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Algebraic Characterization of the Topological σ - Model

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ABSTRACT

The basic cohomology for a Kahlerian topological non linear σ -model is defined by combining the Friedan non linear splitting symmetry with the BRS symmetry introduced by Witten. It turns out that the basic cohomology can be identified with the De-Rham cohomology of the target manifold leading to the observables of the model.

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1. Introduction

The topological non-linear σ -model has been proposed by Witten [1] in the context of the topological field theories [2]. The model is built by means of a BRS symmetry which leads to a cohomological trivial classical lagrangian; i.e. it is a pure BRS variation. In spite of this, the model possesses a non trivial class of observables [1] which, as indicated by Witten, are of a global topological nature and are given by the De-Rham cohomology of the target manifold. However, as shown recently by Ouvry, Stora and Van Baal [3] in the case of topological gauge theories, the definition of the observables involves what is called the basic cohomology of the BRS operator rather than its usual cohomology. This is due to the fact that the BRS operator has trivial cohomology while, as shown in [3] the Witten observables belong to its basic cohomology. A similar situation is met in the case of the topological σ -model, where, on the space of forms, the BRS operator acts like the exterior derivative and hence the Poincare lemma ensures that locally its cohomology is trivial; in other words, to recover the De-Rham cohomology of the target manifold, it seems necessary to introduce some global information. This is achieved by combining the Witten BRS symmetry with the Friedan non linear shift symmetry [4] which arises when the coordinates used to characterize the model belong to the tangent space of the target manifold, as in the case of the normal coordinates. Indeed, as shown by Friedan [4], the non linear shift symmetry has a geometrical interpretation as a non linear flat connection in the tangent space of the target manifold and can be used to define global quantities. The aim of this work is to show that the Friedan symmetry can be combined with the Witten symmetry to define a basic cohomology which turns out to be the De-Rham cohomology and then, in analogy with the case of topological gauge theories, it allows us to recover the Witten global observables of the model. The work is organized as follows. In section 2. we introduce the Friedan non linear symmetry and we combine it with the Witten symmetry; section 3. is devoted to computing the basic cohomology of the BRS operator. The concluding Appendix is devoted to showing that the BRS operator has trivial cohomology.

2. The Model

Let Σ be the two-dimensional flat space-time and M a Kahlerian n -dimensional compact manifold. The bosonic field of the model is defined by the map :

$$\Phi : \Sigma \rightarrow M \quad (2.1)$$

and described locally by the functions $\phi^i(x)$ (x denotes a point in Σ). Introducing the normal coordinates, the field ϕ^i splits as:

$$\phi^i(x) = \varphi^i + \pi^i(\varphi, \xi) \quad (2.2)$$

where φ^i is the classical constant background and $\xi^i(x)$ is taken as the quantum field. The functions $\pi^i(\varphi, \xi)$ are formal power series in ξ^i :

$$\pi^i(\varphi, \xi) = \xi^i - \sum_{n=2}^{\infty} \frac{1}{n!} \xi^{j_1} \dots \xi^{j_n} \Gamma_{j_1 \dots j_n}^i(\varphi) \quad (2.3)$$

with

$$\Gamma_{j_1 \dots j_n}^i(\varphi) = \tilde{\nabla}_{j_1} \dots \tilde{\nabla}_{j_{n-2}} \Gamma_{j_{n-1}, j_n}^i(\varphi) \quad (2.4)$$

where $\tilde{\nabla}$ indicates that the covariant derivative is taken only with respect to the lower indices of the Riemann connection $\Gamma_{jk}^i(\varphi)$. The non linear splitting (2.2) allows us to introduce a non linear shift symmetry. This symmetry derives from the possibility of varying the fields φ^i and ξ^i in a way such that the total variation of ϕ^i vanishes. Let us consider, indeed, the transformation :

$$\begin{aligned} \delta_v \varphi^i &= v^i \\ \delta_v \xi^i &= v^k F_k^i(\varphi, \xi) \end{aligned} \quad (2.5)$$

where v^i are constant parameters and $F_k^i(\varphi, \xi)$ is determined by the requirement that ϕ^i be invariant, i.e. :

$$\delta_k^i + \frac{\partial \pi^i}{\partial \varphi^k} + F_k^l \frac{\partial \pi^i}{\partial \xi^l} = 0 \quad (2.6)$$

The equation (2.6) implies that :

$$\left(\frac{\partial F_k^i}{\partial \varphi^j} - \frac{\partial F_j^i}{\partial \varphi^k} \right) + \left(\frac{\partial F_k^i}{\partial \xi^l} F_j^l - \frac{\partial F_j^i}{\partial \xi^l} F_k^l \right) = 0 \quad (2.7)$$

and, by (2.3) :

$$F_k^i(\varphi, 0) = -\delta_k^i \quad (2.8)$$

The condition (2.7) shows that the transformation (2.5) is Abelian. The field $\xi^i(x)$ is a vector belonging to the tangent space T_φ in the point of M whose coordinates are given by φ^i . Its introduction allows us to introduce functions and tensor-valued functions which are defined on T_φ , i.e. which depend both on φ^i and ξ^i . As shown by Friedan [4], $F_k^i(\varphi, \xi)$ has a geometrical interpretation as a non linear flat connection in the tangent space T_φ and can be used to define global quantities on M. Indeed if a tensor-valued function $\tilde{\vartheta}_{i_1 \dots i_n}(\varphi, \xi)$ satisfies the condition :

$$\frac{\partial \tilde{\vartheta}_{i_1 \dots i_n}}{\partial \varphi^k} + F_k^q \frac{\partial \tilde{\vartheta}_{i_1 \dots i_n}}{\partial \xi^q} + \tilde{\vartheta}_{q \dots i_n} \frac{\partial F_k^q}{\partial \xi^{i_1}} + \dots + \tilde{\vartheta}_{i_1 \dots i_{n-1} q} \frac{\partial F_k^q}{\partial \xi^n} = 0 \quad (2.9)$$

then $\tilde{\vartheta}_{i_1 \dots i_n}$ is the expression in coordinates of a unique globally defined tensor $\tilde{\vartheta}$ on M. For example, the metric $\tilde{g}_{ij}(\varphi, \xi)$ and the complex structure $\tilde{J}_{ij}(\varphi, \xi)$, being global quantities on M, satisfy (2.9) :

$$\frac{\partial \tilde{g}_{ij}}{\partial \varphi^k} + F_k^q \frac{\partial \tilde{g}_{ij}}{\partial \xi^q} + \tilde{g}_{qj} \frac{\partial F_k^q}{\partial \xi^i} + \tilde{g}_{iq} \frac{\partial F_k^q}{\partial \xi^j} = 0 \quad (2.10)$$

$$\frac{\partial \tilde{J}_{ij}}{\partial \varphi^k} + F_k^q \frac{\partial \tilde{J}_{ij}}{\partial \xi^q} + \tilde{J}_{qj} \frac{\partial F_k^q}{\partial \xi^i} + \tilde{J}_{iq} \frac{\partial F_k^q}{\partial \xi^j} = 0 \quad (2.11)$$

For the Riemann connection :

$$\tilde{\Gamma}_{jk}^i(\varphi, \xi) = \frac{1}{2} \tilde{g}^{il} \left(\frac{\partial \tilde{g}_{kl}}{\partial \xi^j} + \frac{\partial \tilde{g}_{jl}}{\partial \xi^k} - \frac{\partial \tilde{g}_{jk}}{\partial \xi^l} \right) \quad (2.12)$$

one finds:

$$\frac{\partial \tilde{\Gamma}_{jm}^l}{\partial \varphi^i} + F_i^k \frac{\partial \tilde{\Gamma}_{jm}^l}{\partial \xi^k} + \tilde{\Gamma}_{km}^l \frac{\partial F_i^k}{\partial \xi^j} + \tilde{\Gamma}_{jk}^l \frac{\partial F_i^k}{\partial \xi^m} - \tilde{\Gamma}_{jm}^k \frac{\partial F_i^l}{\partial \xi^k} + \frac{\partial^2 F_i^l}{\partial \xi^j \partial \xi^m} = 0 \quad (2.13)$$

The normal coordinate expansion of F_i^j , $\tilde{g}_{ij}$, $\tilde{\Gamma}_{jk}^i$ reads:

$$F_i^j = -\delta_i^j - \Gamma_{ik}^j(\varphi) \xi^k - \frac{1}{3} R_{kli}^j(\varphi) \xi^k \xi^l + \dots \quad (2.14)$$

$$\tilde{g}_{ij}(\varphi, \xi) = g_{ij}(\varphi) + \frac{1}{3} R_{iklj}(\varphi) \xi^k \xi^l + \dots \quad (2.15)$$

$$\tilde{\Gamma}_{jk}^i(\varphi, \xi) = \frac{1}{3} (R_{jnk}^i(\varphi) + R_{knj}^i(\varphi)) \xi^n + \dots \quad (2.16)$$

where $\Gamma_{ik}^j(\varphi)$, $R_{jnk}^i(\varphi)$ are the Riemann connection and the Riemann tensor built with

$g_{ij}(\varphi)$. One introduces a fermionic field λ^i :

$$\lambda^i = \eta^i + \chi^i(x) \quad (2.17)$$

where η^i is, in analogy with (2.2), a constant fermionic background and $\chi^i(x)$ is the quantum fluctuation. One introduces also the self-dual fields

$$\begin{aligned} \rho_\alpha^i &= \varepsilon_\alpha^\beta \tilde{J}_j^i(\varphi, \xi) \rho_\beta^j \\ H_\alpha^i &= \varepsilon_\alpha^\beta \tilde{J}_j^i(\varphi, \xi) H_\beta^j \end{aligned} \quad (2.18)$$

where ε_α^β is the complex structure of Σ

$$\varepsilon_\beta^\alpha \varepsilon_\gamma^\beta = -\delta_\gamma^\alpha \quad (2.19)$$

The mass dimension of the field $\lambda^i, \rho_\alpha^i, H_\alpha^i$ are respectively 0, 1, 1 and the assigned ghost charge +1, -1, 0. The field content of the model allows us to write the local BRS symmetry:

$$\begin{aligned} s\varphi^i &= i\eta^i \\ s\xi^i &= i(\eta^i + \chi^i) + iF_l^i \eta^l \\ s\eta^i &= 0 \\ s\chi^i &= -i \frac{\partial F_l^i}{\partial \xi^m} (\eta^m + \chi^m) \eta^l \\ s\rho_\alpha^i &= H_\alpha^i - i\tilde{\Gamma}_{jk}^i \lambda^j \rho_\alpha^k + i \frac{\partial F_m^i}{\partial \xi^k} \eta^m \rho_\alpha^k \\ sH_\alpha^i &= -\frac{1}{2} \tilde{R}_{tkl}^i \rho_\alpha^t \lambda^k \lambda^l - i\tilde{\Gamma}_{jk}^i \lambda^j H_\alpha^k + i \frac{\partial F_m^i}{\partial \xi^k} \eta^m H_\alpha^k \end{aligned} \quad (2.20)$$

where

$$\tilde{R}_{qjp}^i(\varphi, \xi) = \frac{\partial \tilde{\Gamma}_{pq}^i}{\partial \xi^j} - \frac{\partial \tilde{\Gamma}_{jq}^i}{\partial \xi^p} + \tilde{\Gamma}_{jk}^i \tilde{\Gamma}_{pq}^k - \tilde{\Gamma}_{pk}^i \tilde{\Gamma}_{jq}^k \quad (2.21)$$

is the Riemann tensor corresponding to $\tilde{g}_{ij}(\varphi, \xi)$ which satisfies the globality condition :

$$\frac{\partial \tilde{R}_{imnl}}{\partial \varphi^k} + F_k^p \frac{\partial \tilde{R}_{imnl}}{\partial \xi^p} + \tilde{R}_{pmnl} \frac{\partial F_k^p}{\partial \xi^i} + \tilde{R}_{ipnl} \frac{\partial F_k^p}{\partial \xi^m} + \tilde{R}_{impl} \frac{\partial F_k^p}{\partial \xi^n} + \tilde{R}_{imnp} \frac{\partial F_k^p}{\partial \xi^l} = 0 \quad (2.22)$$

It is not difficult to show, using (2.7), (2.13), (2.22) and the Bianchi identity for $\tilde{R}^{ijkl}$, that

s is nilpotent. A lagrangian for such a model is given by taking the BRS variation of

$$\int d^2x (\tilde{g}_{ij} \rho^{i\alpha} \partial_\alpha \xi^j - \frac{1}{4} \tilde{g}_{ij} \rho^{i\alpha} H_\alpha^j) \quad (2.23)$$

i.e.

$$S = \int d^2x (\tilde{g}_{ij} H^{i\alpha} \partial_\alpha \xi^j - \frac{1}{4} \tilde{g}_{ij} H^{i\alpha} H_\alpha^j - i \tilde{g}_{ij} \rho^{i\alpha} \tilde{D}_\alpha \lambda^j - \frac{1}{8} \tilde{R}_{itkl} \rho^{i\alpha} \rho_\alpha^t \lambda^k \lambda^l) \quad (2.24)$$

where

$$\tilde{D}_\alpha \lambda^j = \partial_\alpha \lambda^j + \tilde{\Gamma}_{km}^j \lambda^k \partial_\alpha \xi^m \quad (2.25)$$

S coincides with the Witten action in normal coordinates. One has to note that the fields η^i and χ^i enter into (2.24) only through the combination λ^i and that the derivative of $H^{i\alpha}$ does not appear in (2.24), so $H^{i\alpha}$ is an auxiliary field which can be eliminated by using the equation of motion

$$H_\alpha^i = \partial_\alpha \xi^i + \epsilon_\alpha^\beta \tilde{J}_j^i \partial_\beta \xi^j \quad (2.26)$$

As in the standard cases it is necessary to ensure the off shell nilpotency of the BRS operator. If we define the action of the operator δ_v in (2.5) on the fields $\eta^i, \chi^i, \rho_\alpha^i, H_\alpha^i$ as:

$$\begin{aligned} \delta_v \eta^i &= 0 \\ \delta_v \chi^i &= v^k \frac{\partial F_k^i}{\partial \xi^l} (\eta^l + \chi^l) \\ \delta_v \rho_\alpha^i &= v^k \frac{\partial F_k^i}{\partial \xi^l} \rho_\alpha^l \\ \delta_v H_\alpha^i &= v^k \frac{\partial F_k^i}{\partial \xi^l} H_\alpha^l \end{aligned} \quad (2.27)$$

one has :

$$\begin{aligned} [\delta_v, \delta_\mu] &= 0 \\ [\delta_v, s] &= 0 \end{aligned} \quad (2.28)$$

and from (2.27)

$$\delta_v \lambda^i = v^k \frac{\partial F_k^i}{\partial \xi^l} \lambda^l \quad (2.29)$$

which implies that

$$\delta_v S = 0 \quad (2.30)$$

meaning that the action S is globally defined. We now introduce the operator ι_v whose

action on the fields of the model is :

$$\begin{aligned}
\iota_v \varphi^i &= 0 \\
\iota_v \xi^i &= 0 \\
\iota_v \rho_\alpha^i &= 0 \\
\iota_v H_\alpha^i &= 0 \\
\iota_v \eta^i &= -i v^i \\
\iota_v \chi^i &= i v^i
\end{aligned} \tag{2.31}$$

From (2.31) it follows that

$$\iota_v \iota_\mu + \iota_\mu \iota_v = 0 \tag{2.32}$$

and the operator δ_v decomposes as

$$\delta_v = s \iota_v + \iota_v s \tag{2.33}$$

The operators s, ι_μ, δ_v satisfy the algebra :

$$\begin{aligned}
[\delta_v, \delta_\mu] &= 0 \\
[\delta_v, \iota_\mu] &= 0 \\
[\delta_v, s] &= 0 \\
\iota_v \iota_\mu + \iota_\mu \iota_v &= 0 \\
\delta_v &= s \iota_v + \iota_v s
\end{aligned} \tag{2.34}$$

and

$$\delta_v S = \iota_\mu S = s S = 0 \tag{2.35}$$

The algebra (2.34) is the Abelian version of that introduced in [3] and, as we will see in the next section, it allows us to define a basic cohomology for the BRS operator.

3. Basic cohomology

To recover the Witten observables [1], we study the operators s , ι_μ and δ_v in the class of integrated local dimensionless functionals which depend on the variables $\varphi^i, \eta^i, \xi^i, \chi^i$, i.e:

$$X = \int d^2x A(\varphi, \eta, \xi(x), \chi(x)) \quad (3.1)$$

where analyticity in η, ξ, χ has to be understood. On this space there is no difference between the local and integrated cohomology, so we can consider s , ι_μ and δ_v as ordinary differential operators acting on the local space A . As ordinary differential operators s , ι_μ and δ_v read :

$$s = i\eta^i \frac{\partial}{\partial \varphi^i} + i(\eta^i + \chi^i) \frac{\partial}{\partial \xi^i} + i\eta^l F_l^i \frac{\partial}{\partial \xi^i} - i \frac{\partial F_l^i}{\partial \xi^m} (\eta^m + \chi^m) \eta^l \frac{\partial}{\partial \chi^i} \quad (3.2)$$

$$\iota_\mu = i\mu^i \frac{\partial}{\partial \chi^i} - i\mu^i \frac{\partial}{\partial \eta^i} \quad (3.3)$$

$$\delta_v = v^i \frac{\partial}{\partial \varphi^i} + v^i F_i^l \frac{\partial}{\partial \xi^l} + v^i \frac{\partial F_i^p}{\partial \xi^l} \lambda^l \frac{\partial}{\partial \chi^p} \quad (3.4)$$

It is not difficult to show that, on the local space A , the cohomology of s is trivial (see the Appendix). Now we define the basic cohomology of s as :

$$H^{basic} = (\theta \in A; s\theta = \iota_\mu \theta = \delta_v \theta = 0 \text{ and } \exists \text{ no } \gamma \in A \text{ such that } \theta = s\gamma \text{ and } \iota_\mu \gamma = \delta_v \gamma = 0) \quad (3.5)$$

One has to note that, due to the triviality of the cohomology of s , every quantity invariant under s, δ_v, ι_μ can be written as an exact cocycle of s . However, it is not true that every s -exact cocycle is invariant under δ_v, ι_μ , meaning that quantities which are trivial in the cohomology of s can be non trivial in H^{basic} . Let us examine first the condition

$$\iota_\mu \theta = 0 \quad (3.6)$$

From (3.3) it follows that (3.6) implies that θ depends only on the combination $\lambda^i = \eta^i + \chi^i$,

i.e $\theta = \theta(\varphi, \xi, \lambda)$. Analyticity in λ implies that

$$\begin{aligned}\theta &= \tilde{\theta}_{i_1 \dots i_n}(\varphi, \xi) \lambda^{i_1} \dots \lambda^{i_n} \\ n &= 0, 1, \dots, \dim M\end{aligned}\tag{3.7}$$

where $\tilde{\theta}_{i_1 \dots i_n}$ is a form-valued function on T_φ . The condition $\delta_v \theta = 0$ implies that

$$\frac{\partial \tilde{\theta}_{i_1 \dots i_n}}{\partial \varphi^k} + F_k^q \frac{\partial \tilde{\theta}_{i_1 \dots i_n}}{\partial \xi^q} + \tilde{\theta}_{q i_2 \dots i_n} \frac{\partial F_k^q}{\partial \xi^{i_1}} + \dots + \tilde{\theta}_{i_1 \dots i_{n-1} q} \frac{\partial F_k^q}{\partial \xi^{i_n}} = 0\tag{3.8}$$

i.e. $\tilde{\theta}_{i_1 \dots i_n}$ is a globally defined form-valued function. Finally the condition $s\theta = 0$ means that $\tilde{\theta}_{i_1 \dots i_n}$ is closed since, by (3.7) and (3.8),

$$s\theta = i\lambda^i \frac{\partial \tilde{\theta}_{i_1 \dots i_n}}{\partial \xi^i} \lambda^{i_1} \dots \lambda^{i_n}\tag{3.9}$$

Then H^{basic} is given by globally defined closed forms which cannot be written in a global way as exact form-valued functions, i.e H^{basic} can be identified with the De-Rham cohomology of the target manifold M , so leading to the Witten observables of the model.

4. Conclusion

The introduction of the Friedan non linear shift symmetry allows us to characterize the topological σ -model in terms of the basic cohomology of the BRS operator. It turns out that, as in the case of the gauge theories, the basic cohomology seems the most adequate algebraic approach to define the theory.

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APPENDIX Cohomology of the BRS operator

To study the cohomology of the operator s on the local space A we introduce the counting operator

$$N = \xi^k \frac{\partial}{\partial \xi^k} + \eta^k \frac{\partial}{\partial \eta^k} + \chi^k \frac{\partial}{\partial \chi^k} \quad (1)$$

N decomposes the local space A into subspaces $A^{(\nu)}$ according to its eigenvalue $\nu = 0, 1, 2, \dots$; and induces a separation in s

$$s = \sum_{n=0}^{\infty} s^{(n)} \quad (2)$$

with

$$[N, s^{(n)}] = n s^{(n)} \quad (3)$$

From (3.2) $s^{(0)}$ and $s^{(1)}$ are given by

$$s^{(0)} = i \chi^i \frac{\partial}{\partial \xi^i} \quad (4)$$

$$s^{(1)} = i \eta^i \frac{\partial}{\partial \varphi^i} - i \eta^l \Gamma_{lm}^i(\varphi) \xi^m \frac{\partial}{\partial \xi^i} + i \Gamma_{lm}^i(\varphi) (\eta^m + \chi^m) \eta^l \frac{\partial}{\partial \chi^i} \quad (5)$$

The nilpotency of s implies that

$$\sum_{p=0}^n s^{(p)} s^{(n-p)} = 0 \quad (6)$$

$$n = 0, 1, \dots$$

from which it follows that $s^{(0)}$ is nilpotent :

$$s^{(0)} s^{(0)} = 0 \quad (7)$$

According to the general theory [5] the cohomology of s is isomorphic to a subspace of the cohomology of $s^{(0)}$, so the first step consists of the identification of the cohomology space of $s^{(0)}$. From (4) it is apparent that the cohomology space of $s^{(0)}$ does not contain the

variables χ^i and ξ^i and hence it can be parametrized by a function $\Omega(\varphi, \eta)$ of the variables φ^i, η^i . Analyticity in η^i implies that

$$\begin{aligned}\Omega &= \Omega_{i_1 \dots i_n}(\varphi) \eta^{i_1} \dots \eta^{i_n} \\ n &= 0, 1, \dots, \dim M\end{aligned}\tag{8}$$

Having determined the cohomology of $s^{(0)}$ we turn to the equation

$$sY(\varphi, \eta, \xi, \chi) = 0\tag{9}$$

We shall show that the general solution of (9) is given by

$$Y = \text{const} + s\hat{Y}\tag{10}$$

which means that the cohomology of s is trivial. To prove (10) we proceed by induction with respect to the eigenvalues of N . Decomposing Y according to the operator N ,

$$Y = \sum_{n=0}^{\infty} Y^{(n)}\tag{11}$$

and using (2) one has the equation

$$\begin{aligned}\sum_{p=0}^n s^{(p)} Y^{(n-p)} &= 0 \\ n &= 0, 1, \dots\end{aligned}\tag{12}$$

At the level $n=0$ (12) gives

$$s^{(0)} Y^{(0)} = 0\tag{13}$$

whose general solution is

$$Y^{(0)} = \tau(\varphi) + s^{(0)} \hat{Y}^{(0)}\tag{14}$$

where $\tau(\varphi)$ is an arbitrary function of the background variables φ^i . Next at $n=1$

$$s^{(0)} Y^{(1)} + s^{(1)} Y^{(0)} = 0\tag{15}$$

Inserting (14) and using the equation (6) with $n=1$

$$s^{(0)}(Y^{(1)} - s^{(1)} \hat{Y}^{(0)}) = -s^{(1)} \tau\tag{16}$$

From (5)

$$s^{(1)} = i\eta^i \frac{\partial \tau}{\partial \varphi^i}\tag{17}$$

This equation shows that $(s^{(1)} \tau)$ belongs to the cohomology of $s^{(0)}$ in the $N=1$ sector and

then (16) implies

$$\frac{\partial \tau}{\partial \varphi^i} = 0 \quad (18)$$

i.e. τ is a constant. From (18) it follows that

$$Y^{(1)} = \eta^l v_l(\varphi) + s^{(1)} \hat{Y}^{(0)} + s^{(0)} \hat{Y}^{(1)} \quad (19)$$

where $v_l(\varphi)$ is a vector valued function on M . At $n=2$

$$s^{(2)} Y^{(0)} + s^{(1)} Y^{(1)} + s^{(0)} Y^{(2)} = 0 \quad (20)$$

Inserting the expression for $Y^{(0)}$ and $Y^{(1)}$ and using (6) with $n=2$

$$s^{(0)}(Y^{(2)} - s^{(1)} \hat{Y}^{(1)} - s^{(2)} \hat{Y}^{(0)}) = -s^{(1)} \eta^l v_l(\varphi) \quad (21)$$

Since

$$s^{(1)} \eta^l v_l(\varphi) = i \eta^m \eta^l \frac{\partial v_l}{\partial \varphi^m} \quad (22)$$

it follows that $(s^{(1)} \eta^l v_l)$ belongs to the cohomology of $s^{(0)}$ in the $N=2$ sector; (21) implies that

$$\frac{\partial v_l}{\partial \varphi^m} - \frac{\partial v_m}{\partial \varphi^l} = 0 \quad (23)$$

and the Poincare lemma ensures that, locally,

$$v_l = \frac{\partial \omega(\varphi)}{\partial \varphi^l} \quad (24)$$

The expression for $Y^{(1)}$ in (19) becomes

$$Y^{(1)} = s^{(1)}(\hat{Y}^{(0)} - i\omega(\varphi)) + s^{(0)} \hat{Y}^{(1)} \quad (25)$$

meaning that the sector $N=1$ is completely trivial. It is not difficult now to iterate the procedure and to show that all the sectors with filtration number ≥ 2 are trivial, recovering then the expression (10) for Y .

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