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EFFECTIVE BOUNDARY ACTION IN THE FERMIONIC
STRING THEORY

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Abstract

The Polyakov path integral on Riemann surfaces with boundaries for the Neveu-Schwarz-Ramond (NSR) fermionic string is computed in the case of a special choice of the local boundary conditions for the fermionic fields. Effective boundary action is written explicitly by means of the functions defined on the double of the surface. The obtained expressions are used to study the behaviour of the string propagator in the configuration space. The generalization of the effective fermionic action to the case of nonarchimedean strings is discussed.

The aim of this letter is to study the properties of the effective boundary action in the $D = 10$ NSR fermionic string theory. The effective boundary action appears when the Polyakov path integral $[I]$ on Riemann surface with boundaries is computed in the case of local fermionic boundary conditions preserving $N = 1$ supersymmetry of the string action. The result of the calculations is the functional of the boundary conditions, which can be considered as a wave function in some space. The path integral over the fields of matter supermultiplet has the form:

$$\mathcal{Z} = \int_{\Sigma} D\psi \exp\left(-\int_{\Sigma} \bar{\psi} \partial \psi\right) \int_{\Sigma} D\psi D\bar{\psi} \exp\left(\int_{\Sigma} \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi}\right) S \dots S \quad (I)$$

where Σ is Riemann surface with boundaries; $\bar{\partial}, \partial$ are operators locally written as $\partial/\partial\bar{z}$, $\partial/\partial z$ correspondingly; $S \dots S$ - the product of the supercurrents. Let us consider the case when the supercurrents are absent (for example, due to boundary conditions). Then $\mathcal{Z}$ in (I) can be factorized into two factors depending upon the metric on the surface and boundary conditions:

$$\mathcal{Z} = \mathcal{Z}_B \cdot \mathcal{Z}_F \quad (2)$$

The bosonic part $\mathcal{Z}_B$ was considered in Refs. [2 - 6]. Let us compute:

$$\mathcal{Z}_F = \int_{\Sigma} D\psi D\bar{\psi} \exp\left(\int_{\Sigma} \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi}\right) \quad (3)$$

where the following local boundary conditions [7] are satisfied:

$$(\psi + \bar{\psi})|_{\partial\Sigma_i} = \delta_i, \quad i = 1, \dots, M, \quad \partial\Sigma = \bigcup_{i=1}^M \partial\Sigma_i \quad (4)$$

(M is the number of the components of the boundary). The boun-

dary condition (4) can be specified in the following way. Consider the local coordinates near the boundary: $(z_{(i)}, \bar{z}_{(i)})$ in which boundary can be determined by:

$$z_{(i)} \bar{z}_{(i)} = 1, \quad i = 1, \dots, M \quad (5)$$

It corresponds to the case when our Riemann surface Σ is obtained by cutting out M discs from the closed Riemann surface $\tilde{\Sigma}$. The centers of these discs are situated at the points P_i : $z_{(i)}(P_i) = 0$, and the discs are determined by $|z_{(i)}| < 1$. The boundary conditions on fermions ($1/2$ differentials) depend on the choice of the trivializing section, which can be chosen with zero divisor on Σ (but not on $\tilde{\Sigma}$). When the trivialization has the form $(dz_{(i)})^{1/2}$ near $\partial\Sigma_i$, (4) can be written explicitly as:

$$\psi dz_{(i)}^{1/2} + \bar{\psi} d\bar{z}_{(i)}^{1/2} = \gamma_i dz_{(i)}^{1/2} \quad (6)$$

Let us change the integration variables in (3): $\psi \rightarrow \psi + \psi_{ce}$, $\bar{\psi} \rightarrow \bar{\psi} + \bar{\psi}_{ce}$, where

$$\psi_{ce}(z) = \frac{1}{2\pi} \int_{\partial\Sigma} dw \gamma^+(w) \mathcal{S}(z, w)$$

$$\left[\psi_{ce} + \bar{\psi}_{ce} \left(\frac{d\bar{z}_{(i)}}{dz_{(i)}} \right)^{1/2} \right] \Big|_{\partial\Sigma_i} = \gamma_i(z_{(i)}) \quad (7)$$

($\gamma^+(w)$ is the part of γ , which can be homomorphically continued on Σ , the definition of $\mathcal{S}(z, w)$ see below). Then

$$\begin{aligned} Z_F &= \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[\int_{\Sigma} (\psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi}) + \int_{\partial\Sigma} \gamma (\psi - \bar{\psi}) \right] \\ &\quad (\psi + \bar{\psi}) \Big|_{\partial\Sigma_i} = 0 \end{aligned} \quad (8)$$

$$\gamma(z_{(i)}) = \gamma_i(z_{(i)}), \quad z_{(i)} \in \partial\Sigma_i$$

The boundary condition in (8) becomes zero, but the following

linear in δ term appears:

$$\int_{\partial \Sigma} \delta(\psi - \bar{\psi}) = \sum_{j=1}^M \left(-\frac{i}{2}\right) \int_{\partial \Sigma_j} \left\{ d z_{(j)} \delta_j \psi(z_{(j)}) - d \bar{z}_{(j)} \left(\frac{d z_{(j)}}{d \bar{z}_{(j)}}\right)^{1/2} \delta_j \bar{\psi}(\bar{z}_{(j)}) \right\} \quad (9)$$

To get rid of the linear term, one should make the substitution:

$$\psi \rightarrow \psi + \frac{1}{2\pi} \int_{\partial \Sigma} dw \delta(w) S(z, w) \quad (10)$$

and the corresponding substitution for $\bar{\psi}$ variable. (Note that by means of (7) we add a holomorphic piece to ψ but the change (10) is quite different). The object $S(z, w)$ in (7), (10) is well known Szego kernel (see for example Ref. [8]), defined on the double $\hat{\Sigma}$ of the surface Σ . In the simplest case (Σ -disc, $\hat{\Sigma}$ -sphere): $S(z, w) = \frac{1}{z-w}$. In general:

$$S(z, w) = \frac{\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] \left(\begin{smallmatrix} z \\ w \end{smallmatrix} \right)}{\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] (0) E(z, w)} \underset{z \rightarrow w}{\sim} \frac{1}{z-w} + \dots \quad (11)$$

where $\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]$ is some even spin structure on $\hat{\Sigma}$ (induced from Σ), $E(z, w)$ is Prime form, $\vec{\omega}$ is the canonical set of holomorphic 1-differentials (see Ref. [8] for definitions). Using (9), (10) for (8) one obtains:

$$\begin{aligned} Z_F &= e^{-I^*} \int_{\Sigma} d\psi d\bar{\psi} \exp \left(\int_{\Sigma} \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} \right) = \\ &= e^{-I^*} \left(\det \bar{\partial}_{1/2} \det \partial_{1/2} \right)^{1/2} \end{aligned} \quad (12)$$

where

$$I^* = \frac{1}{8\pi} \int_{\partial\Sigma} dz \int_{\partial\Sigma} dz' \delta(z) S(z, z') \delta(z') =$$

$$= \frac{1}{8\pi} \sum_{i,j} \int_{\partial\Sigma_i} dz_{(i)} \int_{\partial\Sigma_j} dz'_{(j)} \delta_i(z_{(i)}) S(z_{(i)}, z'_{(j)}) \delta_j(z'_{(j)}) \quad (I3)$$

The expressions (I2), (I3) can be written explicitly for the case of cylinder when the fermions are in the NSxNS sector (i.e. antiperiodic when passing around the noncontractible cycle). For the RxR sector (periodic fermions) the expression (I2) as written above vanishes (due to the presence of the zero mode). For this expression to be of any use it should be modified. For example, one can fix $\psi + \bar{\psi}$ on the one component of the boundary and $\psi - \bar{\psi}$ on the other component. The expression (I2) cannot be immediately generalized to the case of the NSxR sector. So, in the NSxNS sector one has:

$$I^* = \frac{1}{8\pi} \left[\int_{C_q^-} dz \int_{C_q^-} dw \delta_1(z) \delta_1(w) S(z, w) + \int_{C_1^+} dz \int_{C_1^+} dw \delta_2(z) \delta_2(w) S(z, w) + \right.$$

$$\left. + 2 \int_{C_q^-} dz \int_{C_1^+} dw \delta_1(z) \delta_2(w) S(z, w) \right] \quad (I4)$$

where

$$S(z, w) = \frac{1}{2\pi i \sqrt{zw}} \frac{\theta \left[\begin{smallmatrix} 1/2 \\ 0 \end{smallmatrix} \right] \left(\frac{1}{2\pi i} \log \frac{z}{w} \middle| 2it \right) \theta \left[\begin{smallmatrix} 1/2 \\ 1/2 \end{smallmatrix} \right]' (0 \middle| 2it)}{\theta \left[\begin{smallmatrix} 1/2 \\ 0 \end{smallmatrix} \right] (0 \middle| 2it) \theta \left[\begin{smallmatrix} 1/2 \\ 1/2 \end{smallmatrix} \right] \left(\frac{1}{2\pi i} \log \frac{z}{w} \middle| 2it \right)} =$$

$$= \frac{1}{z-w} \frac{z+w}{2\sqrt{zw}} \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^2 \frac{(1+q^{2n} \frac{w}{z})}{(1-q^{2n} \frac{w}{z})} \frac{(1+q^{2n} \frac{z}{w})}{(1-q^{2n} \frac{z}{w})} \quad (I5)$$

(The representation of the cylinder as the annulus $1 > |z| > q = e^{-2\pi t}$ is used. $C_R^{\pm}$ are circumferences of the radius R , $\pm$ refers to their orientations). The functions δ_i can have poles when $B_{(i)} = 0$ ($i=1, \dots, m$) and in the global coordinates have

the form:

$$\gamma_1(z) = \sum_{n \leq 0} \gamma_{n+\frac{1}{2}}^{(1)} q^{-n-\frac{1}{2}} z^n, \quad \gamma_2(z) = i \sum_{n \leq 0} \gamma_{n+\frac{1}{2}}^{(2)} z^{-n-1} \quad (I6)$$

The determinant in (I2) depends upon the choice of the metric on the surface. The standard choice is $ds^2 = \left| \frac{dz}{2\pi i \bar{z}} \right|^2$ on the annulus $I > |z| > q$ (induced by the flat metric on the cylinder $0 < \xi_1 < I, 0 < \xi_2 < t$). The result for the determinant is then:

$$[\det \bar{\partial}_{1/2} \det \partial_{1/2}]^{1/2} = q^{-\frac{1}{24}} \prod_{k=0}^{\infty} (1 - q^{2k+1}) = \lambda(q) \quad (I7)$$

The result for the general metric is equal to $\lambda(q)$ up to the exponent of the Liouville action.

Now one can study the expression for the string propagator Let us raise $Z = Z_B \cdot Z_F$ to the power $D=10$ and, corresponding to the chosen boundary conditions, integrate over the metrics. The Liouville factors will be cancelled and if one takes the consistent choice of the ghost boundary conditions the integration measure over the modular parameter of the cylinder will be proportional to the $8 = D-2$ power of the computed expression (only the physical degrees of freedom will contribute). Taking Z_B as calculated in [2], one gets:

$$G = \int_0^\infty \frac{dt}{t^5} \left[\frac{\lambda(q)}{\eta(q^2)} \right]^8 \exp [-I_B - I^*] \quad (I8)$$

where $\lambda(q), I^*$ are defined by (I7), (I5); Dedekind function is $\eta(q^2) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^{2n})$. Let us consider (I8), when the following "invariant" boundary conditions are taken:

$$x|_{\partial \Sigma_i} = x_i = \text{const} \Big|_{\partial \Sigma_i}, \quad \delta_i(z_{(i)}) dz_{(i)}^{1/2} = \delta_i dz_{(i)}^{1/2} = \text{const} \Big|_{\partial \Sigma_i} dz_{(i)}^{1/2} \quad (I9)$$

Then:

$$I_B = \frac{(\Delta x)^2}{2t}, \quad I^* = 0 \quad (20)$$

on every genus Riemann surface Σ (with two components of the boundary), where it should be equal to the "central" pure imaginary [8] element of the period matrix of the double surface $\hat{\Sigma}$. This fact is the consequence of the general form of the Laplace equation solutions on Σ ([5]):

$$\chi_\alpha = \sum_{i=1}^M \sum_{n=1}^{\infty} a_i^{(n)} f_i^{(n)}(z) + \vec{b} \int_{\tilde{\Sigma}} \vec{\omega} + c.c. \quad (21)$$

where $f_i^{(n)}(z)$ is the basis of meromorphic (with poles when $z_{(i)}=0$) functions on $\tilde{\Sigma}$, $\vec{\omega}$ - the set of holomorphic 1-differentials on the double $\hat{\Sigma}$. It can be easily shown that the contribution to the constant values $\chi|_{\partial\Sigma_i}$ is given by the only differential $\hat{\omega}$, normalized on the cycle homologically equal to a component of the boundary:

$$\oint_{\hat{a}} \hat{\omega} = \oint_{\partial\Sigma_1} \hat{\omega} = 1, \text{ then } \oint_{\hat{b}} \hat{\omega} = it \text{ (when } \hat{a}\hat{b}=1) \quad (22)$$

is the "central" matrix element of the period matrix of $\hat{\Sigma}$.

Now let us return to (18) and to study the behaviour of the integral when the integration variable tends to zero. One has:

$$\lambda(q) = e^{-\pi/24t} \prod_{n=1}^{\infty} (1 + e^{-\frac{\pi n}{t}}) \underset{t \rightarrow 0}{\sim} e^{-\pi/24t} (1 + \dots)$$

$$\eta(q^2) = \frac{e^{-\pi/24t}}{\sqrt{t}} \prod_{n=1}^{\infty} (1 - e^{-\frac{\pi n}{t}}) \underset{t \rightarrow 0}{\sim} \frac{e^{-\pi/24t}}{\sqrt{t}} (1 + \dots) \quad (23)$$

Then, using (20), (23) and restoring factor $2\pi\alpha'$, for (18) we obtain:

$$\int_0^{\infty} \frac{dt}{t} \exp\left[-\frac{(\Delta x)^2}{4\pi\alpha' t}\right] \sim \log \frac{(\Delta x)^2}{\alpha'} \quad (24)$$

If we know the behaviour of the propagator in x -space we can make a conclusion about the causal properties of the theory [9, 10]. It was shown there that the string propagator (in the pseudoeuclidean space) can have singularities in the space-like domain. The maximal domain of the singularities in the space-like direction (determined by the condition $(\Delta x)^2 = \ell^2 = 4\pi^2\alpha'$) appears only in the particular case of the boundary conditions for x and ψ variables. Changing the choice of boundary conditions one can contract the domain of singularities out of the lighth-cone. It can be seen from (24) that the singularity domain is absent if one takes supersymmetrical boundary conditions as (4), i.e. supersymmetry eliminates the singularities out of the lighth-cone. This fact may be explained with the help of the "dual" ($t \rightarrow 1/t$) sector. From this point of view the calculated expression corresponds to the one-loop partition function in the Ramond sector of the open string. Then the absence of uncausal singularity in the propagator (24) corresponds to the absence of the tachyon in the spectrum of the Ramond model, i.e. to the consequence of the supersymmetry.

It should be also pointed out that one can connect the expression (24) with the study of the critical behaviour in the three-dimensional Ising model, following the ideology of Ref. [11]. The chosen conditions (4), (19) are successful in the

sence of the independence of the parametrization of the boundaries. However, the asymptotic (24) is true only in $D=10$, and for the 3-dimensional fermionic string one should take into account the corrections, connected with the Liouville modes contribution.

At last, let us notice that the effective boundary action can be used to compute the amplitudes in the open fermionic string theory. Therefore we can use the generalization of the obtained expressions in the nonarchimedean string theory [12]. For example, the disc amplitudes appear if one takes the effective action I^* simply with the Cauchy kernel. Then, the p-adic generalization will be:

$$I^*_{(p,\tau)} \sim \int_{Q_p} dx \int_{Q_p} dy \delta(x) \frac{\text{sign}_\tau(x-y)}{|x-y|_p} \delta(y)$$

where the sign character should satisfy the condition: $\text{sign}_\tau(-x) = -\text{sign}_\tau(x)$. We can also consider the one-loop fermionic string amplitudes by analogy with the bosonic case [13], generalizing (15) as:

$$\Delta_{p,\tau}(x,y) \sim \frac{\text{sign}_\tau(x-y)}{|x-y|_p} \frac{|x+y|_p}{2\sqrt{|x|_p|y|_p}} \left| \prod_{n=1}^{\infty} \left(\frac{1-q^{2n}}{1+q^{2n}} \right)^2 \frac{(1+q^{2n}\frac{y}{x})(1+q^{2n}\frac{x}{y})}{(1-q^{2n}\frac{y}{x})(1-q^{2n}\frac{x}{y})} \right|_p$$

The problem of the p-adic generalization of the fermionic strings will be considered in detail in Ref. [14].

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