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A PROPOSAL TO USE A PLASMA LENS
TO ACHIEVE A 12 NM SPOT AT THE END OF CLIC

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ABSTRACT

We review the use of a wake-field plasma lens as the final focusing element at the interaction point of the CERN Linear Collider scheme (CLIC). Taking longitudinal and transverse aberrations into account we propose a set of parameters to achieve the design specification of 12 nanometer spot radius.

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1. INTRODUCTION

We first describe the principle of the wake field plasma lens. It is well known that when a relativistic bunch of charged particles moves in vacuum the repulsive force due to the space charge of the bunch is cancelled (to the order of $1/\gamma^2$) by the attractive force due to the magnetic field produced by the bunch.

When such a bunch moves through a plasma the situation is different. First let us consider the force on the electrons of the plasma. The plasma electrons feel the full space-charge of the beam and this repulsive force is not balanced by a magnetic force because the plasma electrons are stationary.

As the electron bunch enters the plasma the background electrons are repelled by this space charge and if a positron bunch enters the plasma they are attracted to its axis.

Returning to examine the forces seen by the moving particles in the bunch we find that in both cases the effect of this redistribution of charge in the plasma is to cancel the space-charge force on a particle in the bunch from its neighbours. Consequently, in contrast to the situation in vacuo, the attractive self-field magnetic force of the beam is not cancelled. Ampere's law allows us to calculate this focusing force in the case of a cylindrical symmetry:

$$F_{\perp} = -2\pi e^2 n_b r, \quad (1)$$

where n_b is the beam number density and r the radial coordinate. $F_{\perp}$ acts only in the radial direction. The negative sign in Eq. (1) indicates it is a focusing force for the bunch. Note that this effect is the same for both electrons and positrons.

In the next section we review an analytic model of the plasma wake fields in a linear fluid theory, in order to obtain the right conditions for Eq. (1) to apply : references [1] and [2] are the basis of this model.

2. ANALYTIC MODEL FOR PLASMA LENSES

In the plasma wake field concept a relativistic charged particle beam travels in a plasma with a velocity βc and leaves a plasma perturbation in its wake. We define the wake field $\vec{W}$ as the electromagnetic field associated with the beam-plasma system:

$$\vec{W}_{\parallel}(\xi, r) = \vec{E}_{1z}(\xi, r) \quad (2)$$

$$\vec{W}_{\perp}(\xi, r) = \vec{E}_{1r}(\xi, r) + \vec{\beta} \times \vec{B}_{1\varphi}(\xi, r), \quad (3)$$

where $\xi = z - \beta ct \approx z - ct$. $\vec{E}_{1z}$ is longitudinal (with respect to the beam motion) and $\vec{E}_{1r}$ and $\vec{B}_{1\varphi}$ are the transverse field of the beam-plasma system.

According to references [1] and [2], the linearized Lorentz-Maxwell system for the plasma wake field model is given by:

$$-\partial_{\xi} \vec{v}_1 = -\frac{e}{mc} \vec{E}_1 \quad (4)$$

$$-c\partial_{\xi} n_1 + n_0 \vec{\nabla} \cdot \vec{v}_1 = 0 \quad (5)$$

$$\nabla^2 \varphi_1 = -4\pi \rho_1 \quad (6)$$

$$\nabla^2 \vec{A}_1 = -\frac{4\pi}{c} \vec{J}_1 - \nabla \partial_{\xi} \varphi_1, \quad (7)$$

where $\vec{v}_1 = \vec{v}_1(\vec{x})$ is the fluid plasma velocity perturbation, n_0 is the unperturbed plasma number density and $n_1 = n_1(\vec{x})$ is the plasma number density perturbation (electron perturbation) satisfying the following condition

$$n_1 \ll n_0 \quad (8)$$

and $\vec{\nabla} = \partial_{\xi} \vec{e}_{\parallel} + \vec{\nabla}_{\perp}$. In Eqs. (6)-(7) we have introduced the four potential $(\vec{A}_1, \varphi_1)$ of the beam-plasma system. Furthermore, for these equations we have to define the sources

$$\rho_1(\vec{x}) \equiv -e n_1(\vec{x}) + e \sigma(\vec{x}) \quad (9)$$

$$J_1(\vec{x}) \equiv -e n_0 \vec{v}_1(\vec{x}) + e \vec{c} \sigma(\vec{x}), \quad (10)$$

namely the charge density and the current density of the beam-plasma system, respectively for both electrons (sign -) and positrons (sign +), and by expressing (2) and (3) in terms of the four potential:

$$W_{\parallel}(\xi, r) = \partial_{\xi}(A_{1z} - \varphi_1) \quad (11)$$

$$W_{\perp}(\xi, r) = \partial_r(A_{1z} - \varphi_1). \quad (12)$$

In Eqs. (9) and (10) $\sigma(\vec{x})$ is the beam number density.

Defining $f(r)$ and $g(\xi)$ as the radial and longitudinal beam density profiles we assume the following factorized form for $\sigma(\vec{x})$:

$$\sigma(\vec{x}) = n_b f(r) g(\xi). \quad (13)$$

We can only reduce the system (4)-(7) to only two-coupled equation:

$$\partial_{\xi}^2 n_1 + k_p^2 n_1 = \mp k_p^2 \sigma(\vec{x}) \quad (14)$$

$$(\nabla_{\perp}^2 - k_p^2)(A_{1z} - \varphi_1) = -4\pi e n_1, \quad (15)$$

where

$$k_p = \left(\frac{4\pi e^2 n_0}{mc^2} \right)^{1/2} = \omega_p / c = 1.88 \times 10^{-6} \left(n_0 [\text{cm}^{-3}] \right)^{1/2} \quad (16)$$

is the plasma wave number (ω_p is the plasma frequency).

First we solve for n_1 and then for the wake potential $A_{1z} - \varphi_1$. So, by introducing the function

$$G(\xi) \equiv k_p \int_{\xi}^{\infty} d\xi' g(\xi') \sin k_p(\xi' - \xi) \quad (17)$$

we obtain the following solution for (14):

$$n_1(\vec{x}) = \pm n_b f(r) G(\xi). \quad (18)$$

Thus, by solving (15) for $A_{1z} - \varphi_1$, we find:

$$A_{1z} - \varphi_1 = \mp \frac{4\pi e}{k_p^2} n_b G(\xi) F(r), \quad (19)$$

where

$$F(r) \equiv k_p^2 \int_0^r r' dr' f(r') I_0(k_p r') K_0(k_p r') + \int_r^\infty r' dr' f(r') I_0(k_p r) K_0(k_p r'). \quad (20)$$

Then, (11) and (12) give, in straightforward way, the wake fields:

$$W_{\parallel} = \mp \frac{4\pi e n_b}{k_p^2} G'(\xi) F(r) \quad (21)$$

$$W_{\perp} = \mp \frac{4\pi e n_b}{k_p^2} G(\xi) F'(r) \quad (22)$$

(the primes represent the derivative of function with respect to their own variable).

If we consider G as a very slowly varying function, $W_{\parallel}$ can be neglected. So, we concentrate our attention only on the transverse radial field $W_{\perp}$. Furthermore, we consider the case of a confined bunch to a limited region, namely we consider $\xi < 0$, $r < a$, where a is assumed as a beam radius.

By taking a parabolic radial profile:

$$f(r) = 1 - r^2/a^2 \quad r < a \quad (23)$$

it can be shown that: [2]

$$W_{\perp} = \mp \frac{8\pi n_b}{k} \left[I_1(k_p r) K_2(k_p a) - \frac{r}{k_p a^2} \right] G(\xi). \quad (24)$$

In general this expression shows that the transverse wake field is not

linear with respect to the radial dependence. But, if we take the approximation

$$k_{pa} \ll 1, \quad (25)$$

which means a is smaller than the skin depth c/ω_p , the quantity in square brackets becomes almost $-k_{pr}$ ($k_{pr} \ll k_{pa} \ll 1$) and for e^- and e^+ the transverse force is given by:

$$F_{\perp} = \mp eW_{\perp} = -[2we^2n_bG(\xi)]r. \quad (26)$$

Apart from the presence of $G(\xi)$ Eq. (26) is identical to Eq. (1). This means the wake field model describes correctly the self focusing of the beam and also gives a model for plasma lenses.

We note that the present fluid linear theory for a wake field plasma lens is only consistent with the condition

$$n_0 \gg n_b. \quad (27)$$

In fact, from condition (8) and from (18) we get

$$n_0 \gg n_b f(r) G(\xi).$$

which gives also Eq. (27).

This regime is usually referred to as "the linear plasma lens".

Considering a parabolic longitudinal profile:

$$g(\xi) = 1 - \left(\frac{\xi + b}{b} \right)^2 \quad -2b \leq \xi \leq 0 \quad (28)$$

we find from (17):

$$\begin{aligned} G(\xi) &= 1 - \left(\frac{\xi + b}{b} \right)^2 + \frac{2}{k_p b} \operatorname{sink}_p \xi + \frac{2}{k_p^2 b^2} (1 - \cos k_p \xi) \\ &= g(\xi) + \frac{2}{k_p b} \operatorname{sink}_p \xi + \frac{2}{k_p^2 b^2} (1 - \cos k_p \xi). \end{aligned} \quad (29)$$

So we can distinguish two different limits as follows: $k_p b \ll 1$, referred

to as "round beam limit" (RBL) and $k_{pb} \gg 1$, referred to as "long beam limit" (LBL).

Thus we find from Eq. (26):

$$F_{\perp} \approx \left[\frac{2}{3} \pi e^2 n_b \frac{k_p^2}{b} \xi^3 \right] r \quad (\text{RBL}) \quad (30)$$

(note that $\xi < 0$).

$$F_{\perp} \approx -2\pi e^2 n_b g(\xi) r \quad (\text{LBL}) \quad (31)$$

The property given by Eq. (31) in LBL has been verified for other shapes of the longitudinal profile [3]. We can assume that in LBL $G(\xi) \approx g(\xi)$ also for a longitudinal gaussian profile.

3. NONLINEAR PLASMA LENS

There is another mechanism for the beam focusing which works in the regime

$$n_0 \ll n_b. \quad (32)$$

Usually it is referred to as "the nonlinear plasma lens" because condition (32) is not consistent with the linear theory for the wake fields described above (see Eq. (27)). Consequently, the focusing mechanism must be changed. The physical picture for such a kind of focusing is shown in the following steps:

- a) An electron bunch enters a thin plasma slab (plasma lens) with density $n_b \gg n_0$.
- b) The beam pushes the plasma electrons outwards because of its space charge.
- c) A cylinder of positive charge at density n_0 is so created.
- d) The beam space charge is not completely balanced by the plasma ions.
- e) The resulting focusing force may be obtained by using Gauss' law:

$$F_{\perp} = -2\pi n_0 e^2 r. \quad (33)$$

Note that, by comparing eqs (33) and (1) the only difference consists in the density. Equation (33) is obtained by (1) provided the substitution $n_b \rightarrow n_0$.

But what about positrons ? When a positron bunch enters the plasma with $n_b \gg n_0$, the plasma electrons are not sufficient to neutralize the positron beam. So the focusing force is less linear than in the regime $n_0 \gg n_b$. The positrons are not well focused. That means that the non-linear plasma lens is only useful for electron beams. While in the linear plasma lens there is no difference between the e^+ and e^- cases, here some differences appear.

In the case of e^+ , because of the strong attraction between the positrons and plasma electrons, many plasma electrons are dragged along and accelerated out of the plasma. In the case of e^- , the ions are not dragged because of their great mass, and the radial repulsive electric force due to B_φ is balanced (as in vacuum).

4. BEAM OPTICS

It is well known that the envelope equation (spot-size equation) for a beam of charged particles in the presence of a focusing force [5] is

$$\frac{d^2\sigma}{dz^2} + K\sigma - \epsilon^2\sigma^{-3} = 0, \quad (34)$$

where ϵ is the beam emittance and

$$K = \left| \frac{F_\perp r}{m\gamma\beta^2 c^2} \right| \quad (35)$$

is the focusing strength. For a linear plasma lens (apart from the longitudinal density profile) Eq. (35) gives (see Eq. 1):

$$K = \frac{2\pi e^2 n_b}{m\gamma\beta^2 c^2} = \frac{1}{2\gamma} \frac{\omega_p^2}{c^2} \approx 1.76 \times 10^{-12} \frac{n_b}{\gamma} [\text{cm}^{-2}], \quad (36)$$

where we have approximated $\beta c \approx c$ and expressed n_b in cm^{-3} . Note that ω_p is the "plasma frequency" associated with the beam.

In Eq. (34) $\sigma = \sigma(z)$ is the beam spot-size. With the following initial conditions

$$\sigma_0 = \sigma(z=0) \quad (37)$$

$$(d\sigma/dz)_{z=0} = 0 \quad (38)$$

Equation (34) gives the solution

$$\sigma^2(z) = \frac{\sigma_0^2}{2} \left[\left(1 - \frac{\epsilon^2}{K\sigma_0^4} \right) \cos 2\sqrt{K} z + \left(1 + \frac{\epsilon^2}{K\sigma_0^4} \right) \right]. \quad (39)$$

Equation (39) describes a general caustic including both the focusing and the thermal spread due to the emittance (diffraction effect in competition with the focusing).

In what follows it is useful to introduce the curvature radius $\rho(z)$ of the surface normal to every particle trajectory (at any z) as

$$\frac{1}{\rho(z)} = \frac{1}{\sigma(z)} \frac{d\sigma(z)}{dz} = \frac{1}{2\sigma^2(z)} \frac{d\sigma^2(z)}{dz}. \quad (40)$$

By using (39) in (40) we immediately find

$$\rho(z) = - \frac{(1 - \epsilon^2/K\sigma_0^4) \cos 2\sqrt{K} z + (1 + \epsilon^2/K\sigma_0^4)}{\sqrt{K} (1 - \epsilon^2/K\sigma_0^4) \sin 2\sqrt{K} z}. \quad (41)$$

In order to treat the beam optics for a linear collider we separately analyze solutions (39) and (40) in two different regions: in the lenses and in vacuum between the lenses (final focus).

4.1 In the Lenses

According to Fig. 1 a charged beam (e^- or e^+) enters a plasma slab (plasma lens) with thickness ℓ .

We define the lens focal length f as:

$$f = |\rho(z=\ell)|, \quad (42)$$

Fig. 1

f is the distance where the beam would be reduced at a point F , in a vacuum, after passing the lens in the absence of emittance spread. So

$$f = - \frac{(1 - \epsilon^2/K\sigma_0^4) \cos 2\sqrt{K} l + (1 + \epsilon^2/K\sigma_0^4)}{\sqrt{K} (1 - \epsilon^2/K\sigma_0^4) \sin 2\sqrt{K} l}. \quad (43)$$

However, we are interested in thin lens which is defined by the condition

$$\sqrt{K} l \ll 1 \quad (44)$$

Consequently, in this approximation Eq. (43) gives:

$$f \approx \frac{1}{|(\epsilon^2/K\sigma_0^4) - 1| K l}, \quad (45)$$

and, in addition, for a strong focusing, namely

$$\epsilon^2/K\sigma_0^4 \ll 1, \quad (46)$$

it gives the usual thin lens equation:

$$f = 1/Kl. \quad (47)$$

Under the condition (46), Eq. (39) becomes

$$\sigma^2(z) \approx \sigma_0^2(1 - Kz^2). \quad (48)$$

Note that under the condition (44) the reduction factor at the end of the lens

$$1 - Kl^2 = \sigma^2(l)/\sigma_0^2$$

is almost 1. So we can put $\sigma(l) \approx \sigma_0$.

4.2 In the Final Focus

According to the solution for the e^-e^+ final focus linear collider is from Eq. (39)

$$\sigma^2(z) = \sigma^{*2}[1 + z^2/\beta^{*2}], \quad (49)$$

where σ^* is the beam spot-size at the interaction point (i.p.), defined by $z = 0$ and

$$\beta^* = \sigma^{*2}/\epsilon \quad (50)$$

(the Rayleigh length).

Fig. 2

Defining the β -function $\beta(z)$ as

$$\beta(z) = \sigma^2(z)/\epsilon \quad (51)$$

we can write

$$\beta_0(z) = \beta^*(1 + s^2/\beta^{*2}), \quad (52)$$

and β^* represents the β -function at the i.p. (s is the final focus semi-dimension) and $\beta_0 = \beta(z = \pm l)$.

Now Eq. (41) becomes

$$\varrho(z) = \frac{1 + z^2/\beta^{*2}}{z/\beta^{*2}}. \quad (53)$$

So we can match the focal length of the plasma lenses f with the quantity $|\varrho(z=s)|$ as follows

$$f = |\varrho(z=s)|$$

namely,

$$f = \frac{1 + s^2/\beta^{*2}}{s/\beta^{*2}}. \quad (54)$$

Combining (52) and (54) we get:

$$\beta^* = \frac{\beta_0}{1 + \beta_0^2/f^2} \quad (55)$$

for strong focusing $\beta^* \ll s, \beta_0$. Equations (52) and (55) give

$$f \approx s \approx (\beta^*\beta_0)^{1/2} = \frac{\sigma_0}{\epsilon^{1/2}} (\beta^*)^{1/2}. \quad (56)$$

It can be easily seen that (56) is also satisfied if $f \ll \beta_0$ in eq. (55). So we can write what usually occurs in a linear collider when β^* is very small

$$\beta^* \ll f \ll \beta_0. \quad (57)$$

5. LENS PARAMETERS FOR CLIC

The main parameters of interest for the CERN Linear Collider (CLIC) are reported in Table 1. See also Ref. [4].

Table I

Energy/mc ²	γ	2×10^6 (1 TeV)
Luminosity at i.p.	$\mathcal{L}^*$	$1.1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$
Spot size at i.p.	σ^*	12 nm
Pinch enhancement	H	2.37
Repetition rate	ν	1.69 kHz
Bunch population	N	0.5×10^{10}
Bunch length	σ_z	200 μm
Distruption	D	3.3
Emittance (normalized)	ϵ_n	10^{-6} rad.m ($\epsilon = \epsilon_n/\gamma = 0.5 \times 10^{-12} \text{ rad,m}$)
Beta-function at i.p.	β^*	282 μm

After some iterations to ensure a useful demagnification with modest aberrations, we choose the following input parameters:

Initial beam spot-size	$\sigma_0 = 0.63 \mu\text{m}$
Plasma number density	$n_0 = 10^{19} \text{ cm}^{-3}$
Beam number density	$n_b = 10^{18} \text{ cm}^{-3}$

So the condition (27) is satisfied and we are working in the linear lens regime which does not make difference between electron and positron focusing.

Using the relation

$$\beta_0 = \sigma_0^2 / \epsilon$$

we find

$$\beta_0 = 0.8 \text{ m.}$$

Equation (16) gives

$$k_p^{-1} = c/\omega_p = 1.68 \text{ mm.}$$

Thus

$$k_p \sigma_z \gg 1$$

and the focusing mechanism works in the LBL regime.

Consequently, we can assume Eq. (31) as an expression for the focusing force.

The LBL regime must be consistent with the linear regime condition ($\kappa_p \sigma_0 \ll 1$). This is true in our case since $\sigma_0 \ll \sigma_z$:

$$\sigma_0 \ll 1.68 \mu\text{m} \ll \sigma_z.$$

By introducing the values of β^* and β_0 in eq. (56) we find

$$f \sim s \approx 1.5 \text{ cm.}$$

It can be seen that in this case the condition (57) is satisfied. Equation (36) allows us to calculate the focusing strength giving

$$K \approx 1.2 \text{ cm}^{-2} \quad (W_{\perp}/r \sim 4.2 \text{ Mgauss/cm})$$

and, in turn, by means of the thin lens equation we get the lens thickness

$$l \approx 0.5 \text{ cm.}$$

It can be easily seen that condition (44) is also almost satisfied.

Finally, by using the value of K , σ_0 and ϵ we verify the condition (46) which shows that the system works in the strong focusing regime:

$$\frac{\epsilon^2}{K\sigma_0^4} \approx 8.1 \times 10^{-4}.$$

Note that without pinch enhancement effect the usual expression for the luminosity at i.p. is given by:

$$\mathcal{L}^* = \frac{\nu N^2}{4\pi\epsilon\beta^*} = \frac{\nu N^2}{4\pi\epsilon\beta_0} (1 + \beta_0^2/f^2), \quad (58)$$

where we have used Eq. (55). Furthermore, by using condition (57) we obtain

$$\mathcal{L}^* \approx \frac{\nu N^2}{4\pi\beta_0} \left(\frac{\beta_0}{f} \right)^2. \quad (59)$$

In the ideal case f is perfectly constant. Unfortunately, in reality

f is not a constant since it varies because of the intrinsic focusing mechanism of the plasma lens. This focal length variation gives the aberrations. f depends on the initial position of the beam particles. So the particles are focused at different points. This makes an increased beam spot-size at the interaction point, a larger σ^* and, consequently, lower the luminosity. The amount of this dilation can be estimated by $|\Delta f/f|$ where Δf is the focal length variation.

For the following we distinguish two kinds of aberrations: the longitudinal aberration and the spherical aberration.

6. LONGITUDINAL ABERRATIONS

The simple expression of the longitudinal aberrations in a linear plasma lens is given for our case by Eq. (31), where the focusing force depends on the initial position of the beam particles (note that $\xi = z - \beta ct = z_0$, where z_0 is the particle longitudinal position). So it is very easy to prove that the focal length depends on z_0 and it is given by

$$f(z_0) = \frac{1}{K\ell} \frac{1}{g(z_0)} \equiv \frac{f_0}{g(z_0)}, \quad (60)$$

where

$$f_0 = 1/K\ell \quad (61)$$

is the "unperturbed focal length" according to Eq. (47). That means that the particles in the tails of the longitudinal distribution are less focused than those in the main region of the distribution. By using Eq. (59) we have now

$$\sigma^*(z_0) = \frac{\sqrt{N^2}}{4\pi\beta_0} \frac{\beta_0}{f_0} [g(z_0)]^2. \quad (62)$$

Equation (62) implies that the "effective luminosity" is determined only in the main region of the longitudinal distribution. In order to calculate such an effective luminosity $(\mathcal{L}^*_{\text{eff}})_{\text{LONG}}$, we make an average of Eq. (62) by $g(z_0)$

$$(\mathcal{L}_{\text{eff}}^*)_{\text{LONG}} \equiv \langle \mathcal{L}^*(z_0) \rangle = \frac{\int \mathcal{L}^*(z_0) g(z_0) dz_0}{\int g(z_0) dz_0}. \quad (63)$$

By introducing the quantity

$$\delta_L \equiv \frac{\int [g(z_0)]^3 dz_0}{\int g(z_0) dz_0}$$

we get

$$(\mathcal{L}_{\text{eff}}^*)_{\text{LONG}} = \delta_L \mathcal{L}_0^*, \quad (64)$$

where

$$\mathcal{L}_0^* \equiv \frac{vN^2}{4\pi\epsilon\beta_0} \left(\frac{\beta_0}{f_0} \right)^2$$

is the "unperturbed luminosity at i.p." according to (59). For a longitudinal parabolic profile

$$\delta_L = 0.68 \quad (65)$$

and $(\mathcal{L}_{\text{eff}}^*)_{\text{LONG}}$ is 32% less than $\mathcal{L}_0^*$.

The longitudinal aberrations are important because of the strong focusing.

In fact, strong focusing means very small β^* , namely $f \ll \beta_0$. So the term $[g(z_0)] \beta_0/f_0]^2$ becomes large with respect to the unity in Eq. (58). Consequently, the longitudinal aberrations are introduced. On the other hand, if $\beta_0 \ll f_0$ the effect of $g(z_0)$ cannot be very important, but this situation corresponds to a weaker focusing and a larger β^* .

7. SPHERICAL ABERRATIONS

Spherical aberrations occur when the transverse force is not perfectly linear (with respect to r): a weak defocusing force then appears. In fact, a more careful analysis [3] gives (apart from the longitudinal profile):

$$F'(r) \approx -2\pi e^2 n_b \left[1 - \frac{1}{8} \frac{r^2}{\sigma_0^2} \right] r \quad (66)$$

$$\equiv -k_1 r + k_2 r^3,$$

where $|k_2 r^3| \ll |k_1 r|$.

It can be easily shown that the focal length is no more a constant, but changes with the radial initial positions of the particles r_0 . So

$$f(r_0) = f_0 \left(1 + \frac{k_2}{k_1} r_0^2 \right). \quad (67)$$

Consequently, from (55) and (57) we find:

$$\begin{aligned} \frac{1}{\beta^*(r_0)} &= \frac{1}{\beta_0} \left[1 + \frac{\beta_0^2}{f_0^2} \left(1 - \frac{2k_2}{k_1} r_0^2 \right) \right] \\ &\approx \frac{1}{\beta_0} \frac{\beta_0^2}{f_0^2} \left(1 - \frac{2k_2}{k_1} r_0^2 \right). \end{aligned}$$

So the luminosity at i.p. is now

$$\mathcal{L}^*(r_0) = \frac{v N^2}{4\pi e \beta_0} \frac{\beta_0^2}{f_0^2} \left(1 - \frac{2k_2}{k_1} r_0^2 \right),$$

and the effective luminosity defined as

$$(\mathcal{L}^*_{\text{eff}})_{\text{SPH}} \equiv \frac{\int \mathcal{L}^*(r_0) f(r_0) r_0 \, dr_0}{\int f(r_0) r_0 \, dr_0} \quad (68)$$

is now

$$(\mathcal{L}^*_{\text{eff}})_{\text{SPH}} = \left(1 - \frac{2k_2}{k_1} \sigma_0^2 \right) \mathcal{L}^*_0, \quad (69)$$

where we have introduced the beam spotsizes by

$$\sigma_0^2 = \frac{\int r_0^2 f(r_0) r_0 \, dr_0}{\int f(r_0) r_0 \, dr_0}. \quad (70)$$

Putting

$$\delta_S \equiv 1 - \frac{2k_2}{k_1} \sigma_0^2 \quad (71)$$

Eq. (66) gives

$$\delta_S = 0.75 . \quad (72)$$

So $(\mathcal{L}^*_{\text{eff}})_{\text{SPH}}$ is 25% less than $\mathcal{L}^*_0$.

It is very simple to see that if we take into account both longitudinal and spherical aberrations, we have:

$$\mathcal{L}^*_{\text{eff}} = \delta_S \delta_L \mathcal{L}^*_0 \equiv 0.51 \mathcal{L}^*_0 . \quad (73)$$

So $\mathcal{L}^*_{\text{eff}}$ is now 49% less than $\mathcal{L}^*_0$.

Note that in the case of a nonlinear plasma lens, the aberrations would be smaller. In fact, in Eq. (33), instead of the beam density, we have the plasma density. If the plasma density is homogeneous enough the focusing force is much more linear (in r) than in the linear regime. It means that the spherical aberrations would be reduced. The longitudinal aberration would be also reduced if the ion background is homogeneous. Unfortunately, this kind of lens does not work for positrons.

8. CONCLUSIONS

We find that a preliminar analysis of the longitudinal and transverse aberrations in a wake field plasma lens still allows us to focus down from 630 to 12 nm in both planes. The luminosity is roughly half the theoretical value to be expected from 12 nm radius spot when these aberrations are taken into account.

Remaining problems include designing the detectors to veto background from the beam-plasma events and design of a conventional telescope to bring the spot to a cricle of radius 630 nm. Synchrotron radiation effects due to the deflection which may independently impose a limit, have not been considered here.

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