CERN - TH. 5261/88

HANDLING NON-CRITICAL RANDOM SURFACES

- A combinatorial approach to
wormhole formation

J. M. Alberty

CERN - Geneva

ABSTRACT

Reparametrization invariance of triangulated surfaces is extended to allow for fluctuations in topology. As a side result "foamy" configurations appear. The proposed parametrizations include the possibility of repeating labels. The distribution of different topologies is estimated. The topological term in the Einstein action (in two dimensions) is recovered from a conditional probability distribution.

1. Introduction

Random surface dynamics is by now an established approach [1] for studying the behaviour of bosonic strings [2,3]. Special Monte-Carlo (MC) techniques were developed [4,5,6] to study the Polyakov string [7], where the intrinsic metric is dynamical. Currently the surface is triangulated [8] and each triangulation is considered as a particular (discretized) configuration in an internal parameter space. Hence, the functional integral over the intrinsic metric becomes a sum over all possible triangulations. The MC method consists thus of two parts, namely, one where the thermalization of embedding fields (if any) takes place in the background of a fixed lattice and the other where the lattice itself is thermalized (1).

These lattice simulations are likely to revive some of the traditional numerical problems encountered when trying to extract the physics at the continuum limit. Such problems [3,4,5,9,10] (i.e., existence of a continuum limit, determination of critical properties, etc) will not be considered in this work. They are related to the short-distance behaviour of the surfaces.

Maintaining the surface topology fixed throughout the entire MC simulations, the numerical studies of random surfaces have so far ignored one of the major effects due to the interactions. In fact, although the correlation functions can in principle be separately measured for different topologies, combining them is not the right way to obtain the full correlation functions, since the topological fluctuations were obstructed.

In a different context, it has been recently argued that topological fluctuations, occurring through the creation and annihilation of wormholes, might be the mechanism responsible for the vanishing of the cosmological constant [11]. Loss of quantum coherence could be another consequence of the formation of non-trivial topologies [12].

In this letter we will be concerned with topological properties, which arise mainly from the physics at large distances (2).

Here we present the basic combinatorics of a simple toy-model which is able to generate precisely labelled triangulations of orientable and compact surfaces with

(1) For a continuum limit to exist it is necessary that the number of lattice sites diverges at some critical value of the coupling(s). Thus, the lattice thermalization should be done in the framework of a Grand-canonical Ensemble in the number of sites [5].

(2) This may be controversial: for instance, a cut-off in the topology (i.e., a highest genus) on a **finite** lattice can be removed by either removing the ultra-violet or the infra-red cut-offs, or both [13].

arbitrary genus, by exploring more completely the freedom to parametrize the surfaces at different points. As a side result, a much larger number of configurations ("foam") appears. Those extra configurations do not represent regular surfaces.

Partial loss of information is an essential ingredient of the proposed labelling method.

Starting from no action, we determine the relative weights of different topologies (for orientable surfaces) and recover the usual Einstein term in two-dimensional euclidean pure gravity, from a conditional probability distribution.

We end with a few qualitative comments on mean "free" paths and average distances on the labelled surfaces.

2. The Model

Consider a triangulated surface with the topology of a sphere and a very high number (P) of sites on it. The lattice associated with this surface is supposed to be fixed (i.e. non-dynamical). Let us call it the Ur-lattice (Fig. 1.a). It consists of the P ur-sites, connected by (ur) **links**, and the shortest closed circuit, formed by three links (triangle) defines a **plaquette**. The links have fixed length (say, equal to 1). Furthermore, we will assume that the ur-lattice has no sites with coordination number less than three.

This lattice may eventually be embedded in a D -dimensional space-time, or be considered as representing a 2-dimensional space-time lattice itself, on which various fields can be defined, etc.

Next, to parametrize the "surface", an observer visits all the P Ur-lattice sites and assigns to each one of them an integer label running from 1 to N (with $N \geq P$).

Since the physics does not depend on the particular parametrization chosen, there are $N!/(N-P)!$ possible different labellings (also called parametrizations or labelled triangulations) of the Ur-lattice.

Besides this freedom in the label assignments, the observer has an extra degree of freedom in this model ; namely, he can **forget** that a particular label has already been used on a site and may use it again, with the same probability as before (Fig.1.b).

Furthermore, if the lattice is embedded in space-time or if it represents a

regularized 2-dimensional space-time with fields living on it, we require that there exists a map

$$i \xrightarrow{\quad} \varphi_i \quad (2.1)$$

from the set of labels onto the set of fields (or coordinates). Hence, to distinct Ur-sites, which were identically labelled, there must correspond the same value of the field variable (Fig.1.c). Consequently, if the lattice is embedded in space-time, co-incident labels will imply the formation of self-intersections of the triangulated surface. Alternatively, if the surface represents 2-dimensional space-time, the self-intersections will occur at identically labelled sites, and the coincidence of the field values implied by (2.1) is, in a sense, a primordial symmetry.

Finally we will hereafter assume that each label (from 1 to N) is equally likely to occur at each particular site, and that each site will not be labelled more than once.

This model is sufficiently simple to allow for a straightforward counting of configurations. Hence e.g. the probability for an embedded surface to have a certain topology can be determined.

Here we will not consider possible embeddings of the lattice in space-time or fields defined on it and concentrate on the combinatorics of the labelling.

3. Counting Configurations

We start by counting the number of configurations with exactly m pairs of identically labelled (Ur) sites, corresponding to m self-intersections. More precisely, we count the number of configurations in which a first arbitrary pair of sites gets a common (and arbitrary) label, a second pair gets a second (and distinct) label, and so on, and the remaining $P-2m$ sites remain distinctly labelled. That number is

$$2^{-m} \cdot \frac{N!}{(N-P+m)!} \cdot \frac{P!}{(P-2m+1)!} \underset{(P \rightarrow \infty)}{\approx} 2^{-m} \frac{N!}{(N-P+m)!} e^{(2m-1) \cdot \ln P} \quad (3.1)$$

The important factor to be remembered is the exponential dependence on m with a P -dependent coefficient,

We should stress that the triangulations previously counted merely self-intersect at various, isolated points. Thus, the surface they triangulate are generally pinched-off at those points and will not represent genuine 2-manifolds.

To obtain a triangulated surface with genuine handles or wormholes (1) we must "blow-up" the pinching points and make, at least, **thin handles**. A thin handle forms when (Fig.2):

- i) Two sets of three ur-sites (each) are assigned to identical labels;
- ii) In each set, the three sites are nearest neighbours to each other;
- iii) The labelling orientation on one set (say clockwise) is the opposite of the orientation along the other (**orientation preserving** labelling);
- iv) None of the sites in one set is nearest neighbour to any site in the other set.

A triple coincidence of labels (to produce a thin handle) is certainly much less likely to occur than a double coincidence. On the other hand there are many more ways in which it can occur. This entropy effect generally more than compensates the decrease in the probability for triple coincidences. If we include orientation-preserving as well as non-preserving labellings, the number of configurations with a single thin-handle on a lattice of sites with uniform coordination number q (just to simplify the argument temporarily) is given by

$$\binom{P}{2} \cdot \binom{q}{2}^2 \cdot \frac{N!}{(N-P+3)!} \approx \frac{N!}{(P \rightarrow \infty)^2 \cdot (N-P+3)!} \binom{q}{2}^2 \cdot e^{2 \cdot \ln P} \quad (3.2)$$

The generalization of this result to configurations with m thin-handles is straightforward. In the large- P limit, the number of configurations with m thin-handles is

$$\frac{N!}{(N-P+3m)!} \cdot e^{\{2 \cdot \ln [q \cdot (q-1)] - 3\} \cdot m + O(1/P)} \cdot e^{2m \cdot \ln P} \quad (3.3)$$

which shows essentially the same logarithmic dependence on P as (3.1) does.

(1) Since we will only consider the topological decorations of the original **connected** ur-lattice (obtained by the parametrizations) we are allowed to identify handles with wormholes

In the last counting an extra assumption was made, namely, that the different thin handles did not interfere with each other (i.e. **dilute-gas** approximation for thin handles).

In this way we see that starting from an original, unlabelled spherical ur-lattice, we were able to construct higher topologies through special coincidences in labelling sites of an intermediate lattice of parameters.

Hence the topology is allowed to fluctuate and we generate an Ensemble of different topologies.

An equivalent way to understand the topological fluctuations is to say that the model can generate fluctuations in the curvature density. Generally however these fluctuations can be divided into two categories: those which preserve the topology and those which do not. The first category includes the configurations in which coincident labels affect only nearest-neighbour sites of the ur-lattice. When a pair of nearest-neighbours coalesce, the resulting site inherits all the nearest-neighbours of the original pair. Thus, its curvature decreases with respect to the curvatures at the original pair. This effect is accompanied by a one-unit decrease in the effective number of sites and the increase of curvature at points located near the two coalescing sites. These two effects exactly cancel each other so that the genus of the surface remains constant.

The mechanism just described was proposed in [4,6] as a way to reduce the number of sites of a triangulated surface and generate a Grand-canonical Ensemble of surfaces with fixed genus. In the same references one can find the opposite mechanism, i.e., one which increases the number of sites. No such mechanism however is proposed in the present model. Instead, all labelled triangulations descend from the same ur-lattice by applying the labelling procedure.

The curvature-density fluctuations in the second category generalize the mechanism proposed in [4,6] in the sense that coincident labels may be assigned to non-nearest-neighbour sites, in a fairly arbitrary way.

As a result of the "chaotic" labelling proposed here, a vast majority of configurations obtained will not represent triangulations of any 2-manifold (Fig.3).

Before commenting on these irregular configurations, let us come back to the problem of counting regular configurations with a fixed number of handles.

The partition function for a general compact orientable triangulated surface includes, as usual, a (formal) sum over all topologies. For each topology however, a sum over all sizes (thickness) and all matching angles of the handles must be included as well.

So far we have seen how to generate the thin(nest) handles. Handles of arbitrary sizes appear when the sites sitting on a thin loop acquire labels which match the labels of another set of sites sitting on another thin loop with identical perimeter (and in the same sequence), elsewhere on the lattice (Fig.4).

A **plaquette** (i.e. a triangle formed by three nearest-neighbour sites) is the simplest example of a thin loop. On the lattice **dual** to the ur-lattice, its image is a single point.

On the other hand, the **dual-image** of a generic thin loop is a **polymer line** or even a **tree**, made of links of the dual lattice (dual links).

The restriction to pairings of thin loops (rather than arbitrary loops) eliminates from the counting most of the configurations which do not represent triangulations of a surface. Essentially, the only undesirable configurations remaining are those whose label-sequence along one of the thin loops cannot be transformed into the label-sequence along the other ("matching") thin loop by any circular permutation of those labels. These undesirable configurations include Klein bottles, etc.

Thus, the problem of counting the configurations with a given topology and handles of fixed sizes reduces to the counting, on the dual lattice, of the number of non-intersecting pairs of trees with the same size, each tree carrying a degeneracy factor associated with the circular permutations of the labels along the corresponding thin loop.

Let $\tilde{P}$ denote the number of points on the dual lattice. Then the number of trees with exactly L links is given by

$$Q(L) = \frac{\tilde{A}(L)}{L+1} \cdot \tilde{P} \quad (3.4)$$

where $\tilde{A}(L)$ counts the number of general trees with L links, passing through a **fixed** point of the dual lattice. Then the total number of **sets of two trees** with L links each (tree) is approximated by

$$\tilde{A}_1(L) \approx \frac{1}{2} \cdot \tilde{P} \cdot [\tilde{P} - 2 \cdot (2+L)] \cdot \left[\frac{\tilde{A}(L)}{L+1} \right]^2 \quad (3.5)$$

This expression already takes into account the constraint that between two **different** trees, the nearest points should always be at least two links distant from each other. Otherwise there would be no reason to say that the trees were distinct — rather they would be two parts of the same tree.

Nevertheless the expression (3.1) still neglects excluded-volume effects arising in the special cases when the two trees of a pair are separated by a minimal distance of two links. This will however be an improbable situation if the number of tree-links is much smaller than the total number of links on the dual lattice — **dilute-gas approximation (DGA) for the trees.**

In this approximation, the **number of sets** containing **one pair of trees with L_1 links each, another pair of trees with L_2 links each and so on, up to an m -th pair of trees with L_m links each,** is given by

$$\begin{aligned} \tilde{A}_m(L_1, \dots, L_m) &\approx \\ &\approx 2^{-m} \cdot \prod_{k=1}^m \left[\tilde{P} - 4 \sum_{j=1}^{k-1} (2+L_j) \right] \cdot \left[\tilde{P} - 2 \cdot (2+L_k) - \right. \\ &\quad \left. - 4 \cdot \sum_{j=1}^{k-1} (2+L_j) \right] \cdot \left[\frac{\tilde{A}(L_k)}{L_k+1} \right]^2 \end{aligned} \quad (3.6)$$

Now we can come back to the direct (labelled) lattice and write the combinatorial factor $\tilde{A}_m(L_1, \dots, L_m)$ as

$$\begin{aligned} A_m(P_1, \dots, P_m) &\approx 2^m \cdot \prod_{k=1}^m \left[P - \chi_0 + 2(k-1) - \right. \\ &\quad \left. - 2 \sum_{j=1}^{k-1} P_j \right] \cdot \left[P - \chi_0 + 2(k-1) - 2 \sum_{j=1}^{k-1} P_j - (P_k - 1) \right] \cdot \left[\frac{A(P_k)}{P_k - 2} \right]^2 \end{aligned} \quad (3.7)$$

where χ_0 is the Euler characteristic of the ur-lattice ($\chi_0 = +2$) and $A(P_k)$, defined as

$$A(P_k) \equiv \tilde{A}(L_k)$$

counts the **number of thin-loops visiting P_k sites of the direct lattice.**

In (3.7) we used the following relation between a thin loop and its dual tree:

$$P_j = 3 + L_j \quad (3.8)$$

together with the identity

$$\tilde{P} = 2 \cdot (P - \chi_o) \quad (3.9)$$

between the number of faces of the direct lattice and the number of dual-lattice sites.

Expression (3.7) completes the counting of sets of pairs of trees, as indicated. The second part of the counting problem consists of the label combinatorics and follows along lines similar to those previously described.

In the DGA, the total number of configurations with **genus m** and handles with sizes up to $P_{\max}$, preserving the original orientation of the triangulation is finally given by

$$\mathcal{D}_m = \sum_{P_1=3}^{P_{\max}} \dots \sum_{P_m=3}^{P_{\max}} \frac{N!}{(N - P + \sum_{k=1}^m P_k)!} P_1 \dots P_m \cdot \mathcal{A}_m(P_1, \dots, P_m) \quad (3.10)$$

Even though we are working in the DGA, there is an intermediate regime where $L_k \gg 1$ (i.e., $1 \ll L_k \ll P$) and the number of trees is given by the asymptotic expression [14,15] (independent of the particular details of the lattice):

$$\tilde{A}(L_k) \sim r^{L_k} \cdot L_k^{\mu} \quad (3.11)$$

where r and μ are universal parameters, depending only on the dimensionality of the lattice. In two dimensions we have [15]

$$\begin{aligned} r &\approx 5 \\ \mu &= -1 \end{aligned} \quad (3.12)$$

From the relation (3.7) we obtain

$$A_m(P_1, \dots, P_m) \approx (2P^2)^m \cdot n^{2P} \cdot \prod_{k=1}^m P_k^{2(\mu-1)} \quad (3.13.a)$$

$$P \equiv \sum_{k=1}^m P_k \quad (3.13.b)$$

This is the crudest approximation to $A_m(P_1, \dots, P_m)$ in the sense that IP was simply neglected with respect to P . Eq.(3.10) can then be approximated by

$$D_m \approx (2P^2)^m \cdot N! \cdot \sum_{P=m \cdot P_{\min}}^{m \cdot P_{\max}} \frac{n^{2P} \cdot M(P, m)}{(N-P+P)!} \cdot \Omega_{m\mu}(P) \quad (3.14)$$

where $IP_{\min}$ is set equal to 3 but could in principle be left arbitrary (although not large). $M(IP, m)$ denotes the number of different ways of writing IP as a sum of m positive integers (equal or distinct) and has the asymptotic form

$$M(IP, m) \approx M_0(m) + M_1(m) \cdot P^{m-1} \quad (3.15)$$

with

$$M_0(m) \equiv (-1)^{m-1} \cdot m$$

$$M_1(m) \equiv m \cdot \sum_{k=0}^{m-2} \frac{(-1)^k \cdot (m-1-k)^{m-1}}{k! \cdot (m-k)!} \quad (3.16)$$

$\Omega_{m\mu}(IP)$ is the constrained sum

$$\Omega_{m\mu}(P) \equiv \sum_{P_1=P_{\min}}^{P_{\max}} \dots \sum_{P_m=P_{\min}}^{P_{\max}} \prod_{k=1}^m P_k^{2\mu-1} \quad (3.17)$$

$$(P_1 + \dots + P_m = P)$$

A straightforward evaluation of (3.14) does not seem to be easy due to the

constraint in (3.17). However, for $lP_{\max}$ large enough, we can use the continuous variables $\Phi(x)$ and x , coming from the substitutions

$$\begin{aligned} P_K &\longrightarrow \phi(x) \\ K &\longrightarrow x \end{aligned}$$

and use the Euler summation-formula to approximate (3.14) by

$$\mathcal{D}_m \approx N! 2^m P^{2m} \cdot \int [d\phi(x)] e^{-\Gamma_{\text{eff}}[\phi]} \quad (3.18)$$

with

$$\Gamma_{\text{eff}}[\phi] = \ln \Gamma(1+N-P + \int_1^m dx \phi(x)) - \int_1^m dx [2 \ln \pi \cdot \phi(x) + (2\mu-1) \ln \phi(x)] \quad (3.19)$$

and the measure $[d\phi(x)]$ coming from the substitution

$$\prod_K P_K \longrightarrow [d\phi(x)]$$

The integral in (3.18) could easily be estimated semi-classically if the contributions of the uniform configurations

$$\Phi(x) = \Phi_0 = \text{Constant} \quad (3.20)$$

were dominant. $\Gamma_{\text{eff}}[\Phi(x)]$ then becomes simply (see Fig. 5)

$$\begin{aligned} \Gamma_{\text{eff}}[\Phi_0] &= \ln(1+N-P + m\Phi_0) - \\ &\quad - m \cdot [2 \ln \pi \cdot \Phi_0 + (2\mu-1) \ln \Phi_0] \end{aligned} \quad (3.21)$$

Notice that, for $m \geq 1$, $\Gamma_{\text{eff}}[\Phi_0]$ runs to $-\infty$ as $\Phi_0 \rightarrow 0$. Hence, the preferred uniform configurations are formed by wormholes squeezed to their smallest size in parameter space (the ultra-violet cut-off $lP_{\min} = 3$). This agrees qualitatively with what one expects when extremizing the action in classical gravity with wormholes, where they would be pinched.

This obvious preference of higher topologies for thin handles might be a

consequence of the fact that the restriction to uniform sizes (for the handles) makes the probability for each configuration with reasonably thick handles so small that even the larger entropy coming from the lattice volume (and from the distortions of the thin loops) is unable to compensate for it. Only a more precise estimate of (3.14) would tell us whether or not the thin-handle dominance is a genuine effect, common to most of the unrestricted configurations as well.

By selecting these uniform configurations, one easily obtains a lower bound for (3.14). An upper bound is trivial to get by removing the constraint on (3.17),

$$P! 2^m P^{2m} \cdot m^{-(3m+1)} \cdot \Gamma(3m-1) \cdot \mu(\lambda^2, 3m, 0) < \mathcal{D}_m$$

$$\Omega_{m,-1}(\mathbb{P}) < e^{-2.56... \cdot m} \quad (3.22)$$

in the case $N = P$ and with $IP_{\min} = 3$. Here we find the function [16]

$$\mu(x, \beta, \alpha) \equiv \int_0^\infty dt \frac{x^{\alpha+t} t^\beta}{\Gamma(\beta+1) \cdot \Gamma(\alpha+t+1)}, \quad \left(\begin{array}{l} x \neq 0 \\ \Re \beta > -1 \end{array} \right) \quad (3.23)$$

From (3.14) it is obvious that, in the D.G.A., the exponential dependence on the topology m , already found in (3.1) and (3.3), is not going to be spoiled by the process of summing over all sizes, shapes, positions and "twists" of the m handles, since no matter what function of $IP_{\max}$ comes out of the summation over IP , its dependence on the large $IP_{\max}$ cannot beat the dependence on the (much) larger (logarithm of) P , already present in the m -dependent exponent.

On the other hand however, this exponential growth of with m , indicated by (3.14), seems to contradict the usual exponential suppression of the probability of higher topologies resulting from the presence of the Einstein term (- $\chi\chi$) in the action for pure gravity in two dimensions.

In the framework of the present model, all (regular) configurations are equally likely to occur — the topological index is just a traditional way of grouping those configurations. Hence, modulo a normalization factor, the **a priori** probability of each configuration does not depend on the configuration. However, the **conditional** probability that a particular configuration occurs — given that it must have a certain topology m — will be inversely proportional to the number of configurations with that topology that is:

$$\mathcal{D}_m^{-1} \sim e^{-2m \cdot \ln P}$$

Thus, modulo factors which are either dependent on $IP_{\max}$ or independent of m , we recover the usual suppression factor $\exp(\gamma\chi)$ as a conditional probability distribution.

The question of the **absolute** probability of configurations, in the present model, involves the existence of a **maximal topology**, $m_{\max}$, since the normalization factor $\mathcal{N}$ will be

$$\frac{1}{\mathcal{N}} \sim \frac{1}{\mathcal{D}_0 + \mathcal{D}_1 + \dots + \mathcal{D}_{m_{\max}}} \quad (3.24)$$

and $m_{\max}$ will, in turn, be bounded by some increasing function of P .

4. Mean Free Distance and Recurrent Events

Consider a sequence of visits to the (ur) sites, starting from an arbitrary site and proceeding to a nearest neighbour. Every site gets a label as described above.

A **failure** is defined to be the choice of a label which has already been picked up before, irrespective of its actual value.

The distribution of failures after a large number of visits will be related to the importance of higher topologies and the foam, at large distances.

In fact, each visit constitutes a trial with only two possible outcomes: failure, with probability p , and success with probability $1-p$. However these probabilities do not remain constant throughout the sequence of trials. Instead, the increasing size of the sample makes more probable a next coincidence of labels. Hence, the consecutive waiting times between failures are neither independent of nor subject to a common distribution.

This makes the problem of the distribution of coincident labels a hard question in Renewal Theory [17]. Qualitatively however, one may expect that the probability of a long sequence without the occurrence of a single failure (i.e. without going back to some previously visited point) will decrease quickly with the number of visits. And since long-distance travels (on regular configurations) must be associated to long sequences of visits without failures we conclude that either the mean free distance is rather small in the ensemble of all possible configurations, or that

large regular configurations are highly improbable ones, which we already know is the case.

On the other hand, the probability that a coincidence occurs in a sequence of n successive trials, irrespective of the value of the coinciding labels and also irrespective of whether or not that label has already appeared before that n -sequence — let us call this an **n -restricted failure** — increases with n but is otherwise independent of where the sequence started. Thus, if we keep n with a value giving a reasonably small probability for restricted failures, then after a sufficiently large number of n -sequences, we will be able to visit sites arbitrarily distant from the first one, even though the trip has not been smooth: we have diffused through the foam by going first around an enormous number of small "bubbles" with typical sizes of the order of n .

Acknowledgements

We thank Peter Ruback for a suggestion on the combinatorics and Holger Nielsen for a nice discussion a long time ago.

References:

- [1] - A.B. Zamolodchikov, Phys.Lett. **117B** (1982), p. 87.
T. Eguchi and H. Kawai, Phys.Lett. **114B** (1982), p. 247.
T. Eguchi and H. Kawai, Phys.Lett. **110B** (1982), p. 143.
D. Weingarten, Phys.Lett. **90B** (1980), p. 280.
M. Bander and C. Itzykson, "Fields on a Random lattice II. Random Surfaces: A search for a discrete model", in "Non-linear equations in classical and quantum field theory", ed. by N.Sanchez, 1983/84.
- [2] - J. Ambjorn, "Random Surfaces and Strings", preprint NBI-HE-87-53
J. Ambjorn, Ph. de Forcrand, F. Koukiou and D. Petritis, Phys. Lett. **B197** (1987), p.548.
J. Ambjorn, B. Durhuus and T. Jonsson, Phys. Rev. Lett. **58** (1987), p.2619.
- [3] - J. Ambjorn, B. Durhuus, J. Fröhlich and P. Orland, Nucl. Phys. **B270** [FS 16] (1986), p.457.
J. Ambjorn, B. Durhuus and J. Fröhlich, Nucl. Phys. **B275** [FS 17] (1986), p.161.
- [4] - V.V. Zmushko and A.A. Migdal, Sov. J. Nucl. Phys. **45** (5) (1987), p.929.
- [5] - V.D. Boulatov and V.A. Kazakov, Phys. Lett. **B184** (1987), p.247.
- [6] - J. Jurkiewicz, A. Krzywicki and B. Petersson, Phys. Lett. **B177** (1986), p.89.
A. Krzywicki, "Computer experiments with Random Surfaces", preprint LPTHE-Orsay 87-71 (Nov.87), Talk given at the International Symposium on Field Theory on the Lattice, 28 Sep.-2 Oct. 1987, Seillac (France), ed. by A. Billoire, R. Lacaze, A. Morel, O. Napoly and J. Zinn-Justin.
D.V. Boulatov, V.A. Kazakov, I.K. Kostov and A.A. Migdal, Nucl. Phys. **B257** (1986), p.641.
D.V. Boulatov, V.A. Kazakov, I.K. Kostov and A.A. Migdal, Phys. Lett. **B174** (1986), p.87.
- [7] - A.M. Polyakov, Phys.Lett. **103B** (1981) 207.

- [8] - D.J. Gross, Phys. Lett. **138B** (1984), p. 185.
 A. Billoire, D.J. Gross and E. Marinari, Phys. Lett. **139B** (1984), p.75.
 T. Eguchi and T. Filk, Univ. Tokyo preprint UT-425 (1984)
 T. Eguchi, R. Nakayama and S.K. Yang, Univ. Tokyo prep. (1984).
 V.A. Kazakov, Phys. Lett. **150B** (1985), p.282.
- [9] - B. Durhuus, J. Fröhlich and T. Jonsson, Nucl. Phys. **B240** (1984), p.453
 F. David, Nucl. Phys. **B257** (1985), p.543.
 D. Espriu, Phys. Lett. **B194** (1987), p.271.
 J. Ambjorn, B. Durhuus and J. Fröhlich, Nucl. Phys. **B257** (1985), p.433.
 A. Kazakov, I.K. Kostov and A.A. Migdal, Phys. Lett. **157B** (1985), p.295.
- [10] - I.K. Kostov and M.L. Mehta, Phys. Lett. **B189** (1987), p.118.
 I.K. Kostov and A. Krzywicki, Phys. Lett. **B187** (1987), p.149.
- [11] - S.W. Hawking, Phys. Lett. **134B** (1984), p.403.
 S. Giddings and A. Strominger, Nucl. Phys. **B306** (1988), p.890.
 E. Baum, Phys. Lett. **133B** (1983), p.185.
 S. Coleman, "Why there is Nothing rather than Something: A theory of the Cosmological Constant", Harvard Univ. preprint HUTP-88/A022.
- [12] - S.W. Hawking, Phys. Lett. **B195** (1987), p. 337.
 G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, Mod. Phys. Lett. A, vol.3 (1988), p.1231.
 A. Strominger, Phys. Rev. Lett. **52** (1984), p.1733.
 S. Coleman, Nucl. Phys. **B307** (1988), p.867.
 S.W. Hawking, Phys. Rev. **D37** (1988), p.904.
- [13] - R. Schrader, J. Stat. Phys. **40** (1985), p.533.
- [14] - D.S. Gaunt, J.E.G. Lipson, J.L. Martin, M.F. Sykes, G.M. Torrie, S.G. Whittington and M.K. Wilkinson, J.Phys.A: Math. Gen. **17** (1984) p.211.
- [15] - J. Fröhlich, with A. Bovier and U. Glaus, "Mathematical Aspects of the Physics of Disordered systems", in "Phenomenes critiques, systemes aleatoires, theories de jauge", Les Houches 1984, ed. by K. Osterwalder and R. Stora.

[16] – A. Erdelyi, "Higher transcendental functions", Vol. III, p.217, McGraw-Hill, 1953.

[17] – W. Feller, "An Introduction to Probability Theory and its Applications", 3rd ed., Vol.I, John Wiley & Sons, 1968.

Figure Captions

Figure 1.

- a) The Ur-Lattice.
- b) The Labelled Lattice — a possible configuration (Label l appears repeated)
- c) Effect of the map (2.1) on the embedded labelled lattice.

Figure 2.

- a) The sequences of labels around two matching triangles (simplexes) run in opposite directions, to preserve the surface orientation.
- b) Formation of an oriented thin handle.

Figure 3.

A few examples of configurations which are not associated to any triangulated orientable 2-surface:

- a) Sequences of coincident labels with identical orientation around triangles lead to non-orientable surfaces with thin handles (Klein bottles);
- b) Coincidences at isolated sites;
- c) Formation of handles with sections;
- d) Multiple (≥ 3) occurrence of the same label.

Figure 4.

Two matching thin loops (solid lines) on the lattice (thin lines) and their dual images (dotted lines) — a branched and a non-branched polymer (The use of a regular honeycomb lattice was arbitrary).

Figure 5.

The exponent of the integrand in (3.18) as a function of the handle size Φ_0 and of the topology m , for the case of uniform regular configurations with $N = P$. Small values of Φ_0 are generally favoured. The lower topologies $m = 1$ and $m = 2$ exhibit a local maximum of $-\Gamma_{\text{eff}}[\Phi_0]$ above the cut-off $IP_{\text{min}}=3$.

a)

b)

c)

Fig. 1

a)

b)

Fig. 2

a)

b)

c)

d)

Fig. 3

Fig. 4

Fig. 5