated and is independent of the transverse shape of the bunch. There are also terms of order Λ^{-1} , including the neglected terms of relative order Λ^{-2} with respect to the leading one. They would require a more complicated calculation but they can be neglected at this stage.

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FIGURE CAPTIONS

- 1-a Pair production by a photon in a bunch of electrons (positrons).
- 1-b Pair production by an electron in a bunch of positrons.

Fig. 1