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MULTI-LOOP NO-RENORMALIZATION THEOREMS FOR A CLASS OF
FOUR-DIMENSIONAL STRING MODELS

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A B S T R A C T

We demonstrate the vanishing of one-, two- and three-point functions in a class of four-dimensional string models, which implies the non-renormalization of the Kähler potential and gauge kinetic term characterizing the low-energy effective field theory. Previous results are extended rigorously to genus two. We also discuss the prospects for further extension to arbitrary genus and gauge non-singlet fields.

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It seems likely that for the foreseeable future the task of phenomenological physics will be to explore the low-energy effective field theory derived from string theory. This effective field theory is presumably some form of $d = 4$, $N = 1$ supergravity. Terms which are at most quadratic in the field derivatives, and hence dominant at low energies, may be characterized by two functions of the chiral supermultiplet fields. One is the Kähler potential, which specifies among other features of the supergravity theory the kinetic energy terms and Yukawa couplings of the chiral fields, and the other is an analytic function, which specifies the kinetic terms of the gauge fields [1]. It is therefore vital to specify the Kähler potential and gauge kinetic function derived from string.

The genus-zero (tree-level) contribution to the effective field theory is known [2] to have a Kähler potential of some generalized no-scale type [3], and the gauge kinetic function is known to have a simple universal form. Scaling laws characterizing the general forms of higher-genus ($-loop$) corrections to the Kähler potential and the gauge kinetic function are also known [4]. However, these arguments do not actually tell us whether any of the possible higher-genus contributions are in fact non-zero. Partial results are available which indicate that genus-one (one-loop) string configurations do not renormalize either the Kähler potential or the gauge kinetic function in low-energy effective field theories obtained from the ten-dimensional heterotic string and a restricted class of four-dimensional models [5]. The purpose of this letter is to extend these results to higher-genus configurations in a class of four-dimensional string models, where the left- and right-moving sectors are separately modular invariant.

We first prove that, like the genus-one (one-loop) contributions, genus-two (two-loop) contributions to a set of one-, two- and three-point functions vanish. Next we recall that the vanishing in arbitrary background fields of contributions to one-, two- and three-point functions which are bilinear in the external field momenta are sufficient to show that the Kähler metric and the gauge kinetic function of the effective low-energy field theory are not renormalized. We then combine these arguments to show that there are no purely dilaton-dependent genus-one or -two renormalizations of the Kähler metric or of the gauge kinetic functions. We also discuss the prospects for extending these no-renormalization theorems to gauge non-singlet chiral fields and to higher-genus surfaces.

We first specify the set of four-dimensional models under consideration, namely those in which the left- and right-moving sectors are separately modular invariant. The general form of the modular-invariant genus- g partition function is [6]:

$$Z_g = \int_{\mathcal{M}_g} [\mathcal{D}\Omega] \sum_{\alpha, \beta \in \Xi_g} C[\alpha] X[\beta] \prod_{S \in F} \theta \left[\begin{smallmatrix} \alpha(S) \\ \beta(S) \end{smallmatrix} \right]^{\frac{1}{2}}(0, \Omega), \quad (1)$$

where $\mathcal{M}_g$ is the moduli space with measure $[\mathcal{D}\Omega]$, which includes the determinants coming from integrations over non-zero modes and is described by $3g-3$ complex parameters, Ω is the period matrix, $\theta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]$ are the θ -functions for genus g with characteristics α, β (each of which are g -vectors with entries 0 or 1), $X \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]$ is the contribution of the fermionic ghosts and the integration over the supermoduli, which includes the insertion of $2g-2$ supercurrents (picture-changing operators) coupled to the gravitino zero modes. The coefficients

$$C \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] = \prod_{i=1}^g C \left[\begin{smallmatrix} \alpha_i \\ \beta_i \end{smallmatrix} \right] \quad (2)$$

are determined by modular invariance; their product form (2) in terms of genus-one coefficients $C \left[\begin{smallmatrix} \alpha_i \\ \beta_i \end{smallmatrix} \right]$ ($i = 1, \dots, g$), is required by factorization. In models where modular invariance is imposed separately on the left- and right-movers (commonly called $N = 4$ models), the fermions are assembled in sets of eight species with one coefficient for each factor $\theta^4 \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]$. We thus have a sum

$$\sum_{\alpha, \beta \in \Xi_g} X \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] \theta^4 \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] \delta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] \quad (3)$$

with phases $\delta \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right]$ dictated by modular invariance.

The vanishing of the one-, two- and three-point functions at the genus-one level in this class of models [5] follows from the Riemann identity [7]

$$\sum_{\ell=1}^4 (-1)^{\ell+1} \prod_{a=1}^4 \theta_{\ell}(x_a, \tau) = 2 \prod_{a=1}^4 \theta_1(x'_a, \tau), \quad (4)$$

where $\theta_{\ell}(x, \tau)$ ($\ell=1, \dots, 4$) are the genus-one θ -functions and $x'_{1,2} = \frac{1}{2}(x_1 + x_2 \pm x_3 \pm x_4)$; $x'_{3,4} = \frac{1}{2}(x_1 - x_2 \pm x_3 \pm x_4)$. The Riemann identity is useful here as there are no gravitino zero modes ($g=1$), which makes $X \left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right] = 1$.

The vanishing of one-, two- and three-point functions at the genus-two level in this class of models requires more analysis because there are two gravitino zero modes and hence two insertions of picture-changing operators. We recall that the supercurrent J_S has two contributions

$$\mathcal{J}_S = \mathcal{J}_{gh} + \mathcal{J}_m \quad , \quad (5)$$

where the ghost part is

$$\mathcal{J}_{gh} = -c \partial \beta + \frac{1}{2} \gamma \beta - \frac{3}{2} \partial c \beta \quad (6)$$

and the matter part is

$$\mathcal{J}_m = \mathcal{J}_A + \mathcal{J}_B \quad , \quad (7)$$

where

$$\mathcal{J}_A = -\frac{1}{2} \psi^\mu \partial \chi_\mu \quad (8a)$$

$$\mathcal{J}_B = -\frac{1}{2} \sum_{I=1}^6 \lambda^I y^I \omega^I \quad . \quad (8b)$$

Here b, c are the ghosts and β, γ are the superghosts, X^μ ($\mu=0, \dots, 3$) is the bosonic and ψ^μ is the fermionic part of the matter supermultiplet, and $(\lambda^I, y^I, \omega^I)$ ($I=1, \dots, 6$) are the left-moving fermions constituting the non-linear realization of supersymmetry. The product $J_S(1)J_S(2)$ contains non-zero correlators only for $J_{gh}(1)J_{gh}(2)$ and $J_m(1)J_m(2)$, which are coupled to the gravitino zero-mode wave functions χ_a ($a=1, 2$). Taking these wave functions to be independent of the supermoduli and located at points z_a ($a=1, 2$); i.e., $\chi_a = \delta(z-z_a)$ ($a=1, 2$), we find that the points z_a where the supercurrents are inserted can be shifted around arbitrarily, changing the integrand only by a total derivative in moduli space [8]. We will later on fix these points to be located at the zeroes of the Abelian differential as suggested in Ref. [9]. Using now the hyperelliptic representation [10], which is valid for all genus-two surfaces, we find that the two holomorphic differentials can be written as

$$\omega_1(z) = y^{-1}(z) \quad (9a)$$

$$\omega_2(z) = z y^{-1}(z) \quad , \quad (9b)$$

where the defining equation for the hyperelliptic surface is

$$y^2(z) = \prod_{i=1}^6 (z-a_i) \quad . \quad (10)$$

The branch points a_i ($i=1, \dots, 6$) define three branch cuts in the lower and upper sheets for the function $y(z)$, which represents a generic genus-two Riemann surface. Three of these points can be fixed by Möbius invariance and the other three can be taken to be the moduli. In this hyperelliptic representation the correlation functions for an even spin structure α can be written as [10]

$$\langle \psi(z) \psi(x) \rangle_\alpha = \frac{1}{z-x} \frac{U(z) + U(x)}{2 \sqrt{U(z)U(x)}} , \quad (11)$$

where

$$U(x) = \prod_{i=1}^3 \sqrt{\frac{x-A_i}{x-B_i}} . \quad (12)$$

Here $\{A_k\}$ and $\{B_k\}$ ($k=1,2,3$) are a splitting of the branch points $\{a_i\}$ ($i=1, \dots, 6$) into two non-intersecting sets. This splitting is equivalent to one of the ten possible even spin structures α , and a permutation of the points a_i is equivalent to a change of the spin structure. Thus a modular-invariant combination would be found by symmetrization over the branch points. The non-zero mode contributions to the fermionic determinants for spin structure α can also be written in terms of the branch points. Here we use the Thomae formula [7]

$$Q_\alpha \equiv \partial_\alpha^4(0, \tau) = \pm \det^2 K \prod_{i < j=1}^3 A_{ij} B_{ij} , \quad (13)$$

where K is the period matrix along the A-cycle for the holomorphic differentials (9) and

$$A_{ij} = A_i - A_j \quad ; \quad B_{ij} = B_i - B_j . \quad (14)$$

With this background in mind, we now fix the points z_a ($a=1,2$), where the supercurrents are inserted, to be the two simple zeroes of $\omega_1(z)$; i.e., at the points ∞ on the upper and lower sheet ($z_{1,2} = \infty \pm$). The correlation functions for the matter part of the supercurrent (7) then become [9]:

$$\langle \psi(z_1) \psi(z_2) \rangle_\alpha = \frac{1}{2} \sum_{i=1}^3 (A_i - B_i) . \quad (15)$$

Here ψ is a generic spin- $\frac{1}{2}$ field [ψ^μ or λ^I , y^I , ω^I in J_A or J_B in Eq. (7)]. The correlators with the fields at z_a ($a=1,2$), on the other hand, are [11]

$$\langle \psi(x) \psi(z_1) \rangle_\alpha = \frac{U(x) - 1}{2 \sqrt{U(x)}} \quad (16a)$$

$$\langle \psi(x) \psi(z_2) \rangle_\alpha = \frac{\mathcal{U}(x) + 1}{2 \sqrt{\mathcal{U}(x)}} , \quad (16b)$$

where x is some point on the surface. For the ghost part of the supercurrent [Eq. (6)] we need the correlators of the spin-3/2 field. They can be expressed in terms of derivatives of spin- $\frac{1}{2}$ correlators, which for the above choice of z_a ($a=1,2$) can be written as

$$\langle \partial \psi(z_1) \psi(z_2) \rangle_\alpha = \frac{1}{2} \sum_{i=1}^3 (A_i^2 - B_i^2) . \quad (17)$$

Summing now over the spin structures α (i.e., permuting the branch points a_i) with phases which are uniquely determined by modular invariance [9], we will show that

$$\sum_{\alpha=1}^{10} \eta_\alpha \langle \mathcal{J}_m(z_1) \mathcal{J}_m(z_2) \rangle_\alpha = \sum_{\alpha=1}^{10} \langle \mathcal{J}_{gh}(z_1) \mathcal{J}_{gh}(z_2) \rangle_\alpha \eta_\alpha = 0 \quad (18)$$

The phases η_α as given in Ref. [9] are defined as follows:

$$\eta_1 = -\eta_2 = \eta_3 = -\eta_4 = \eta_5 = -\eta_6 = \eta_7 = \eta_8 = -\eta_9 = \eta_{10} = 1 , \quad (19)$$

where by convention the spin structures Q_α [Eq. (13)] are given by

$$\begin{aligned} Q_1 &= 123/456 , \quad Q_2 = 124/356 , \quad Q_3 = 125/346 , \quad Q_4 = 126/345 \\ Q_5 &= 134/256 , \quad Q_6 = 135/246 , \quad Q_7 = 136/245 \\ Q_8 &= 145/236 , \quad Q_9 = 146/235 , \quad Q_{10} = 156/234 . \end{aligned} \quad (20)$$

Here $ijk/\lambda mn$ is a partition of a_i into the sets A_i and B_i , and $a_1 = A_1$.

To prove Eq. (18) we use (5)-(8), (13)-(15), (17) and the phases (19). The expressions in (18) when written in terms of the branch points a_i are totally antisymmetric and should therefore be proportional to the totally antisymmetric polynomial

$$P(a) = \prod_{1 \leq i < j \leq 6} a_{ij} \quad (21a)$$

$$a_{ij} \equiv a_i - a_j \quad (21b)$$

This polynomial is of order 15 whereas the expressions in Eq. (18) are at most of order nine in the a_i , and therefore cannot be proportional to $P(a)$ with a non-zero coefficient. We can summarize this by a set of identities

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[\sum_{i=1}^3 (A_i^n - B_i^n) \right]^m = 0 \quad \begin{cases} n=2 \text{ for } m=1 \\ n=1 \text{ for } m=1,3 \end{cases} \quad (22)$$

a subset of which ($m=1$) was found in Refs. [11,12]^{*}). Here we find the additional identity $n = 1$, $m = 3$ relevant to the correlators of λ^I , y^I , ω^I in $\langle J_B(z_1) J_B(z_2) \rangle$ [see Eq. (8b)].

The familiar statement that the cosmological constant vanishes for this class of four-dimensional models follows directly from Eq. (18). In a similar way we can prove the vanishing of one-, two- and three-point functions for the emission of gravitons (including the dilaton) and/or gauge bosons. Two-point amplitudes involving scalar non-singlets would also vanish after the sum over spin structures. However, we will see that this sum alone does not guarantee the vanishing of all the three-point functions involving scalar non-singlets.

The vanishing of various amplitudes due to the sum over spin structures follows directly from the following set of identities

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[u(x) u(y) - \frac{(-1)^m}{u(x) u(y)} \right] \left[\sum_{j=1}^3 (A_j - B_j) \right]^m = 0 \quad m=0,1 \quad (23)$$

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[\frac{u(x)}{u(y)} - (-1)^m \frac{u(y)}{u(x)} \right] \left[\sum_{j=1}^3 (A_j - B_j) \right]^m = 0 \quad \begin{cases} n=2 \text{ for } m=1 \\ n=1 \text{ for } m=0,1,3 \end{cases} \quad (24)$$

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[u(x) - \frac{(-1)^m}{u(x)} \right] \left[\sum_{j=1}^3 (A_j - B_j) \right]^m = 0 \quad m=0,1,2 \quad (25)$$

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[\frac{u(v)}{u(x) u(w)} - (-1)^m \frac{u(x) u(w)}{u(v)} \right] \left[\sum_{j=1}^3 (A_j - B_j) \right]^m = 0 \quad m=0,1 \quad (26)$$

^{*}) These identities were given the name Lianzi identities in Refs. [11,12].

Among these identities, the case $m > 1$ is related to the insertion of $J_B(z_1)J_B(z_2)$ in the correlation functions, and the subset in (24)-(26) with $m = 0$ [also $m=1$ in (24)] can be found in Refs. [11,12]. To prove the vanishing of these expressions we simply count the powers of a_i which the left-hand side contains. Thus, apart from (24) ($m=3, n=1$) and (26) ($m=1$) the rest of the expressions on the left-hand side of (23)-(26) are polynomials of order less than 15 and totally antisymmetric. They are therefore zero as they cannot be proportional to $P(a)$ (21) with non-zero coefficients. In (24) ($m=3, n=1$) we have terms of order 15. However, they are all proportional to $\prod_{i=1}^3 A_i B_i = \prod_{j=1}^6 a_i$, which is independent of the spin structure α , and can therefore be factorized out of the sum over α , leaving a totally antisymmetric polynomial of order 9, which must vanish. In a similar way we find in (26) terms of order 16 and 15. The terms of order 16 all contain $\prod_{i=1}^3 A_i B_i$ and thus vanish, as this term can be factorized out of the sum over α , leaving a totally antisymmetric polynomial of order 10. The terms of order 15, on the other hand, do not have the correct structure to be proportional to $P(a)$, and would therefore have to vanish.

Two other expressions whose vanishing would guarantee the vanishing of the three-point functions containing scalar non-singlets do not in fact vanish. We find instead that

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[\frac{1}{u(x)u(v)u(w)} - u(x)u(v)u(w) \right] = \frac{P(a)}{y(x)y(v)y(w)} \quad (27)$$

$$\sum_{\alpha=1}^{10} \eta_{\alpha} Q_{\alpha} \left[\frac{u(v)}{u(x)u(w)} - \frac{u(x)u(w)}{u(x)} \right] \left[\sum_{j=1}^3 (A_j - B_j) \right]^2 = \frac{4P(a)(v-w)(x-v)}{y(x)y(v)y(w)}, \quad (28)$$

where $y(x)$ is the defining expression for the hyperelliptic surface (10). Both (27) and (28) appear in the contractions of the scalar non-singlets with the supercurrents $J_B(z_1)J_B(z_2)$. Thus the sum over the spin-structures alone does not guarantee the vanishing of all the three-point functions. However, by taking all possible permutations of fields in $J_B(z_1)$ and $J_B(z_2)$ [see Eq. (8b)], we find that these three-point functions do in fact vanish. To see this, we note that the only dependence of the three-point functions on the points (x,v,w) where the vertex operators are inserted is through the function y [Eq. (10)]. This is obvious in (27), but we will immediately see that it is correct also for the correlators contributing to (28). We analyze (27) first. Taking the two-point correlation functions between fermionic fields appearing in the vertex operators of the scalar non-singlets [13] with $\lambda, y, w, \varepsilon \in J_B$ we find, e.g., combinations of the form

$$\sum_{i,j=1}^6 \langle \psi(x) \lambda^i(z_1) \rangle_\alpha \langle \psi(x) \lambda^j(z_2) \rangle_\alpha \langle \psi(v) y^i(z_1) \rangle_\alpha \langle \psi(v) y^j(z_2) \rangle_\alpha \\ * \langle \psi(w) \omega^i(z_1) \rangle_\alpha \langle \psi(w) \omega^j(z_2) \rangle_\alpha \quad (29)$$

(where the ψ are generic fermionic fields which need not be the same in all the correlators) which after the sum over spin structures is equal to (27). Here we use (16) and (26) ($m=0$) to eliminate the rest of the terms appearing in this sum. We note that the two-point correlators in (29) are non-zero only when the indices fit and we have the same type of field (these are all free fields). If, for example, $\psi(x) = \lambda^M(x)$ is the fermionic field contributed by the scalar non-singlet, then the two-point correlator would be non-zero only for equal indices: $\langle \lambda^M(x) \lambda^I(z_{1,2}) \rangle = \delta^{MI}$. Permuting now the fermionic fields in $J_B(z_1) J_B(z_2)$, we would find that the sum is zero, as all the contributions are equal to (27) up to a sign. Here it is the symmetric form of (27) which is important for the proof.

In a similar way we find (28) in, e.g.:

$$\sum_{i,j=1}^6 \langle \psi(x) \psi(v) \rangle_\alpha \langle \psi(w) \psi(v) \rangle_\alpha \langle \psi(w) \lambda^i(z_1) \rangle_\alpha \langle \psi(v) \lambda^j(z_2) \rangle_\alpha \\ * \langle y^i(z_1) \omega^i(z_1) y^j(z_2) \omega^j(z_2) \rangle_\alpha \quad (30)$$

and the rest of the terms sum up to zero in view of the identities (23)-(26). Here we need to divide (28) by $4(x-v)(v-w)$ [see Eq. (11)] to get the correlator of (30), thus getting an expression depending on x, v, w symmetrically as in (27). Permuting now the fermionic fields in $J_B(z_1) J_B(z_2)$ we find that the sum is zero. Other possible combinations of two-point correlators which could appear in three-point functions involving scalar fields would vanish either because of the identities (23)-(26) or after taking all permutations over the fermionic fields as explained above. We have thus shown that all the one-, two- and three-point functions in the class of four-dimensional models (3) that we have studied are zero for genus two^{*)}.

To generalize the proof of the vanishing of the n -point functions ($n \leq 3$) to higher genus, one might use the conjecture

$$\sum_{\varepsilon=0,1} (-1)^\varepsilon \chi \left[\begin{smallmatrix} \varepsilon \\ 0 \end{smallmatrix} \right] = 0 \quad (31)$$

*) We note that the vanishing of these n -point functions ($n \leq 3$) for the class of models (3) was studied also in Ref. [14], with the conclusion that the Riemann identity which involves the sum over spin structures guarantees their vanishing. As we have noted above, this sum alone is not sufficient for the matter part of the supercurrent. As for the ghost part, permutations over all branch points in the partition function alone are not enough. One needs to take into account the dependence of the correlation functions (11) and (16) on the branch points, and for that we needed the larger set of identities (23)-(26).

which was first suggested in Ref. [15]. It is certainly correct for genus one, as the superghost contribution to $X[\frac{\alpha}{\beta}]$ is equal for all spin structures, but its validity for higher genus has to be carefully examined. Once it were known to be correct, a generalization of the Riemann identity would probably be useful for the proof. However, we cannot exclude the possibility that this line of argument might fail, but the higher-genus $n \leq 3$ -point functions might vanish for some other reason.

We would like to examine now the physical consequences of the no-renormalization theorems we have proved above. As was mentioned in the introduction, the Kähler potential K and the gauge kinetic function f fix the kinetic terms of the effective low-energy Lagrangian:

$$\mathcal{L}_K = K_j^i \partial^\mu \varphi^j \partial_\mu \varphi_i^* + \text{Re} f_{ab} F^{\mu\nu a} F_{\mu\nu}^b + \text{Im} f_{ab} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^b, \quad (32)$$

where the ϕ^i are chiral fields, the $F_{\mu\nu}^a$ are gauge field strengths, and

$$K_{ij} = \frac{\partial^2 K}{\partial \varphi^i \partial \varphi^{*j}} \quad (33)$$

is the Kähler metric. We see that the Kähler metric and the gauge kinetic function can be renormalized only if additional interaction terms are generated which are quadratic in the four-momenta k . Terms of higher order in the momenta are irrelevant to the effective low-energy theory.

We note that the chiral field ϕ^i in the class of four-dimensional string theories that we consider can be divided into two categories: the dilaton S which is a remnant of the truncated $N = 4$ graviton supermultiplet, and the other chiral fields $\phi^{\hat{i}}$ which may be gauge non-singlets.

The dependences of K and f on S are simple to discuss. At the genus-zero (tree-level) they are

$$K = -\ln(S + S^*) \quad ; \quad f_{ab} = S \delta_{ab} \quad . \quad (34)$$

The genus-one and -two contributions to all n -point functions $n \leq 3$ are now known to vanish for all values of the loop coupling parameter e^ϕ . Since values ϕ of e^ϕ are in one-to-one correspondence with values of S , this means that any genus-one or -two contribution to K_j^i or f must be independent of S .

The dependences of K_j^i or f on the other chiral fields $\phi^{\hat{i}}$ are more subtle to

discuss. At the genus-zero (tree-level) they are certainly more model-dependent, although generally of the no-scale type.

Some remarks about the genus-one level were made in Ref. [5]. The above arguments indicate that at the genus-two level there are no contributions to two- or three-point functions involving two or three gauge non-singlet fields. However, the extension to higher-point functions is not trivial, as the sums over J_B indices and spin structures do not guarantee vanishing, unlike the case [e.g., Eqs. (29) and (30)] for the three-point functions. We plan to return to these questions in a future paper.

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