CERN-TH.5294/89

SUPERSTRINGS WITH ZERO TENSION

J. Barcelos-Neto[†]

M. Ruiz-Altaba

CERN, Theory Division

CH-1211 Genève 23

ABSTRACT

The superstring in the limit of vanishing tension is analyzed with the help of a phase space Lagrangian. We show that our formulation is the supersymmetric extension of Schild's null string and that all the symmetries of the standard Green-Schwarz theory are maintained. The null superstring thus arises as a new theoretical laboratory, its complexity between the usual superstring and the massless superparticle. Its canonical quantization is carried through in detail.

CERN-TH.5294/89

February 1989

[†] Permanent address: Instituto de Física, Universidade Federal do Rio de Janeiro, RJ 21945, Brasil.

A few years ago, Schild found the tensionless bosonic string[1]. This curious system was quantized much later and shown to have no critical dimension[2, 3]. In this work we introduce and explore, classically and quantum mechanically, the tensionless Green–Schwarz (GS) superstring[4, 5]. The first difficulty to be circumvented is how to take the tension T to zero in a theory whose action is proportional to T . We first show an alternative way of describing the GS string by means of a phase-space Lagrangian (“Palatini formalism”)[6], where the limit $T \rightarrow 0$ can be taken in a consistent way. We define the tensionless superstring action as this limit and study its symmetries and Hamiltonian constraints. We show that these zero-tension superstrings generate light-like or null world-sheets in a supersymmetric target manifold: they constitute an extension of Schild’s null strings[1]. All points on the string move, as in the purely bosonic case, at the speed of light. All the usual symmetries of the superstring survive the dynamical simplification of vanishing tension, including the magic local fermionic[7] and bosonic[8] symmetries. In contrast to the usual superstring, reparametrization invariance holds only on-shell. We introduce the transverse or null gauge (analogue of the conformal gauge), further fix the customary light-cone gauge, and solve the equations of motion explicitly. Finally, we quantize in detail the gauge-fixed null superstring and find that this system has no critical dimension.

1.

The GS action for the $N = 1$ superstring theory is[4]

$$S_{GS} = -T \int d\tau d\sigma \left(\sqrt{-g} + i\epsilon^{ab} Z_a^\mu \bar{\theta} \Gamma_\mu \partial_b \theta \right) \quad (1)$$

where T is the string tension and g the determinant of the induced world-sheet metric $g_{ab} = Z_a^\mu Z_b^\nu \eta_{\mu\nu}$, with $Z_a^\mu = \partial_a X^\mu - i\bar{\theta} \Gamma^\mu \partial_a \theta$. The D variables $X^\mu(\tau, \sigma)$ describe the space-time coordinates of the string, and $\theta_\alpha(\tau, \sigma)$ is a real D -dimensional spinor.[†]

The action (1) is invariant under local reparametrizations and under global supersymmetry. A crucial feature of the action (1) is that it also enjoys Siegel’s local supersymmetry[7] provided the famous Γ -matrix identity $\Gamma_\mu \Psi_{[i} \bar{\Psi}_j \Gamma^\mu \Psi_{k]} = 0$ holds. This occurs only in the following cases: $D = 3$ and θ Majorana, $D = 4$ and θ Majorana or Weyl, $D = 6$ and θ Weyl, and $D = 10$ and θ simultaneously Majorana and Weyl.

The canonical momenta conjugate to X^μ and θ_α are

$$\begin{aligned} \mathcal{P}^\mu &= -T(\sqrt{-g} g^{0a} Z_a^\mu + i\bar{\theta} \Gamma^\mu \theta') \\ \pi_\alpha &= i(\mathcal{P}^\mu - T Z_1^\mu)(\bar{\theta} \Gamma_\mu)_\alpha \end{aligned} \quad (2)$$

[†] Our conventions are as follows. The Minkowski space-time metric is $\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$, $\epsilon^{01} = 1$, and the conformally flat world-sheet metric is $\eta_{ab} = \text{diag}(-1, 1)$.

where we have used left derivatives for fermionic fields and $\theta' = \partial_\sigma \theta$. These momenta lead to the following set of primary constraints[9]:

$$\begin{aligned}\psi_0 &= (\mathcal{P} + T\bar{\theta}\Gamma\theta')^2 + T^2 Z_1^2 \approx 0 \\ \psi_1 &= (\mathcal{P} + T\bar{\theta}\Gamma\theta') \cdot Z_1 \approx 0 \\ \phi_\alpha &= \pi_\alpha - i(\mathcal{P} - TZ_1) \cdot (\bar{\theta}\Gamma)_\alpha \approx 0\end{aligned}\tag{3}$$

We now introduce the phase-space Lagrangian

$$\mathcal{L} = \dot{X}^\mu \mathcal{P}_\mu + \dot{\theta}_\alpha \pi_\alpha + \lambda_a \psi_a + \zeta_\alpha \phi_\alpha\tag{4}$$

The equations of motion for the Lagrange multipliers, λ_a and ζ_α , yield the constraints (3). Let us show that the first-order Lagrangian (4) does contain the GS theory. We first obtain the equations of motion by varying $\mathcal{P}_\mu$ and π_α :

$$\dot{X}^\mu + 2\lambda_0 (\mathcal{P}^\mu + iT\bar{\theta}\Gamma^\mu\theta') + \lambda_1 Z_1^\mu + i\bar{\theta}\Gamma^\mu\zeta = 0\tag{5}$$

$$\dot{\theta}_\alpha + \zeta_\alpha = 0\tag{6}$$

Inserting (6) into (5) and successively contracting the resulting equation with Z_0^μ , Z_1^μ , and $\mathcal{P}^\mu + iT\bar{\theta}\Gamma^\mu\theta'$, we obtain

$$\begin{aligned}Z_0^2 + \lambda_1 Z_1 \cdot Z_0 + 2\lambda_0 (\mathcal{P} + iT\bar{\theta}\Gamma\theta') \cdot Z_0 &= 0 \\ \lambda_1 Z_1^2 + Z_0 \cdot Z_1 &= 0 \\ (\mathcal{P} + iT\bar{\theta}\Gamma\theta') \cdot Z_0 - 2\lambda_0 T^2 Z_1^2 &= 0\end{aligned}\tag{7}$$

These expressions allow us to write the bosonic Lagrange multipliers in terms of the original fields,

$$\begin{aligned}\lambda_0 &= \pm \frac{1}{2T Z_1^2} \left[(Z_0 \cdot Z_1)^2 - Z_0^2 Z_1^2 \right]^{\frac{1}{2}} \\ &= \pm \frac{1}{2T Z_1^2} \sqrt{-g} \\ \lambda_1 &= -\frac{Z_0 \cdot Z_1}{Z_1^2}\end{aligned}\tag{8}$$

Substituting back the constraint relations (3) into (4), we find

$$\begin{aligned}\mathcal{L} &= \dot{X}^\mu \mathcal{P}_\mu + \dot{\theta}_\alpha \pi_\alpha \\ &= \mathcal{P}_\mu Z_0^\mu + iT Z_1^\mu \bar{\theta} \Gamma^\mu \dot{\theta}\end{aligned}\tag{9}$$

Using the last of Eqs.(7) and the expression for λ_0 from (8) (we choose $\lambda_0 < 0$), the Lagrangian (9) reduces to the original one (1).

The point is that the phase-space Lagrangian (4) contains the standard Lagrangian after using the equations of motion for the Lagrange multipliers and for the conjugate

momenta, but not the equations of motion for the dynamical fields X^μ and θ_α . It is, nevertheless, more general. In particular, it allows us to consider the limiting case of vanishing string tension $T \rightarrow 0$ (which could not be done, of course, with the GS action).

2.

The constraints for the superstring with zero tension can be read off from (3) above without any difficulty:

$$\begin{aligned}\psi_0 &= \mathcal{P}^2 \approx 0 \\ \psi_1 &= \mathcal{P} \cdot Z_1 \approx 0 \\ \phi_\alpha &= \pi_\alpha - i\mathcal{P} \cdot (\bar{\theta}\Gamma)_\alpha \approx 0\end{aligned}\tag{10}$$

and the action for the null superstring is precisely Eq.(4) with these new constraints.

Using the fundamental non-vanishing Poisson brackets[10]

$$\begin{aligned}\{X^\mu(\sigma, \tau), \mathcal{P}^\nu(\sigma', \tau)\} &= \eta^{\mu\nu} \delta(\sigma - \sigma') \\ \{\theta_\alpha(\sigma, \tau), \pi_\beta(\sigma', \tau)\} &= -\delta_{\alpha\beta} \delta(\sigma - \sigma')\end{aligned}\tag{11}$$

the classical algebra generated by the constraints (10) of the null superstring can be written down explicitly:

$$\begin{aligned}\{\psi_0(\sigma, \tau), \psi_0(\sigma', \tau)\} &= 0 \\ \{\psi_1(\sigma, \tau), \psi_1(\sigma', \tau)\} &= (\psi_1(\sigma, \tau) + \psi_1(\sigma', \tau)) \delta'(\sigma - \sigma') \\ \{\psi_0(\sigma, \tau), \psi_1(\sigma', \tau)\} &= (\psi_0(\sigma, \tau) + \psi_0(\sigma', \tau)) \delta'(\sigma - \sigma') \\ \{\psi_0(\sigma, \tau), \phi_\alpha(\sigma', \tau)\} &= 0 \\ \{\psi_1(\sigma, \tau), \phi_\alpha(\sigma', \tau)\} &= -2i\mathcal{P}_\mu \left(\bar{\theta}' \Gamma^\mu \right)_\alpha \delta(\sigma - \sigma') \\ \{\phi_\alpha(\sigma, \tau), \phi_\beta(\sigma', \tau)\} &= 2i\mathcal{P}_\mu \left(\Gamma^0 \Gamma^\mu \right)_{\alpha\beta} \delta(\sigma - \sigma')\end{aligned}\tag{12}$$

It does not close because, as we shall discuss below, the primary fermionic constraints ϕ_α contain both first and second class constraints. In order to study the symmetries of the tensionless superstring, it is best to solve them and work with the Lagrangian

$$\mathcal{L} = \mathcal{P} \cdot Z_0 + \lambda_0 \mathcal{P}^2 + \lambda_1 \mathcal{P} \cdot Z_1\tag{13}$$

First of all, this Lagrangian is obviously invariant under a global spacetime supersymmetry transformation, whose infinitesimal form is given by ($\dot{\epsilon} = \epsilon' = 0$):

$$\begin{aligned}\delta\theta &= \epsilon \\ \delta X^\mu &= -i\bar{\theta}\Gamma^\mu \epsilon \\ \delta\mathcal{P}_\mu &= \delta\lambda_0 = \delta\lambda_1 = 0\end{aligned}\tag{14}$$

Under infinitesimal reparametrizations, $\delta\sigma_a = \xi_a$, the action changes by

$$\delta_R S = \int d\tau d\sigma (-\partial_a \xi_a \mathcal{L} + \delta_\xi \mathcal{L}) \quad (15)$$

where, if Φ represents both X^μ and θ_α ,

$$\begin{aligned} \delta_\xi \Phi &= \xi_a \partial_a \Phi \\ \delta_\xi \partial_a \Phi &= (\partial_a \xi_b) \partial_b \Phi \end{aligned} \quad (16)$$

Introducing the derivative operator $\mathcal{D} = \partial_\tau + \lambda_1 \partial_\sigma$, and postulating the transformations

$$\begin{aligned} \delta_\xi \mathcal{P}^\mu &= (\xi'_1 - \lambda_1 \xi'_0) \mathcal{P}^\mu \\ \delta_\xi \lambda_0 &= (\xi_0 - \xi'_1 + 2\lambda_1 \xi'_0) \lambda_0 \\ \delta_\xi \lambda_1 &= -\mathcal{D}\xi_1 + \lambda_1 \mathcal{D}\xi_0 \end{aligned} \quad (17)$$

we arrive at

$$\begin{aligned} \delta_R \mathcal{L} &= -i(\xi_1 - \lambda_1 \xi_0) \mathcal{P}_\mu \bar{\theta}^\dagger \Gamma^\mu \dot{\theta} \\ &= -i\delta_\xi \bar{\theta} \mathcal{P} \cdot \Gamma \mathcal{D}\theta \end{aligned} \quad (18)$$

It is clear then that our action is reparametrization invariant if and only if the spinor θ satisfies the condition

$$\mathcal{P} \cdot \Gamma \mathcal{D}\theta = 0 \quad (19)$$

This issue is intimately connected with the local spacetime (Siegel) supersymmetry[7]

$$\begin{aligned} \delta\theta &= \kappa \\ \delta X^\mu &= i\bar{\theta} \Gamma^\mu \kappa \\ \delta \mathcal{P}_\mu &= \delta\lambda_0 = \delta\lambda_1 = 0 \end{aligned} \quad (20)$$

under which the Lagrangian exhibits a similar behavior:

$$\delta_\kappa \mathcal{L} = -2i\mathcal{P} \cdot \bar{\kappa} \Gamma \mathcal{D}\theta \quad (21)$$

Our action is thus both reparametrization-invariant and locally supersymmetric if the condition (19) holds. As we shall see, (19) is actually the equation of motion for the fermionic field θ . So we do have a sensible theory on-shell, although we do not expect any conserved charges associated with either reparametrization or Siegel invariances. In sharp contrast with the tensionful case, (a) the reparametrization symmetry holds only on-shell and (b) the Siegel symmetry is valid for any space-time dimension.

For completeness, we display the field transformations under the local bosonic (Sierra) symmetry[8] with infinitesimal parameter b :

$$\begin{aligned} \delta X^\mu &= i\bar{\theta} \Gamma^\mu \delta\theta \\ \delta\theta &= b\mathcal{D}\theta \\ \delta \mathcal{P} &= \delta\lambda_0 = \delta\lambda_1 = 0 \end{aligned} \quad (22)$$

under which $\delta_b \mathcal{L} = 0$ even off-shell.

3.

The equations of motion obtained by varying the momenta $\mathcal{P}^\mu$ and π_α can be put together as

$$-2\lambda_0 \mathcal{P}^\mu = Z_0^\mu + \lambda_1 Z_1^\mu \quad (23)$$

from which equations similar to (7) can be obtained:

$$\begin{aligned} Z_0^2 + \lambda_1 Z_1 \cdot Z_0 + 2\lambda_0 \mathcal{P} \cdot Z_0 &= 0 \\ \lambda_1 Z_1^2 + Z_0 \cdot Z_1 &= 0 \\ \mathcal{P} \cdot Z_0 &= 0 \end{aligned} \quad (24)$$

The induced metric can thus be written as

$$g = Z_1^2 \begin{pmatrix} \lambda_1^2 & -\lambda_1 \\ -\lambda_1 & 1 \end{pmatrix} \quad (25)$$

which clearly exhibits that the world-sheet is a null surface (this is the reason for calling the superstring with zero tension a null superstring):

$$\det g = Z_0^2 Z_1^2 - (Z_0 \cdot Z_1)^2 = 0 \quad (26)$$

We have succeeded in constructing, then, a manifestly supersymmetric generalization of Schild's null string.

The equations of motion for X^μ and θ_α are

$$\begin{aligned} \dot{\mathcal{P}}^\mu + (\lambda_1 \mathcal{P}^\mu)' &= 0 \\ \dot{\mathcal{P}} \cdot \Gamma \theta + 2\mathcal{P} \cdot \Gamma \dot{\theta} &= 0 \end{aligned} \quad (27)$$

with $\mathcal{P}^\mu$ given by Eq. (23). In addition to the physical fields, it involves the Lagrange multipliers λ_0 and λ_1 , which actually should be thought of as the independent degrees of freedom of the (null) induced metric on the world-sheet. It is interesting to note that under a reparametrization (17), λ_1 is the only field which does not transform into something times itself. In fact, $\delta_\xi \lambda_1 = -\dot{\xi}_1 + \lambda_1 \times \dots$. This means that we may choose to partially fix the reparametrization symmetry and impose[†] everywhere on the world-sheet

$$\lambda_1 = 0, \quad (28)$$

In this gauge, every single point of the string moves along a null (super)geodesic ($Z_0^2 = 0$), in complete analogy to the purely bosonic case[1, 11]. This statement is stronger (but also

[†] Other simple gauge choices are $\lambda_1 = \pm 1$, which exhibit some sort of chirality.

gauge-dependent) than the statement that the world-sheet is a null surface. Similarly, transversity of the tensionless superstring holds ($Z_0 \cdot Z_1 = 0$). It is easy to see also that the quantity Z_1^μ is space-like, *i.e.*, $Z_1^2 > 0$.

On the other hand, the fact that the Lagrange multiplier λ_0 is undetermined means that $\mathcal{P}^\mu$ is related to the velocity only modulo an arbitrary (gauge-dependent) constant. We completely fix the reparametrization symmetry of the theory by fixing it to be a *constant* factor which defines the overall normalization between momentum and velocity. We choose

$$\lambda_0 = -\frac{1}{2} \quad (29)$$

as suggested by the form of (23). These choices of λ_1 and λ_0 constitute the “transverse” (or “null”) gauge, the equivalent in our case of the conformal gauge for the usual superstring.

In this fully fixed transverse (or null) gauge, the equations of motion for the physical fields become

$$\begin{aligned} \dot{Z}_0^\mu &= 0 \\ Z_0 \cdot \Gamma \dot{\theta} &= 0 \end{aligned} \quad (30)$$

which imply that every single point of the superstring propagates as if it were a free superparticle. In this gauge, the condition (19) on the fermionic fields is just the fermionic equation of motion, as advertised. We shall solve the equations of motion in the light-cone gauge below, after motivating the need for further gauge-fixing.

4.

To quantize the null superstring, it is necessary to use the full first-order Lagrangian (4), which includes the fermionic momenta and constraints. The quantity $\mathcal{P}_\mu \left(\bar{\theta}' \Gamma^\mu \right)_\alpha$ is weakly zero because, in addition to the first-class constraints associated with the local supersymmetry, ϕ_α contains second class constraints, which express the standard relationship among coordinates and momenta for the fermionic degrees of freedom. This can be seen by trying to invert the matrix $C_{\alpha\beta} = \{\phi_\alpha, \phi_\beta\}$ with the information in the constraint algebra (12):

$$C_{\alpha\beta}^{-1} = -\frac{i}{2} (\mathcal{P}^2)^{-1} \mathcal{P}_\mu \left(\Gamma^\mu \Gamma^0 \right)_{\alpha\beta} \delta(\sigma - \sigma') \quad (31)$$

This result is ill-defined, of course, because of the constraint ψ_0 in the denominator.

The covariant separation of the primary constraints ϕ_α into first and second class constraints is complicated and constitutes the main obstacle toward the covariant quantization of the null superstring, in complete analogy to the usual superstring. Presumably, it requires an infinite sequence of ghosts for ghosts[12, 13, 14]. It is interesting that the null superstring shares this complication with the usual $T \neq 0$ GS superstring.

We avoid the problem of covariantly separating first and second class constraints by working in the light-cone gauge, *i.e.*, we impose

$$\begin{aligned} X^+ &= c\tau \\ \mathcal{P}^+ &= p^+ \\ \Gamma^+\theta &= 0 \end{aligned} \tag{32}$$

where $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^{D-1})$, etc. This gauge-fixing is done in addition to the transverse conditions $\lambda_0 = -\frac{1}{2}$ and $\lambda_1 = 0$ discussed above, as a consequence of which the two constants c and p^+ are actually related to each other. This can be easily seen by considering Eqs.(23), (24) and the gauge conditions (32), from which $-2\lambda_0 p^+ = Z_0^+ = \dot{X}^+ = c$ so that in fact $c = p^+$.

Concerning the fermionic degrees of freedom, we first note that although the matrices Γ^+ and Γ^- are nilpotent, their sum is non-singular. It follows that exactly half of the eigenvalues of each must be zero[5]. Therefore, the gauge choice $\Gamma^+\theta = 0$ in (32) fixes half of the components of θ to zero and the propagating degrees of freedom are only $\Gamma^-\theta$. We do not change notation and still use the full θ_α after this gauge-fixing, with the understanding that the subindex α really runs over only half of the original values.

In the light-cone (and transverse) gauge, then,

$$\begin{aligned} Z_a^+ &= p^+ \delta_{a0} \\ Z_a^- &= \partial_a X^- - i\bar{\theta}\Gamma^- \partial_a \theta \\ Z_a^I &= \partial_a X^I \end{aligned} \tag{33}$$

where only X^I and $\Gamma^-\theta$ are independent degrees of freedom. In terms of these physical fields, the gauge-fixed Lagrangian is

$$\begin{aligned} \mathcal{L} &= \dot{\theta}_\alpha \pi_\alpha + \dot{X}^I \mathcal{P}^I \\ &= \dot{X}^I \left(\dot{X}^I - i\bar{\theta}\Gamma^I \dot{\theta} \right) \end{aligned} \tag{34}$$

The equations of motion for the independent degrees of freedom simplify significantly:

$$\begin{aligned} \ddot{X}^I &= 0 \\ \dot{\theta} &= 0 \end{aligned} \tag{35}$$

(The second expression can be trivially derived using $\{\Gamma^+, \Gamma^-\} = 2$.) These equations can be solved to yield

$$\begin{aligned} \vec{X} &= \vec{Y}(\sigma) + \tau \vec{P}(\sigma) \\ \theta &= \theta(\sigma) \end{aligned} \tag{36}$$

The dynamics of the null superstring appears deceptively simple. Even though the tension is zero, the different points of the null string (all propagating along null geodesics) still keep a memory of their neighbors, as is obvious from the expression for the dependent variable

$$X^- = y^-(\sigma) + \frac{\tau}{2p^+} \vec{P}^2 \quad (37)$$

with the condition

$$y^{-'} = \frac{1}{p^+} \vec{P} \cdot \vec{Y}' + i\bar{\theta}\Gamma^-\theta' \quad (38)$$

Clearly, then, all the fun is tucked away into X^- , where the shapes (in σ) of the bosonic and fermionic components are correlated.

We may now turn, finally, to the quantization of the null superstring. To do this in the light-cone gauge, where all the constraints are first class, we have two alternative (but equivalent) procedures[9, 15]. The first is to incorporate the constraints in the canonical quantization formalism and to use Dirac brackets instead of Poisson brackets for the transition to quantum dynamics[3]. The second one is to solve the constraints and write the dependent variables in terms of the independent ones; quantization follows then directly from the Poisson brackets[2]. We adopt the second possibility for the bosonic variables. Even though we have chosen the second avenue, the Dirac bracket formalism must still be used to obtain $\{\theta_\alpha, \theta_\beta\}$.

The bosonic variables are quantized starting from the standard Poisson bracket for the independent degrees of freedom

$$\{X^I(\sigma, \tau), \mathcal{P}^J(\sigma', \tau)\} = \delta^{IJ} \delta(\sigma - \sigma') \quad (39)$$

The dependent quantities $\mathcal{P}^+$ and X^+ are given by the gauge-fixing conditions (32), $\mathcal{P}^-$ follows from the constraint ψ_0 :

$$\mathcal{P}^- = \frac{1}{2p^+} \mathcal{P}^I \mathcal{P}^I = \frac{1}{2p^+} \vec{P}^2 \quad (40)$$

and $X^{-'}$ follows from ψ_1 :

$$X^{-'} = i\bar{\theta}\Gamma^-\theta' + \frac{1}{p^+} \vec{P} \cdot \vec{X}' \quad (41)$$

To obtain X^- we introduce the quantity[†]

$$q^-(\tau) = \int_0^1 d\sigma X^-(\tau, \sigma) \quad (42)$$

[†] We are taking the range of σ to be $0 \leq \sigma \leq 1$, as was already implicit in Eq. (32). The whole discussion holds for both open and closed null superstrings.

and use the identity

$$X^-(\tau, \sigma) = q^-(\tau) + \left[\int_0^\sigma - \int_0^1 d\sigma' \int_0^{\sigma'} \right] d\sigma'' X^-(\tau, \sigma'') \quad (43)$$

so that

$$X^- = q^-(\tau) + \left[\int_0^\sigma d\sigma'' - \int_0^1 d\sigma' \int_0^{\sigma'} d\sigma'' \right] \left(\frac{1}{p^+} \mathcal{P}^I \frac{\partial X^I}{\partial \sigma''} + i \bar{\theta} \Gamma^- \frac{\partial \theta}{\partial \sigma''} \right) \quad (44)$$

Notice that, even though the equation of motion for the independent bosonic variables $\vec{X}$ is completely disconnected from the fermionic ones, the same is not true of the dependent variable X^- . So the Poincaré algebra might be nontrivial.

We also have to consider the bracket

$$\{q_0^-, p^+\} = -1 \quad (45)$$

where $q^-(\tau) = q_0^- + p^+ \tau$.

In the light-cone gauge, the fermionic momentum constraint reads as

$$\phi'_\alpha = \pi_\alpha + i \mathcal{P}^+ (\bar{\theta} \Gamma^-)_\alpha - i \mathcal{P}^I (\bar{\theta} \Gamma^I)_\alpha \approx 0 \quad (46)$$

The fundamental Poisson bracket (11) is not altered if we redefine the fermionic momentum

$$\pi_\alpha \rightarrow \pi'_\alpha = \pi_\alpha + i \mathcal{P}^+ (\bar{\theta} \Gamma^-)_\alpha - i \mathcal{P}^I (\bar{\theta} \Gamma^I)_\alpha \quad (47)$$

The fermionic constraint is now

$$\phi'_\alpha = \pi'_\alpha - i \mathcal{P}^I (\bar{\theta} \Gamma^I)_\alpha \approx 0 \quad (48)$$

so that $C_{\alpha\beta} = \{\phi'_\alpha, \phi'_\beta\}$ can be inverted, in contrast to (31):

$$C_{\alpha\beta}^{-1} = \frac{i}{2} (\vec{\mathcal{P}}^2)^{-1} \mathcal{P}^I (\Gamma^I \Gamma^0)_{\alpha\beta} \delta(\sigma - \sigma') \quad (49)$$

The bracket $\{\theta_\alpha, \theta_\beta\}$ (for the non-zero θ_α only) then follows straightforwardly,

$$\begin{aligned} \{\theta_\alpha(\sigma, \tau), \theta_\beta(\sigma', \tau)\} &= \{\theta_\alpha, \phi'_\gamma\} C_{\gamma\delta}^{-1} \{\phi'_\delta, \theta_\beta\} \\ &= \frac{i}{2} \frac{\mathcal{P}^I}{(\vec{\mathcal{P}}^2)} (\Gamma^0 \Gamma^I)_{\alpha\beta} \delta(\sigma - \sigma') \end{aligned} \quad (50)$$

The remaining non-zero Poisson brackets, involving dependent variables, can be (laboriously) checked to be the following. No confusion should arise between the fermionic variables $\theta_\alpha(\sigma)$ (or rather, $(\Gamma^- \theta(\sigma))_\alpha$) and the step function $\theta(\sigma - \sigma')$:

$$\{X^I(\sigma), \mathcal{P}^-(\sigma')\} = \frac{1}{p^+} \mathcal{P}^I \delta(\sigma - \sigma') \quad (51)$$

$$\{X^-(\sigma), \mathcal{P}^I(\sigma')\} = \frac{1}{p^+} \frac{\partial}{\partial \sigma'} [\mathcal{P}^I(\sigma')(\theta(\sigma' - \sigma) - \sigma')] \quad (52)$$

$$\{X^-(\sigma), \mathcal{P}^-(\sigma')\} = \frac{1}{p^+} \frac{\partial}{\partial \sigma'} [\mathcal{P}^-(\sigma')(\theta(\sigma' - \sigma) - \sigma')] + \frac{1}{p^+} \mathcal{P}^- \delta(\sigma - \sigma') \quad (53)$$

$$\{X^-(\sigma), \mathcal{P}^+(\sigma')\} = -1 \quad (54)$$

$$\{X^-(\sigma), X^I(\sigma')\} = \frac{1}{p^+} [\theta(\sigma' - \sigma) - \sigma'] \frac{\partial}{\partial \sigma'} X^I \quad (55)$$

$$\{X^-(\sigma), \theta_\alpha(\sigma')\} = \frac{\mathcal{P}^I(\sigma')}{\bar{p}^2} \left[\frac{1}{2} \left((\Gamma^I \Gamma^- \theta(\sigma'))_\alpha (1 - \delta(\sigma - \sigma')) + \sigma' \left((\Gamma^I \Gamma^- \frac{\partial \theta}{\partial \sigma'})_\alpha \right) \right] \quad (56)$$

$$\{X^-(\sigma), X^-(\sigma')\} = \frac{1}{p^+} \left[(\theta(\sigma' - \sigma) - \sigma') \frac{\partial X^-}{\partial \sigma'} - (\theta(\sigma - \sigma') - \sigma) \frac{\partial X^-}{\partial \sigma} \right] \quad (57)$$

This last bracket is the hardest to check. Crucially, the fermionic contributions to it cancel and thus no critical dimension will arise for the null superstring from the requirement of closure of the Poincaré superalgebra, just as for the bosonic null string[2, 3]. The lack of a critical dimension can be understood from the continuous nature of the spectrum.

Conclusions

In this letter we have studied the tensionless superstring. We have shown that it is a natural extension of Schild's null string and that all the symmetries of the GS theory remain. Canonical quantization in the light-cone gauge proves that the system possesses no critical dimension.

Tensionless strings are to tensionful strings like massless particles to massive ones. It is not clear to us what role, if any, will the null superstring play in the physics of string theory. However, it certainly constitutes an interesting theoretical laboratory, with more structure than the point particle, for clarifying open questions in string theory. A detailed study of its spectrum-generating algebra should help to clarify its possible equivalence to the recently discovered null spinning string[16, 17]. It seems likely to us that critical dimensions will arise from requiring the equivalence of these two theories. Also, the covariant quantization of the null superstring is currently under investigation[18].

Acknowledgements. J. B.-N. is supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brasil, and thanks the CERN Theory Division for hospitality during this work.

References

- [1] A. Schild, *Phys. Rev.* **D16** (1977) 1722.
- [2] F. Lizzi, B. Rai, G. Sparano and A. Srivastava, *Phys. Lett.* **182B** (1986) 326.
- [3] R. Amorim and J. Barcelos-Neto, *Z. Phys.* **C38** (1988) 643.
- [4] M.B. Green and J.H. Schwarz, *Phys. Lett.* **136B** (1984) 367; *Nucl. Phys.* **B243** (1984) 285.
- [5] For an in-depth review, see M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Cambridge Univ. Press (1987) New York.
- [6] R. Marnelius, *Acta Phys. Pol.* **B13** (1982) 669; H. Hwang and R. Marnelius, *Nucl. Phys.* **B272** (1986) 389.
- [7] W. Siegel, *Phys. Lett.* **128B** (1983) 397; *Class. Quantum Grav.* **2** (1985) L95.
- [8] G. Sierra, *Class. Quantum Grav.* **3** (1986) L67.
- [9] P.A.M. Dirac, *Can. J. Math.* **2** (1950) 129; *Lectures on Quantum Mechanics*, Yeshiva University (1964) New York.
- [10] R. Casalbuoni, *Nuovo Cim.* **A33** (1976) 115.
- [11] A. Karlhede and U. Lindström, *Class. Quantum Grav.* **3** (1986) L73.
- [12] R. Kallosh, *Phys. Lett.* **195B** (1987) 369; R. Kallosh and M. Rahmanov, *Phys. Lett.* **214B** (1988) 549.
- [13] E. Nissimov, S. Pacheva and S. Solomon, *Nucl. Phys.* **B297** (1988) 349.
- [14] A.H. Díaz and F. Toppan, *Phys. Lett.* **211B** (1988) 285.
- [15] A. Hanson, T. Regge and C. Teitelboim, *Hamiltonian Constrained Systems*, Accademia dei Lincei (1976) Roma.
- [16] J. Gamboa and M. Ruiz-Altaba, to appear in *Proceedings Summer Chile Workshop 1989*, eds. N. Bralić *et al.*
- [17] J. Gamboa, C. Ramírez and M. Ruiz-Altaba, CERN-TH preprint (February 1989).
- [18] J. Barcelos-Neto, C. Ramírez and M. Ruiz-Altaba, in preparation.