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QCD Corrections to the $b \rightarrow s\gamma$ Decay - Appearance of Double Log Terms

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ABSTRACT

A two-loop Feynman diagram calculation for the $b \rightarrow s\gamma$ amplitude to the order α_s is presented. We find an $\alpha_s \log^2$ contribution, a term missing in previous calculations. However, owing to a small coefficient of the double log term, the $\alpha_s \log$ terms found previously still dominate numerically. The present approach throws new light on a recent controversy concerning QCD corrections in $b \rightarrow s\gamma$.

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1. Introduction

A significant impact of QCD corrections on weak radiative B-decays provides an important test of loop effects in the standard model (SM)^[1]. The basic contribution to the magnetic moment operator

$$\bar{s}\sigma^{\mu\nu}(m_b R + m_s L)b F_{\mu\nu}, \quad (1.1)$$

(causing real photon emission) arises on account of Fig.1a. As already shown for penguin operators^[2], including gluons (Fig.1b) relaxes the GIM cancellation from power-like to logarithmic. A straightforward generalization of the calculation for $s \rightarrow d\gamma$ ^[3] gives the α_s -order amplitude of Fig.1b

$$M_U^\mu(b \rightarrow s\gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{4\pi^2} 2Q_U \frac{\alpha_s}{\pi} \ln \frac{m_t^2}{\mu^2} (m_b \bar{s}\sigma^{\mu\nu} i q_\nu R b). \quad (1.2)$$

Taking $m_t \simeq 80\text{GeV}$ and $\mu \simeq 5\text{GeV}$ gives the prediction

$$BR(b \rightarrow s\gamma) \simeq 3 \times 10^{-4}. \quad (1.3)$$

This value can be compared with the present experimental limit

$$BR(b \rightarrow s\gamma)_{expt} \leq (6 - 50) \times 10^{-4}, \quad (1.4)$$

obtained by combining the upper bound reached by ARGUS^[4] $BR(B \rightarrow K^*\gamma) \leq 2.4 \times 10^{-4}$, and the predicted range^[5] for the ratio $\frac{\Gamma(B \rightarrow K^*\gamma)}{\Gamma(b \rightarrow s\gamma)} \simeq (0.05 - 0.40)$. It follows from Refs.3,4 that the improvement by an order of magnitude on the experimental side seems to enable one to test the apparently large QCD corrections for the process under consideration.

Still, there is at present a controversy in the literature^[6,7], about the precise values of these corrections. (Numerically, there is a factor ~ 2 discrepancy in the amplitude). In an attempt to resolve this controversy we discover some subtle features of the dynamics governing the $b \rightarrow s\gamma$ (or $s \rightarrow d\gamma$) decay. In the following we present how we, at least in principle, disagree with both Ref. 6 and Ref. 7.

2. Appearance of the Double Log Contribution to the $b \rightarrow s\gamma$ Transition

The controversy mentioned above arises in the procedure of generalizing the leading $\log \alpha_s \log$ (eq.(1.2)), to all orders, $\sum (\alpha_s \log)^n$. Both Ref.6 and Ref.7 use a standard procedure of renormalization-group equation (RGE) summing the logarithmically divergent diagrams in the truncated theory, involving the operator mixing. The divergent truncated theory of Refs.3,6,7 corresponds to the diagrams with W-exchange shrunk to a point. Furthermore, in Refs.6,7 the W-boson and the t-quark are both integrated out at the W-scale. Then the GIM mechanism is lost, and the truncated theory is rather divergent. The authors of Ref.7 ascribe their disagreement with the result of Ref.6 to the inadequacy of dimensional regularization (-which is in fact due to the problems with γ_5 in divergent loop integrals of the truncated theory). This critique does not seem to be accepted by the authors of Ref.6.

Here we attempt to resolve the controversy in a different way. We want to point out that Feynman diagrams giving contributions of the form (1.1) to $b \rightarrow s\gamma$ are finite in the full $SU(3) \times SU(2) \times U(1)$ theory. The RGE calculations of Refs.6,7 should, when expanded in powers of α_s , reproduce the result of the full theory. A nice calculation illustrating this point is given by Khatsymovsky^[8] in resolving a previous controversy on the penguin contribution to the $\Delta S = 1$ transition.

A two-loop diagram which complements those represented in Fig.1b is displayed in Fig.2. Such diagrams contain the one-loop penguin amplitude as a subloop:

$$\epsilon^a(G)^\mu f_P(p^2 g_{\mu\nu} - p_\mu p_\nu)(\bar{s}\gamma^\nu t^a Lb)C_P + (terms \sim k, k^2) \quad (2.1)$$

where $f_P = iG_F(\sqrt{2}/3)g_s/(4\pi^2)$, t^a is a colour matrix, $\epsilon(G)$ is the gluon polarization tensor, and C_P is some logarithmic function of the gluon momentum p , the external quark momenta k and $k+p$, and of the involved quark- and W- masses^[9]. Using this expression in the subsequent loop in Fig.2, we obtain the amplitude for a down-quark type emitted photon:

$$M_D^\mu(b \rightarrow s\gamma) = if_P g_s e Q_D \frac{i}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} q_\nu (\bar{s}k_\alpha \gamma_\beta t^a Lb) [2\ln \frac{m_t^2}{\mu^2} + \frac{1}{2}(\ln \frac{m_t^2}{\mu^2})^2], \quad (2.2)$$

where k is the external quark momentum, $\mu^2 \equiv -k^2$, and $\mu \sim m_b$. On account of the

relation (obtained by using the free Dirac equation),

$$i\epsilon^{\mu\nu\alpha\beta}q_\nu(\bar{s}k_\alpha\gamma_\beta Lb) \simeq \frac{1}{2}\bar{s}\sigma^{\mu\nu}(m_b R + m_s L)b iq_\nu \quad (2.3)$$

eq.(2.2) acquires the familiar form of the magnetic dipole operator (1.2):

$$M_D^\mu(b \rightarrow s\gamma) = -\frac{G_F}{\sqrt{2}}\frac{e}{4\pi^2}\frac{Q_D}{9}\frac{\alpha_s}{\pi}\left[2\ln\frac{m_t^2}{\mu^2} + \frac{1}{2}\left(\ln\frac{m_t^2}{\mu^2}\right)^2\right](m_b\bar{s}\sigma^{\mu\nu}iq_\nu Rb) \quad (2.4)$$

Here we want to stress the appearance of the second term containing the double log form in the bracket of (2.4). We can trace this term back to the "non-local" ($\sim p_\mu p_\nu$) part of the effective penguin interaction (2.1). The single log in (2.4) is due to the "local" ($\sim g_{\mu\nu}$) part of (2.1).

Similar $\log^2$ terms have recently been observed^[10,11] as a new entry of the anomalous dimension matrix relevant for $s \rightarrow d\gamma^*$ (γ^* means a virtual photon). However, in Refs.10,11 the $\log^2$ terms appear both for the up- and down-quark emitted photons at first order in α_s , whereas the leading log series starts at the zeroth order for Q_U (the u 's and d 's are numerical coefficients):

$$u_0^*Q_U\ln\frac{m_t^2}{\mu^2} + \alpha_s(u_1^*Q_U + d_1^*Q_D)\left(\ln\frac{m_t^2}{\mu^2}\right)^2. \quad (2.5)$$

In contrast, the real photon emission at hand starts with large logarithms at the α_s order, the sum of eqs. (1.2) and (2.4):

$$\alpha_s\left[(u_1Q_U + d_1Q_D)\ln\frac{m_t^2}{\mu^2} + \frac{1}{4}d_1Q_D\left(\ln\frac{m_t^2}{\mu^2}\right)^2\right]. \quad (2.6)$$

This is in accord with the rule of getting maximally one log per loop. The appearance of such double log terms in the complete theory signals an incompleteness of the truncated-theory calculations^[6,7]. In view of the additional controversy arising from two different results in the reported truncated-theory calculations (Refs. 6 and 7, respectively), we find it appropriate to rely on the full-theory calculation, adopted here.

3. Discussion and Conclusions

Some technical comments should be made:

i) We keep the exact expression for the subloop in (2.1). (Note that the diagrams where the gluon is emitted from an external leg of the non-diagonal $b \rightarrow s$ self-energy have to be included to satisfy the Ward identity for the subloop (2.1).) It is important that the external quark of the subloop, which enters the two-loop diagram, should be kept at arbitrarily high momentum $\sim p$. In order to obtain the amplitude of the form (2.2), we have to extract one factor of q and one factor of k from the diagram in Fig.2. If both q and k are extracted from the part of the diagram where the photon is emitted, the remaining loop integration over the virtual (Euclidean) gluon momentum can, by power counting, be seen to have the (potentially) leading form

$$\int \frac{dp^2}{p^2} C_P(p^2), \quad (3.1)$$

where

$$C_P(p^2) \sim \frac{m_t^2}{p^2}; \quad C_P(p^2) \sim \ln \frac{m_t^2}{p^2} + \text{const.}, \quad (3.2)$$

for $p^2 \gg m_t^2$ and $\mu^2 < p^2 < m_t^2$, respectively. Thus we expect that μ^2 and m_t^2 act as the effective lower and upper cut-offs, which in fact turns out to be the case. Such effective cut-offs of the integral in (3.1) lead to the double log term in (2.4). Similarly, for the diagram in Fig.1b, the $b \rightarrow s\gamma G$ subloop is $\sim q p^2 C_{\gamma P}$, where

$$C_{\gamma P} \sim (p^2 + m_q^2)^{-1} \quad (3.3)$$

for $q = c, t$. (For simplicity we have considered the case where $m_q^2 \ll M_W^2$.) Connecting the gluon(G) in $b \rightarrow s\gamma G$ to one of the external quark lines as in the diagram of Fig.1b, and extracting k , the loop integral is seen to be essentially of the form

$$\int \frac{dp^2}{(p^2 + m_c^2)} - (c \rightarrow t), \quad (3.4)$$

which can only give one single log.

It might happen that the coefficient of the potentially dominant term vanishes for some reason (-technically, because of the Feynman parameter integration or the Dirac algebra). This is the fact for the contribution due to the "local" term ($\sim g_{\mu\nu}$) in (2.1). However, the "non-local" term ($\sim p_\mu p_\nu$) produces a non-vanishing double log contribution. (The "non-local" term also produces a single (subleading) log with a coefficient ~ 1 . This has been omitted in (2.4).)

ii) The external quark momentum k can also be extracted from the penguin subloop, which corresponds to the additional terms $\sim k, k^2$ in (2.1). However, the corresponding contributions are suppressed compared with (2.2). This is also the case when the photon is emitted from the W-boson. Furthermore, there are no mass insertion contributions in the Feynman diagram approach. Note that both the diagrams in Figs.1b and 2 are obtained when a photon is emitted from a self-penguin^[12] which has a $\log^2$ behaviour.

iii) The Dirac expression which emerges naturally in the Feynman diagram calculations is the left-hand side of (2.3), as can be seen from (2.2). The dipole operator containing $\sigma_{\mu\nu}$ only appears after the use of (2.3). The left-hand side of (2.3) contains an external quark momentum k . At operator level this k corresponds to a covariant derivative containing a gluon field. Thus the relation (2.3) is only valid on-shell and with QCD switched off.

iv) As mentioned above, one factor of k (and q) has to be extracted from the loop integral. If one puts $k = 0$ in the remaining integral, the double log will appear as

$$\left(\ln \frac{m_t^2}{m_b m_s}\right)^2 - \left(\ln \frac{m_c^2}{m_b m_s}\right)^2 \quad (3.5)$$

and not as $(\ln(m_t^2/m_c^2))^2$. Thus there is apparently an infrared divergence for $m_b m_s \rightarrow 0$. However, keeping $\mu^2 \equiv -k^2$ in the loop integral, there is no divergence for $m_b m_s \rightarrow 0$.

v) For the case $s \rightarrow d\gamma^*$, both the "local" and the "non-local" part of the penguin subloop (2.1) produce a $\log^2$ term, the "non-local" being $-3/8$ of the "local"^[10]. However, the "local" induced $\log^2$ term disappears for emission of a real photon, while the "non-local" induced $\log^2$ term remains. In Ref.11 the $\log^2$ term for $s \rightarrow d\gamma^*$ is obtained as a one-loop contribution from a four-quark penguin operator of the left-left type. This contribution corresponds to the diagram in Fig.1, with the W-propagator shrunk to a point, and with s, b running in the loop instead of c, t . However, this diagram gives no logs for real photon emission. Moreover, it seems that it only takes into account the effect of local penguin operators, and not the $\log^2$ effect due to the "non-local" penguin term in (2.1).

To conclude, in this note we have presented new SD contributions of the double log type to the $b \rightarrow s \gamma$ ($s \rightarrow d \gamma$) transition. We have obtained them in the complete theory and observe that they cannot appear by expanding the existing results^[6,7] of the truncated theory to the α_s -order. Although leading for the Q_D subclass of diagrams, the double log terms enter with a rather small coefficient. This can be seen from the ratio of (2.4) to (1.2) (keeping only the double log in (2.4)) :

$$\frac{M_D(b \rightarrow s \gamma)}{M_U(b \rightarrow s \gamma)} = -\frac{1}{36} \frac{Q_D}{Q_U} \ln \frac{m_t^2}{\mu^2} \quad (3.6)$$

giving $\frac{1}{13}$ for $m_t \sim 80 \text{ GeV}$, $\mu \sim 5 \text{ GeV}$. Accordingly, the $\alpha_s \log$ term $\sim Q_U$ is numerically dominant for the calculated α_s order.

What could be the reason for the $\log^2$ effect obtained here but which is missing in the truncated theory calculation of Refs.6,7? We will suggest that the root of the discrepancy might be related to the technical comments iii-v) made above. Especially, we suspect that the operator basis used in Refs.6,7 is not complete enough to take into account the numerically small $\log^2$ effect due to the "non-local" penguin. We intend to elaborate further details elsewhere.

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FIGURE CAPTIONS

Fig.1. (a): Bare electromagnetic penguin for the up-quark emitted photon. (b): A QCD correction to (a). Including the crossed versions there are four such diagrams.

Fig.2. A penguin induced down-quark emitted photon diagram. The gluon can also be absorbed and the photon emitted by the b-quark.

Fig. 1a

Fig. 1b

Fig. 2