CERN-TH.5308/89

STABILITY OF EXCITED BOSONIC STELLAR CONFIGURATIONS

Ph. Jetzer

CERN - Geneva

A B S T R A C T

In this paper I study the problem of the dynamical stability of Bose star configurations which correspond to excited states. In particular I show that the eigenvalues of the pulsation equation, which governs the time evolution of the infinitesimal radial oscillations, are real. This, together with the theorem, proved by Lee and Pang, that for a configuration with critical central density ρ_{crit} the pulsation equation has a zero mode and the fact that for central densities much smaller than ρ_{crit} the lowest eigenvalue is positive, leads to the conclusion that the excited Bose star configurations are classically stable for central densities up to ρ_{crit} .

CERN-TH.5308/89
February 1989

Recent developments in particle physics and cosmology suggest that evolving scalar fields may have played an important role in the evolution of the early universe, for instance in primordial phase transitions, and that they act as an important source for the missing matter. This latter possibility raises naturally the question whether cold, gravitational equilibrium configurations of massive scalar fields - Bose stars - may exist and whether they are dynamically stable [1-3]. Spherically symmetric gravitational equilibrium configurations in asymptotically flat space-times for massive complex fields (interacting only gravitationally) have been found by Kaup [4] as well as Ruffini and Bonazzola [5] by solving the coupled system of Einstein-Klein-Gordon equations. They analyzed only the nodeless radial scalar solution, corresponding to the ground state. The results obtained in Refs. [4,5] have been confirmed and extended later on, see Refs. [6,7]. In particular, Friedberg, Lee and Pang [7] have also computed the radial solutions with nodes, which correspond to excited states. It has been found that there is always a critical mass and a critical particle number above which mass gravitational collapse occurs. These quantities for the nodeless solution are respectively given by $M_{\text{crit}} = 0.633 M_{\text{Pl}}^2/m$ and $N_{\text{crit}} = 0.653 M_{\text{Pl}}^2/m^2$, where $M_{\text{Pl}} = G^{-\frac{1}{2}}$ is the Planck mass and m is the mass of the scalar particle. For the excited states the critical mass as well as the critical number grow approximately linearly with the number of nodes [7].

The extension to the self-interacting case, for a potential $V(|\phi|) = 1/4 \tilde{\lambda} |\phi|^4$ ($\tilde{\lambda} > 0$), has been considered in the work of Colpi, Shapiro and Wasserman [8]. The consequence of switching on a quartic self-interaction between the scalar particles is to increase the above limits on the critical mass and particle number. If one plots the mass for the equilibrium configurations of the Bose star against the central density, one finds, except for the scales, a behaviour remarkably similar to those of neutron stars: the mass quickly raises to a maximum (for $\rho = \rho_{\text{crit}}$), drops a little, oscillates and approaches an asymptotic value at large central densities, the same happening for the particle number. These similarities lead naturally to the conjecture [6] that the stability behaviour is similar to the fermionic case, namely that the boundary between stable and unstable configurations is given by $\rho = \rho_{\text{crit}}$.

In a previous paper [1] I analyzed the time evolution of infinitesimal radial oscillations which conserve the total number of particles, starting from the scalar wave equation coupled to the Einstein field equation. Following the method developed by Chandrasekhar [3] I could find an eigenvalue equation which determines the normal modes of the radial oscillations, and a variational principle for determining the eigenfrequencies of the oscillations. A numerical analysis for the case of a nodeless radial scalar solution allowed to find an upper limit for the boundary between stable and unstable configurations, which turned out to be not

much above ρ_{crit} , in both the cases analyzed, namely with and without a quartic self-interacting potential term. In a recent paper, Lee and Pang [2] showed that both M and N as functions of the central density ρ are stationary at the same points and in particular at $\rho = \rho_{\text{crit}}$. Therefore it follows that at these stationary points there is a zero mode of the radial oscillations. This is the same as in the case of a star described by a perfect fluid. Moreover they have also studied the eigenvalues in the non-relativistic limit, corresponding to a very small central density, for the case where there is no quartic self-interaction. For the case where the radial scalar solution has no nodes as well as for the case with nodes it turns out that the lowest eigenvalue is positive in this limit. For the nodeless case it is easily proved that all the eigenvalues must be real [1,2]; thus with the above facts it leads to the conclusion that the equilibrium solutions up to the critical central density ρ_{crit} are classically stable. Above ρ_{crit} the equilibrium solutions are unstable. For $\rho > \rho_{\text{crit}}$ there are other values of ρ for which M and N are stationary and where there is therefore a zero mode. The fact that for these points there is a negative upper bound for the lowest eigenvalue [1] establishes that it must be a higher eigenvalue which becomes zero.

In this paper I address the question whether in the case of an equilibrium solution with nodes the eigenvalues of the pulsation equation are still real. In fact for equilibrium solutions with nodes, the corresponding pulsation equation has singularities. Nevertheless it can still be proved, by analyzing the behaviour of the solution at the singularities, that the eigenvalues are real^{*)}, which means also that the corresponding operator is symmetric as in the nodeless case. Therefore like in the nodeless case we conclude that the excited bose star configurations are classically stable for central densities up to ρ_{crit} .

The system we consider is a complex scalar field coupled to gravity, whose action is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{g^{\mu\nu}}{2} \phi_{;\mu}^* \phi_{;\nu} - \frac{m^2}{2} |\phi|^2 - \frac{\tilde{\lambda}}{4} |\phi|^4 \right] \quad (1)$$

This action is invariant under a global $U(1)$ phase transformation, $\phi \rightarrow e^{i\theta} \phi$, which implies the conservation of its generator N , the number of particles minus the number of antiparticles. For our considerations it is more convenient to write the complex scalar field as $\phi = (\phi_1 + i\phi_2)e^{-i\omega t}$, where ϕ_1 and ϕ_2 are two real fields, which are functions of r and t alone since we consider only spherically symmetric equilibrium configurations. We express the metric in Schwarzschild co-ordinates

*) In Ref. [2] using another approach, leading apparently to a different type of pulsation equation they find numerically also complex eigenvalues in the node case. Most probably these correspond to solutions for which $\delta N \neq 0$.

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \quad (2)$$

where ν and λ are functions of r and t only ($g^{00} = -e^{-\nu}$ and $g^{rr} = e^{-\lambda}$). For $r \rightarrow \infty$, we require $e^{\nu(r,t)} \rightarrow 1$ as well as $e^{\lambda(r,t)} \rightarrow 1$. At the equilibrium configuration, the functions ν , λ are time-independent. In order to recover the equilibrium solutions found in Refs. [4-8] we set $\phi_2 = 0$ as well as $\phi_1(r,t) = \phi_0(r)$. We will denote all the equilibrium quantities by a subscript $_0$. The equilibrium solutions are determined by two Einstein equations

$$(re^{-\lambda_0})' = 1 - 4\pi Gr^2 \left[(m^2 + e^{-\nu_0} \omega^2 + \frac{\tilde{\lambda}}{2} \phi_0^2) \phi_0^2 + e^{-\lambda_0} \phi_0'^2 \right] \quad (3)$$

$$\frac{e^{-\lambda_0}}{r} \nu_0' = \frac{1}{r^2} (1 - e^{-\lambda_0}) + 4\pi G \left[(e^{-\nu_0} \omega^2 - m^2 - \frac{\tilde{\lambda}}{2} \phi_0^2) \phi_0^2 + e^{-\lambda_0} \phi_0'^2 \right] \quad (4)$$

and the scalar wave equation

$$\phi_0'' + \left(\frac{2}{r} + \frac{\nu_0' - \lambda_0'}{2} \right) \phi_0' - e^{\lambda_0} (m^2 - e^{-\nu_0} \omega^2 + \tilde{\lambda} \phi_0^2) \phi_0 = 0. \quad (5)$$

The total particle number is given by

$$N_0 = 4\pi\omega \int_0^\infty dr r^2 \phi_0^2 e^{\frac{1}{2}(\lambda_0 - \nu_0)} \quad (6)$$

The appropriate initial and boundary conditions are: $\phi_0(0) = \text{const}$, $\phi_0'(0) = 0$, $e^{\lambda_0(0)} = 1$, $e^{\nu_0(\infty)} = 1$. One finds then numerically a solution to the system of equations, which constitute an eigenvalue problem for $e^{\nu_0(0)}$ and ω^2 , for any number of nodes in the function $\phi_0(r)$.

Consider now the situation where the equilibrium configuration is perturbed in a way such that the spherical symmetry is still preserved. These perturbations will give rise to motions in the radial direction. The equations governing the small perturbations are obtained by expanding all functions to first order ($\lambda(r,t) = \lambda_0(r) + \delta\lambda(r,t)$, $\nu(r,t) = \nu_0(r) + \delta\nu(r,t)$, $\phi_1(r,t) = \phi_0(r) + \delta\phi_1(r,t)$, $\phi_2(r,t) = \delta\phi_2(r,t)$) and then by linearizing the equations. We obtain the following set of Einstein equations for the perturbed quantities

$$\begin{aligned} (re^{-\lambda_0} \delta\lambda)' = & -8\pi Gr^2 \left[\frac{1}{2} \delta\nu e^{-\nu_0} \omega^2 \phi_0^2 + \frac{1}{2} \delta\lambda e^{-\lambda_0} \phi_0'^2 - e^{-\nu_0} (\omega^2 \phi_0 \delta\phi_1 + \omega \phi_0 \delta\dot{\phi}_2) \right. \\ & \left. - m^2 \phi_0 \delta\phi_1 - \tilde{\lambda} \phi_0^3 \delta\phi_1 - e^{-\lambda_0} \phi_0' \delta\phi_1' \right] \end{aligned} \quad (7)$$

$$\frac{e^{-\lambda_0}}{r} (\delta v' - v_0' \delta \lambda) = \frac{e^{-\lambda_0}}{r^2} \delta \lambda + 8\pi G \left[-\frac{1}{2} \delta v e^{-v_0} \omega^2 \phi_0^2 - \frac{1}{2} \delta \lambda e^{-\lambda_0} \phi_0'^2 + e^{-v_0} (\omega^2 \phi_0 \delta \phi_1 + \omega \phi_0 \delta \dot{\phi}_2) - m^2 \phi_0 \delta \phi_1 - \tilde{\lambda} \phi_0^3 \delta \phi_1 + e^{-\lambda_0} \phi_0' \delta \phi_2' \right] \quad (8)$$

and

$$\delta \dot{\lambda} = 8\pi G r (\phi_0' \delta \dot{\phi}_1 + \omega \phi_0 \delta \phi_2' - \omega \phi_0' \delta \phi_2) \quad (9)$$

The linearized equations for $\delta \phi_1$ and $\delta \phi_2$ are

$$\begin{aligned} \delta \phi_1'' + \left(\frac{2}{r} + \frac{v_0' - \lambda_0'}{2} \right) \delta \phi_1' + \frac{1}{2} \phi_0' (\delta v' - \delta \lambda') + \delta \lambda e^{\lambda_0} (\omega^2 e^{-v_0} \phi_0 - m^2 \phi_0 - \tilde{\lambda} \phi_0^3) \\ + e^{\lambda_0} (\omega^2 e^{-v_0} - m^2 - 3\tilde{\lambda} \phi_0^2) \delta \phi_1 - e^{\lambda_0 - v_0} \phi_0 \omega^2 \delta v - e^{\lambda_0 - v_0} \delta \ddot{\phi}_1 + 2e^{\lambda_0 - v_0} \omega \delta \dot{\phi}_2 = 0 \end{aligned} \quad (10)$$

and

$$\begin{aligned} \delta \phi_2'' + \left(\frac{2}{r} + \frac{v_0' - \lambda_0'}{2} \right) \delta \phi_2' + e^{\lambda_0} (\omega^2 e^{-v_0} - m^2 - \tilde{\lambda} \phi_0^2) \delta \phi_2 - e^{\lambda_0 - v_0} \delta \ddot{\phi}_2 \\ + \frac{1}{2} e^{\lambda_0 - v_0} \omega [\phi_0 (\delta \dot{v} - \delta \dot{\lambda}) - 4 \delta \dot{\phi}_1] = 0 \end{aligned} \quad (11)$$

Not all these equations are independent, for instance, one can show that Eq. (11) is a consequence of the previous equations [1]. The basic system of four equations (7)-(10) can be reduced to a system of two coupled equations involving only $\delta \phi_1$ and $\delta \phi_2$ (for details see [1]). Notice that Eq. (9) integrates directly in time if we write $\delta \phi_2(r, t) \equiv \phi_0(r) \dot{g}(r, t)$. In the following we denote $f_1(r, t) = \delta \phi_1(r, t)$ and $f_2(r, t) = g'(r, t)$ (prime and dot denote differentiation with respect to r and t respectively). We also suppose for f_1 and f_2 a time dependence of the form $e^{i\sigma t}$. This is possible since all coefficients entering the equations are time-independent. We get thus the following eigenvalue or pulsation equation (f_1 and f_2 denote from now on only the radial part)

$$L_{ij} f_j = \sigma^2 M_{ij} f_j \quad i, j = 1, 2 \quad (12)$$

where

$$M_{ij} = e^{v_0 - \lambda_0} \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \quad (13)$$

with $G_1 = r^2 e^{\frac{1}{2}(v_0 - \lambda_0)}$ and $G_2 = r^2 \phi_0^2 e^{\frac{3}{2}(v_0 - \lambda_0)}$

$$L_{ij} = \begin{bmatrix} -\frac{\partial}{\partial r} G_1 \frac{\partial}{\partial r} + G_1 C_1(r) & -2\omega \frac{\partial}{\partial r} (G_1 \phi_0) + G_1 C_3(r) \\ 2\omega G_1 \phi_0 \frac{\partial}{\partial r} + G_1 C_3(r) & -\frac{\partial}{\partial r} G_2 \frac{\partial}{\partial r} + G_2 C_2(r) \end{bmatrix} \quad (14)$$

$$C_1(r) = \left[e^{\lambda_0} (3\omega^2 e^{-\nu_0} + \omega^2 + 3\tilde{\lambda} \phi_0^2) + 16\pi G e^{\lambda_0} r \phi_0' (\omega^2 \phi_0 + \tilde{\lambda} \phi_0^3) - 4\pi G r \phi_0'^2 (\nu_0' - \lambda_0' + \frac{2}{r}) \right] \quad (15a)$$

$$C_2(r) = \left[\frac{2(\lambda_0' - \nu_0')}{r} + \frac{2}{r^2} - \frac{(\nu_0'' - \lambda_0'')}{2} - 2 \frac{\phi_0''}{\phi_0} + 2 \left(\frac{\phi_0'}{\phi_0} \right)^2 + \frac{(\lambda_0' - \nu_0')}{2} \left[(\nu_0' - \lambda_0') + 4 \frac{\phi_0'}{\phi_0} \right] - 4\pi G r \omega^2 \phi_0^2 e^{\lambda_0 - \nu_0} (\nu_0' - \lambda_0' + \frac{2}{r}) \right] \quad (15b)$$

$$C_3(r) = \left[-2\phi_0' \omega + 8\pi G r e^{\lambda_0} (\omega^2 \phi_0 + \tilde{\lambda} \phi_0^3) \omega \phi_0^2 - 4\pi G r \phi_0' (\nu_0' - \lambda_0' + \frac{2}{r}) \omega \phi_0^2 \right]. \quad (15c)$$

We expect the radial perturbations not to change the total particle number N_0 . In fact it turns out that the integrand of δN is a total derivative with respect to r [1], so that

$$\delta N = \delta \pi \left(f_2 r^2 \phi_0^2 e^{(\nu_0 - \lambda_0)/2} \right) \Big|_0^\infty. \quad (16)$$

Therefore the requirement $\delta N = 0$ translates into conditions on the boundary values for f_2 . For $r \rightarrow \infty$ we require $r^2 \phi_0^2 f_2 \rightarrow 0$ and for $r \rightarrow 0$ we have $r^2 f_2 \rightarrow 0$. The boundary conditions for f_1 are: $r \rightarrow \infty$, $f_1 \rightarrow 0$; and $r \rightarrow 0$, $f_1 = \text{const.}$ With these boundary conditions the operators L_{ij} and M_{ij} are symmetric in the case that $\phi_0(r)$ is nodeless [1], and therefore the corresponding eigenvalues are real.

Consider now the case where $\phi_0(r)$ has one or more nodes. Equation (12) is then singular at the points where $\phi_0(r)$ vanishes. This complicates the analysis of the spectrum of the operator L_{ij} . Suppose that there is also a complex eigenvalue σ^2 with corresponding eigenfunction f ($=f_1, f_2$), then also the complex conjugate function $\bar{f}$ is a solution with eigenvalue $\bar{\sigma}^2$, since all the coefficients of the

equation are real. An equation involving the imaginary part of the complex eigenvalue σ^2 can be obtained by evaluating the integral $\int_a^b dr (f_1 L_{1j} \bar{f}_j - \bar{f}_1 L_{1j} f_j) = (\bar{\sigma}^2 - \sigma^2) \int_a^b dr (f_1 M_{1j} \bar{f}_j)$ ($i, j=1,2$) over an arbitrary chosen interval (a,b) of the positive real axis, which does not contain singular points. We get

$$2i(\text{Im } \sigma^2) \int_a^b e^{v_0 - \lambda_0} \{G_1 |f_1|^2 + G_2 |f_2|^2\} dr = G_1 [\bar{f}_1 f_1' - f_1 \bar{f}_1'] \Big|_a^b + G_2 [\bar{f}_2 f_2' - f_2 \bar{f}_2'] \Big|_a^b + 2w G_1 \phi_0 [\bar{f}_1 f_2 - f_1 \bar{f}_2] \Big|_a^b. \quad (17)$$

For convenience, we consider the interval $(0, r_0)$, where r_0 is the position of the first node of $\phi_0(r)$. From the above Eq. (17) we see that the behaviour of the solution of Eq. (12) at the origin and at the point r_0 puts constraints on the value of $(\text{Im } \sigma^2)$. The behaviour of the solution $f = (f_1, f_2)$ at the origin can be studied by inserting in Eq. (12) the following expansion:

$$\text{Re } f_1 = \sum_{n=0}^{\infty} a_n r^{\alpha+n}, \quad \text{Re } f_2 = \sum_{n=0}^{\infty} b_n r^{\beta+n}$$

and similarly for $\text{Im } f_1$ and $\text{Im } f_2$. It turns out that $\text{Re } f_1$ as well as $\text{Im } f_1$ behave as r^{-1} or are regular (constant or r^2); whereas $\text{Re } f_2$ and $\text{Im } f_2$ behave as r^{-2} or are regular (like r). The singular solution, like r^{-1} , for $\text{Re } f_1$ and $\text{Im } f_1$ has to be excluded due to the boundary condition on f_1 , which is for $r \rightarrow 0$ $f_1 = \text{const.}$ Similarly a behaviour as r^{-2} at the origin for $\text{Re } f_2$ and $\text{Im } f_2$ is also excluded by the boundary condition coming from the requirement that $\delta N = 0$. f_1 and f_2 are thus regular at the origin and therefore in Eq. (17) the right-hand side contribution from the origin vanishes due to the fact that both G_1 and G_2 are proportional to r^2 .

To study the behaviour of the solution of Eq. (12) at r_0 , the position of the first node (clearly the results are the same at every node position), it is convenient to use the following expansion:

$$\text{Re } f_1 = \sum_{n=0}^{\infty} a_n \phi_0^{\alpha+n}, \quad \text{Re } f_2 = \sum_{n=0}^{\infty} b_n \phi_0^{\beta+n}$$

and

$$\text{Im } f_1 = \sum_{n=0}^{\infty} A_n \phi_0^{\gamma+n}, \quad \text{Im } f_2 = \sum_{n=0}^{\infty} B_n \phi_0^{\delta+n}.$$

We notice that $\phi_0(r)$ has a simple zero at the nodes, otherwise also $\phi_0'(r)$ vanishes at r_0 then, due to the zero-order Eq. (5), ϕ_0'' and thus all higher derivatives

would vanish at the node, which is clearly inconsistent. Inserting the above expansion into Eq. (12), one gets

$$\sum_{n=0}^{\infty} \left\{ a_n \phi_0^{\alpha+n-2} (\alpha+n)(\alpha+n-1) \phi_0'^2 + a_n \phi_0^{\alpha+n-1} \left[(\alpha+n) \phi_0'' + \left(\frac{z}{r_0} + \frac{(\nu_0' - \lambda_0')}{z} \right) (\alpha+n) \phi_0' \right] \right. \\ \left. + a_n \phi_0^{\alpha+n} \left[e^{\lambda_0} (-3\omega^2 e^{-\nu_0} - \omega^2) + 4\pi G r_0 \phi_0'^2 (\nu_0' - \lambda_0' + \frac{z}{r_0}) + (Re\sigma^2) e^{\lambda_0 - \nu_0} \right] \right. \\ \left. + b_n \phi_0^{\beta+n} \left[2\omega (\beta+n) \phi_0' + 4\omega \phi_0 \right] - (Im\sigma^2) A_n \phi_0^{\gamma+n} e^{\lambda_0 - \nu_0} \right\} = 0 \quad (18)$$

and similarly for Imf_1 by interchanging (a_n, α) , (b_n, β) by (A_n, γ) , (B_n, δ)

$$\sum_{n=0}^{\infty} \left\{ b_n \phi_0^{\beta+n-2} [(\beta+n)(\beta+n-1) - 2] \phi_0'^2 + b_n \phi_0^{\beta+n-1} \left[(\beta+n+2) \phi_0'' + \left(\frac{z}{r_0} + \frac{z}{z} (\nu_0' - \lambda_0') \right) (\beta+n) \phi_0' \right] \right. \\ \left. + 4 \frac{(\nu_0' - \lambda_0')}{z} \phi_0' \right] + b_n \phi_0^{\beta+n} \left[\frac{2(\nu_0' - \lambda_0')}{r_0} - \frac{z}{r_0^2} + \frac{(\nu_0'' - \lambda_0'')}{z} + \frac{(\nu_0' - \lambda_0')^2}{z} + (Re\sigma^2) e^{\lambda_0 - \nu_0} \right] \\ \left. + a_n \phi_0^{\alpha+n-2} [(1-\alpha-n) 2 \phi_0' e^{\lambda_0 - \nu_0} \omega] + a_n \phi_0^{\alpha+n} [4\pi G r_0 \phi_0' (\nu_0' - \lambda_0' + \frac{z}{r_0}) \omega e^{\lambda_0 - \nu_0}] \right. \\ \left. - (Im\sigma^2) B_n \phi_0^{\delta+n} e^{\lambda_0 - \nu_0} \right\} = 0 \quad (19)$$

and similarly for Imf_2 by interchanging (b_n, β) , (a_n, α) by (B_n, δ) , (A_n, γ) . Each power of ϕ_0 , in the above equations, has to vanish separately. This gives constraints which allow us to determine α , β , γ , δ as well as the coefficients a_n , b_n , A_n and B_n . From the constraints it follows that Ref_1 and Imf_1 are regular at r_0 , where they behave like $const$, ϕ_0 or ϕ_0^3 . For Ref_2 , and similarly for Imf_2 , the coefficient in front of the lowest power of ϕ_0 in Eq. (19) is: $b_0(\beta^2 + \beta - 2)$, which suggests that $\beta = -2$ could be a possible solution. Notice that $f_2 = g'$ and that the "physical" quantity is $\delta\phi_2 = \phi_0 \dot{g}$ and so if $f_2 \sim \phi_0^{-2}$ then $\delta\phi_2 \sim const$. But if we take $\beta = -2$, then from Eq. (18) it follows that the coefficient b_1 in the expansion has to vanish ($\alpha > 0$ otherwise $a_0 = 0$). This implies that the coefficient in front of ϕ_0^{-3} in Eq. (19), which has to vanish, is given by: $b_0 [-\phi_0' (\nu_0' - \lambda_0') - (4/r_0) \phi_0']$. Using the zero-order Eqs. (3) and (4) one can write it also as: $-2b_0 [1/r_0 (e^{\lambda_0} + 1)]$, and since $e^{\lambda_0} > 0$ it follows that $b_0 = 0$ itself. The singular solution with $\beta = -2$ is therefore excluded, the same being true for ϕ_0^{-1} . In conclusion also Ref_2 and Imf_2 are regular at the node and behave like ϕ_0 or ϕ_0^2 . Therefore the second and third term on the right-hand side of Eq. (17) vanish at r_0 , since G_2 is proportional to ϕ_0^2 . A non-vanishing contribution on the right-hand side of Eq. (17) can come only from the first term. But since f_1 and f_2 have no singularities at the nodes one can consider as integration limits in Eq. (17) the interval $(0, \infty)$, and at infinity all the functions f_1 , f_2 and their derivatives vanish. Therefore it

follows that $(\text{Im}\sigma^2) = 0$ and thus that the operator, also in the node case, has only real eigenvalues.

In conclusion we have established that for the case with no quartic self-interaction the equilibrium solutions with nodes up to the critical central density ρ_{crit} are classically stable. Above ρ_{crit} the system becomes unstable at least up to the next value of ρ for which M and N are stationary. It seems very likely that also above this point the system, like in the nodeless case, is unstable, although this is still an open point. We expect similar conclusions to hold also for the case where there is a quartic self-interacting term or even for a more general scalar potential, since the proof that the operator L_{ij} has only real eigenvalues does not depend upon the form of the self-interaction.

ACKNOWLEDGEMENTS

I would like to thank W. Amrein, S. Chandrasekhar and N. Straumann for useful discussions.

REFERENCES

- [1] Ph. Jetzer - Fermilab Preprint 66A (1988) to appear in Nucl.Phys. B; Fermilab Preprint 88A (1988) to appear in the Proceedings of the Fifth Marcel Grossmann Meeting, Perth (August 1988).
- [2] T.D. Lee and Y. Pang - Columbia University Preprint, CU-TP-401 (1988).
- [3] S. Chandrasekhar - Phys.Rev.Lett. 12 (1964) 114, 437; Ap.J. 140 (1964) 417.
- [4] D.J. Kaup - Phys.Rev. 172 (1968) 1331.
- [5] R. Ruffini and S. Bonazzola - Phys.Rev. 187 (1969) 1767.
- [6] J.D. Breit, S. Gupta and A. Zaks - Phys.Lett. 140B (1984) 329.
- [7] T.D. Lee - Phys.Rev. D35 (1987) 3637;
R. Friedberg, T.D. Lee and Y. Pang - Phys.Rev. D35 (1987) 3640.
- [8] M. Colpi, S.L. Shapiro and I. Wasserman - Phys.Rev.Lett. 57 (1986) 2485.