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QUANTUM GRAVITY AND THE COLLAPSE OF THE WAVE FUNCTION

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ABSTRACT

We derive a general form of non-unitary modification to the Hamiltonian evolution equation caused by wormholes interacting with a microscopic system. We show that it suppresses elements of the density matrix which are off-diagonal in the configuration space representation, thus causing the wave function to collapse. An order-of-magnitude estimate indicates that the collapse time is very long ($t \sim 10^{37}$ s) for a single elementary particle. However, the effect is enhanced for the centre-of-mass motion of a large aggregate of particles. In particular, the wave function of a body containing Avogadro's number of particles may be localized within a Bohr radius in $t \sim 10^{-7}$ s.

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The current formulation of quantum mechanics is generally thought to be incomplete and unsatisfactory. Although quantum mechanics provides a precise and successful framework for describing microphysics, doubts and questions arise when it is extended to macrophysics. Prominent among these problems are those of the collapse of the wave packet and the transition to a classical description of large objects. It has occasionally been suggested that these phenomena might become tractable within a quantum theory of gravity, but this suggestion has not been developed in a concrete fashion. A semiclassical treatment of quantum effects in topologically non-trivial configurations of space-time, such as in the neighborhood of a black hole,¹⁾ indicates that an initially pure quantum-mechanical state can evolve into a mixed state as information is lost through the event horizon. It has been suggested²⁾ that this macroscopic effect may have a microscopic analogue, with topological fluctuations at the Planck scale providing the possibility of information loss and entropy increase also on a microscopic scale. A definitive exploration of this possibility must probably await the systematic exploration of a fully consistent quantum theory of gravity, such as we believe is provided by string theory. However, in the interim a phenomenological dilute-gas treatment has been developed^{2,3)} for topologically non-trivial space-time configurations (wormholes) which are small compared with contemporary physics scales but much larger than the Planck scale. This introduces a new element of uncertainty into the description of physics, beyond that in conventional quantum mechanics, which results in entropy increase along the lines suggested earlier on general grounds.

In this paper we investigate whether the possibility of entropy increase offered by quantum gravity can resolve the outstanding problems of the collapse of the wave function and the transition from quantum to classical behavior for large systems. We discuss this question in the framework of the dilute-gas approximation for wormholes, although we have reservations concerning the completeness and adequacy of this approach. We argue that although quantum gravitational effects have a negligible effect on off-diagonal elements in the configuration space representation of the density matrix for a single particle, they become important for macroscopic systems. Specifically, we estimate that the wave function for a system with Avogadro's number of microscopic constituents collapses in about 10^{-7} s.

Hawking's original proposal⁴⁾ that there might be an increase in entropy for microscopic systems, and hence an apparent breakdown of quantum mechanics, was based on the idea that information could be lost across microscopic event horizons associated with topological fluctuations in quantum gravity. Calculations of this effect led to the proposal⁴⁾ that the linear transformation between

“in” and “out” density matrices:

$$(\rho_{out})_B^A = S_{BC}^A{}^D (\rho_{in})_D^C \quad (1)$$

might not be factorizable, as in conventional quantum mechanics and quantum field theory, as the product of an S-matrix and its hermitian conjugate:

$$S_{BC}^A{}^D \neq S_C^A S_B^\dagger{}^D \quad (2)$$

In the first part of this paper we re-examine this proposal in the context of the “wormhole calculus” recently introduced by Coleman³⁾ and Hawking²⁾.

We start by considering a single particle with wave function $\phi(x)$. The “in” state consisting of this particle and an a priori unknown wormhole configuration can be written as

$$|1\rangle_{in} = \int dx \int d\alpha \phi(x) f(\alpha) |x, \alpha\rangle \quad (3)$$

where $f(\alpha)$ is the wave function of the wormhole variables $\{\alpha\}$ and $|x, \alpha\rangle$ is a state in the composite Hilbert space $\mathcal{H} \equiv \mathcal{H}_\phi \otimes \mathcal{H}_w$. The observable density matrix in configuration space associated with the “in” state (3) is obtained by integrating over the wormhole variables.

$$\rho_{in}(x', x) \equiv \langle x' | \rho_{in} | x \rangle = \int d\alpha'' dx'' \langle x', \alpha'' | x'', \alpha'' \rangle \langle x'', \alpha'' | x, \alpha \rangle = \phi^*(x') \phi(x) \quad (4)$$

The “in” state is pure, so $Tr_x \rho_{in}^2 = 1$ and the quantum entropy

$$S_{in} = -Tr_x \rho_{in} \ln \rho_{in} = 0 \quad (5)$$

The “out” state after scattering may be written formally as

$$| \rangle_{out} = \int dx \int d\alpha \phi(x) f(\alpha) S_{x,\alpha} |x, \alpha\rangle \quad (6)$$

where $S_{x,\alpha}$ is the scattering matrix which now depends both upon the particle variable x and the wormhole variables $\{\alpha\}$. The observable configurations space density matrix for the “out” state is

$$\begin{aligned} \langle x' | \rho_{out} | x \rangle &= \int d\alpha \langle x', \alpha | \rangle_{out} \langle \rangle_{out} | x, \alpha \rangle \\ &= \int d\alpha \int dx_1 \int dx_2 \int d\alpha_1 \int d\alpha_2 \langle x', \alpha | \phi(x_1) \\ &\quad f(\alpha_1) S_{x_1, \alpha_1} |x_1, \alpha_1\rangle \langle x_2, \alpha_2 | S_{x_2, \alpha_2}^\dagger f(\alpha_2) \phi(x_2) | x, \alpha \rangle \end{aligned} \quad (7)$$

If we assume that the effective interaction is local, then the matrix element

$\langle x, \alpha | S_{x_1, \alpha_1} | x_1, \alpha_1 \rangle$ can be written in the form

$$\langle x, \alpha | S_{x_1, \alpha_1} | x_1, \alpha_1 \rangle = \delta(x - x_1) S_x(\alpha_1, \alpha) \quad (8)$$

In this case, equation (7) can be written in the form

$$\langle x' | \rho_{out} | x \rangle = \rho_{in}(x', x) \int d\alpha \int d\alpha_1 \int d\alpha_2 S_{x'}(\alpha_1, \alpha) S_x^\dagger(\alpha_2, \alpha) f(\alpha_1) f(\alpha_2) \quad (9)$$

It is easily checked that $Tr_x(\rho^{out}(x', x))^2 < 1$ in general, indicating that pure states evolve into mixed states and entropy increases even at the microscopic level. The relation (9) between initial and final density matrices can be written in the general linear form (1), but $\$$ cannot be written as a simple product of S -matrices as in conventional quantum mechanics: here

$$\$ = \int d\alpha \int d\alpha_1 \int d\alpha_2 S_{x'}(\alpha_1, \alpha) S_x^\dagger(\alpha_2, \alpha) f(\alpha_1) f(\alpha_2) \quad (10)$$

The evolution of a pure "in" state into a mixed "out" state is due to this non-factorizability of the $\$$ operator, which is in turn due to the necessity of summing over all wormhole states α . The $\$$ operator remains non-factorizable even if we assume that the interactions do not have off-diagonal matrix elements between wormhole states, so that

$$S_{x'}(\alpha_1, \alpha) = \delta(\alpha_1 - \alpha) S_{x'}(\alpha) \quad (11)$$

in which case

$$\$ = \int d\alpha S_{x'}(\alpha) S_x^\dagger(\alpha) |f(\alpha)|^2 \quad (12)$$

Factorizability is regained, and microscopic entropy conservation restored, only if it is further assumed that the wormhole state is completely defined:

$$f(\alpha) = \delta(\alpha - \alpha_o) \quad (13)$$

The assumption (11) implies that the wormhole state $|\alpha\rangle$ is an eigenstate $|\hat{\alpha}\rangle$ of the interaction Hamiltonian. In particular, if $H_I \propto (a + a^\dagger)$, where the operators $a(a^\dagger)$ annihilate (create) wormholes, then the eigenstates $|\hat{\alpha}\rangle$ defined by $(a + a^\dagger)|\hat{\alpha}\rangle = \hat{\alpha}|\hat{\alpha}\rangle$ are "coherent states". The further assumption (13) implies that $|\text{in}\rangle$ is just such an eigenstate of H_I , i.e., $|\text{in}\rangle = \phi(x)|\hat{\alpha}_o\rangle$ for some $\hat{\alpha}_o$. If $|\text{in}\rangle$ were to be any other state: $f(\hat{\alpha}) \neq \delta(\hat{\alpha} - \hat{\alpha}_o)$, there would still be a loss of coherence because of the need to sum over $\hat{\alpha}$ to obtain $|\text{out}\rangle$.

In equation (3) we have allowed $|\text{in}\rangle$ to be a general linear combination of a complete basis set $|\alpha\rangle$. Hawking²⁾ has suggested taking $|\text{in}\rangle = |0\rangle$: a “no-wormhole” initial state. Coleman^{3,5)} has argued that the wormhole state is likely to be a pure coherent state, as part of his argument for the vanishing of the cosmological constant. However, other authors⁶⁾ have questioned this conclusion, and even described it as “fine-tuning”, and we see no good reason why equation (13) should hold. *A priori*, we would expect the space $\{\alpha\}$ of wormhole parameters to be large, even infinite-dimensional. The argument for the vanishing of the cosmological constant, if correct, would only fix one combination of the parameters $\{\alpha\}$. Analogous arguments might fix more combinations of the $\{\alpha\}$, but it is not obvious the wormhole state would be completely specified as in (13). We therefore retain and explore the non-factorizable evolution equation (7) for the density matrix.

If the characteristic wormhole interaction time τ_W is short compared with the typical evolutionary time-scale of the unperturbed unitary Hamiltonian, then we would expect that there is a dynamical equation for the density matrix averaged over time intervals $\Delta t \gg \tau_W$ which takes the form [7]

$$\frac{\partial \rho}{\partial t} = +i[\rho, H_0] + (\delta \mathbb{H})\rho \quad (14)$$

where the appearance of the new term $\propto (\delta \mathbb{H})$ is due to the non-factorizability of $\S(7,10)$. It was shown in ref. 7 that probability conservation requires

$$\frac{d}{dt} \text{Tr}(\rho^2) = 2\rho_A^B \delta \mathbb{H}_{BC}^A \rho_D^C \leq 0 \quad (15)$$

and hence that $\delta \mathbb{H}$ must be a negative semi-definite matrix. It was further shown that in a simple 2-state system probability and energy conservation required that the non-zero elements of $\delta \mathbb{H}$ take the form

$$\delta \mathbb{H} = \begin{pmatrix} -\alpha & -\beta \\ -\beta & -\gamma \end{pmatrix} : \alpha, \gamma > 0, \alpha\gamma > \beta^2 \quad (16)$$

and that the resulting asymptotic form of the density matrix $\rho(t)$ given by (14) was

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 1 & e^{-(\alpha+\gamma)t/2} e^{-i\Delta Et} \\ e^{-(\alpha+\gamma)t/2} e^{i\Delta Et} & 1 \end{pmatrix} \quad (17)$$

Note that off-diagonal elements in $\rho(t)$ (17) decay exponentially, so that the state becomes completely mixed. It is just such an effect in the infinite-dimensional

configuration-space density matrix $\rho(x', x)$ that leads to the collapse of the wave function. The analogue of equation (17) for small times is

$$\rho_{out}(x', x) = \rho_{in}(x', x)[1 - Dt(x - x')^2 + \dots] \quad (18)$$

corresponding to $\delta H = D(x - x')^2$, and we will now sketch one derivation of such a non-unitary term.

To evaluate ρ_{out} in equation (7) we need the wave function $f(\alpha)$ of the wormhole variables. Since wormholes are closed universes, at the level of classical general relativity they carry zero quantum numbers, and hence no energy, momentum and angular momentum. We assume that quantum wormholes have a wave function which is gaussian in momentum space with zero angular momentum:

$$f(\alpha) \equiv f(k) = \frac{1}{4\pi k^2} \frac{1}{\sqrt{\Delta}} e^{-k^2/\Delta^2} \quad (19)$$

The parameter Δ is the spread in the momentum around $k = 0$, which should be expressible in terms of the relevant mass scales, including the Planck mass m_P and the mass $m \sim 1\text{GeV}$ of a microscopic particle. Since Δ is expected to be small, it can presumably be written as

$$\Delta \simeq A_n \frac{m^{n+1}}{m_P^n}; \quad A_n = O(1) \quad (20)$$

where n is some integer ≥ 1 . In the following we shall assume that $\Delta \sim O(m^2/m_P)$. As for $S_x(k_1, k)$ in equation (8), we assume translational invariance so that

$$S_x(k_1, k) = \hat{S}(k_1, k) e^{i(k_1 - k) \cdot x} \quad (21)$$

The scattering matrix $\hat{S}(k_1, k)$ in momentum space is related to the scattering amplitude $F(k)$ by

$$\hat{S}(k_1, k) = \delta(k_1 - k) \left[1 + \frac{i}{k} F(k) \right] \quad (22)$$

Substituting equations (19, 20, 21) into equation (14) and summing over wormholes quantum numbers w we obtain

$$\begin{aligned} \rho^{out}(x', x) = \rho^{in}(x', x) & \left\{ 1 + \frac{1}{\Delta} \int d^3k e^{-2k^2/\Delta^2} e^{ik \cdot (x' - x)} \sum_w \left[\frac{i}{k} F_w(k) \right. \right. \\ & \left. \left. - \frac{i}{k} F_w^\dagger(k) + \frac{1}{k^2} |F_w(k)|^2 \right] \right\} \end{aligned} \quad (23)$$

The Gaussian factor cuts off the integration for $k \geq \Delta$. Therefore we can expand the exponential $e^{ik \cdot (x' - x)}$ in a Taylor series as long as $|x' - x| < \frac{1}{\Delta} \sim 10^{19} \text{GeV}^{-1} \sim$

$10^4 m$. This condition is amply satisfied in all the quantum systems whose wave function's spread is observable. Therefore we Taylor expand to obtain

$$\rho^{out}(x', x) = \rho^{in}(x', x) \left\{ 1 + \frac{1}{\Delta} \int d^3 k e^{-2k^2/\Delta^2} \sum_w \left[\frac{-2}{k} \text{Im} F_w(k) + \frac{1}{k^2} |F_w(k)|^2 \right] \right. \\ \left. \cdot \left[1 + ik \cdot (x' - x) - \frac{[k \cdot (x' - x)]^2}{2} + \dots \right] \right\} \quad (24)$$

We assume that the individual particle–wormhole scattering events obey conventional quantum mechanics, to the extent that $S(k', k)$ is unitary which implies that

$$\int d^3 k \sum_w \{ |F_w(k)|^2 - 2k \text{Im} F_w(k) \} = 0 \quad (25)$$

and hence in turn that the first term in the expansion of $e^{ik \cdot (x' - x)}$ in equation (24) vanishes. The terms containing $k \cdot (x' - x)$ also vanish when integrated over the 3-dimensional angle. The first non-zero terms in (24) are

$$\rho^{out}(x', x) = \rho^{in}(x', x) \left[1 - \frac{1}{\Delta} \int dk k^2 (x' - x)^2 \sum_w |F_w(k)|^2 e^{-2k^2/\Delta^2} \right] \\ \simeq \rho^{in}(x', x) \left[1 - \Delta^2 \left(\sum_w |F_w|^2 \right) (x' - x)^2 \right] \quad (26)$$

Notice that equation (26) implies $\text{Tr}_x \rho^{out} = \text{Tr}_x \rho^{in}$, ensuring that probability is conserved. It is however clear from equation (24) that this would not have been the case if we had not enforced unitarity of the particle–wormhole S-matrix. The non-diagonal correction $\propto (x' - x)^2$ in equation (26) corresponds to the promised collapse of the quantum–mechanical wave function. The qualitative structure of equation (26) [cf the expectation (18)] seems likely to survive, even if the scenario we have just sketched for deriving it does not.*

We now indicate an order of magnitude estimate for the new off-diagonal term in the density matrix. Consider the interaction of a wormhole with chiral fermions,²⁾ which may be represented by the four-fermion effective interaction

$$\mathcal{L}_I = 0 \left(\frac{1}{m_P^2} \right) (\bar{\psi}_1 \gamma^\mu \psi_1) (\bar{\psi}_2 \gamma_\mu \psi_2) (a_i + a_i^\dagger) \quad (27)$$

where $a_i (a_i^\dagger)$ is a wormhole annihilation (creation) operator. After making a Fierz

* We note in passing that the form of modification to the evolution of the density matrix which we have derived corresponds to that postulated previously in ref. 8 on purely empirical ad hoc grounds.

transformation and using the fermion mass to absorb the fermion chirality, we obtain an effective interaction

$$\mathcal{L}_{eff} = 0\left(\frac{m^3}{m_P^2}\right) \bar{\psi}\psi(a_i + a_i^\dagger) \quad (28)$$

The corresponding dimensionless scattering amplitude off an individual wormhole would be

$$F_w \sim \frac{m^2}{m_P^2} \quad (29)$$

To estimate the total particle–wormhole interaction rate, we assume that the density n_w of wormholes is $0(1)$ per Planck volume, and that the geometrical area σ of the fermion wave function is $0(1/m^2)$. Therefore the total scattering rate in a time interval δt is

$$\sum_{w:t < t_w < t + \delta t} |F_w|^2 = |F_w|^2 n_w \sigma \delta t \sim \left(\frac{m^2}{m_P^2}\right)^2 (m_P^3) \left(\frac{1}{m^2}\right) \delta t \sim \frac{m^2}{m_P} \delta t \quad (30)$$

Substituting $\Delta \sim (m^2/m_P)$ and equation (30) for $\sum_w |F_w|^2$ in equation (26) we obtain the following evolution equation for the configuration–space density matrix.

$$\frac{\partial \rho}{\partial t} \simeq i[\rho, H_0] - \frac{m^6}{m_P^3} (x - x')^2 \rho \quad (31)$$

[cf equation (14)]. The solution of the modified Schrödinger equation (31) can be related to the solution $\rho_0(x', x, t)$ of the conventional Schrödinger equation $\partial \rho_S / \partial t = i[\rho_S, H_0]$ by:

$$\rho(x', x, t) \simeq \rho_S(x', x, t) \exp(-D(x - x')^2 t) \quad (32)$$

where $D \sim m^6/m_P^3$. We note again that our results (31, 32) are likely to be more general than our “derivation”. We emphasize that $Tr_x \rho(x', x, t) = Tr_x \rho_0(x', x, t) = 1$, and hence that probability is conserved, although $Tr_x [\rho(x', x, t)]^2 < 1$ in general.

The structure of the off–diagonal factor in (32) means that conventional quantum–mechanical superposition breaks down when the wave function is spread over a distance $R \equiv |x' - x|$ after a time

$$\delta t \geq \frac{1}{DR^2} \quad (33)$$

has elapsed. This time–scale is probably too long for wave function collapse to be detectable in microscopic systems. For example, in metals we know that valence

electrons are loosely bound and have wave functions at least as broad as several lattice spacings. If we assume the spread of a typical electron wave function to be $R \sim 10^3 a_0$ where a_0 is the Bohr radius, then an individual electron wave function starts to collapse after a time

$$\delta t \sim \frac{1}{DR^2} \sim \frac{m_p^3}{m_e^6} \left(\frac{1}{10^8 \text{GeV}^{-1}} \right)^2 \simeq 10^{37} \text{s} \quad (34)$$

which is unobservably long.

The small magnitude of the fermion–wormhole scattering amplitude (29), the small value of D (32) and the long collapse time–scale δt (34) are traceable to the fact that the light fermions are chiral. It had been pointed out previously² that wormholes could be expected to give non–chiral fermions a mass $\sim m_p$. Only chiral fermions can survive wormhole dynamics with small masses – a gravitational analogue of the “survival hypothesis” of traditional grand unification. We have shown here that chirality is also necessary for the survival of coherence, at least approximately, for microscopic bodies in the real world.

However, the situation is different for a macroscopic body containing many particles with coordinates $x_i (i = 1, 2, \dots, N)$. Let us denote the centre–of–mass coordinate by X and the relative coordinates by Δx_j :

$$x_i = X + c_{ij} \Delta x_j \quad (i = 1, 2, \dots, N) \quad (35)$$

If the centre–of–mass motion can be decoupled from the internal motion, then

$$H = H(X) + H(\Delta x_i) + \sum_{i=1}^N \sum_w H_w(x_i) \quad (36)$$

where $\sum_w H_w(x_i)$ represents the interactions with wormholes, and the density matrix ρ_x for the whole system when the cm motion is decoupled from the ‘internal’ motion may be written as

$$\rho_x = \rho_X \rho_{\Delta x} \quad (37)$$

The equation of motion associated with equation (36) is

$$\begin{aligned} \dot{\rho}_X \simeq i & [[\rho_X, H(X)] \rho_{\Delta x} + [\rho_{\Delta x}, H(\Delta x)] \rho_x] \\ & - \sum_{i=1}^N D(x_i - x'_i)^2 \rho(x'_i, x_i) \end{aligned} \quad (38)$$

Using equation (35) and taking the trace over the internal coordinates Δx_i we

obtain

$$\begin{aligned}\dot{\rho}_X \simeq & i[\rho_X, H(X)] - ND(X' - X)^2 \rho_X \\ & + D \sum_i \int \left[\{c_{ij}c_{il}\Delta x_j \Delta x_l - c_{ik}c_{im}\Delta x_k \Delta x_m\} \right. \\ & \left. - 2X\{c_{ij}\Delta x_i - c_{ik}\Delta x_k\} \right] \rho_{\Delta x} \Pi_i d(\Delta x_i)\end{aligned}\quad (39)$$

where the last term vanishes after integration. Hence the equation of motion for the centre-of-mass density matrix is

$$\dot{\rho}(X', X, t) \simeq i[\rho(X', X, t), H(X)] - ND(X' - X)^2 \rho(X', X, t) \quad (40)$$

which has the solution

$$\rho(X', X, t) \simeq \rho_o(X', X, t) \exp(-ND(X' - X)^2 t) \quad (41)$$

where ρ_o is again the solution of the unmodified Schrödinger equation. The factor of N in equation (41) opens the way to a much more rapid collapse of the wave function for a macroscopic system.

The lower bound on the number of particles required for the centre-of-mass of the system to behave classically is

$$N \geq \frac{1}{D(X' - X)^2 t} \quad (42)$$

To estimate N from equation (42), we take $(X' - X)$ to be the smallest distance over which the spread of the centre-of-mass wave packet could be measured, which we estimated as $(X' - X) \sim \alpha_o \sim 10^6 \text{GeV}^{-1}$. The time t is taken to be the smallest time over which the centre-of-mass motion of a macroscopic object could be measured, which we estimate to be $t \sim 10^{-8} \text{sec} \simeq 10^{16} \text{GeV}^{-1}$. Using the previous order of magnitude estimate $D \sim m^6/m_p^3$ with $m \sim 1 \text{GeV}$, $m_p \sim 10^{18} \text{GeV}$ we estimate from equation (42) that $N \geq 10^{25}$. In other words, objects containing more than Avogadro's number of particles will behave classically after about 10^{-7}s .

Thus Schrödinger's cat is either dead, or not, as the case may be.

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