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SPIN, MASSES AND OTHER BARYONIC OBSERVABLES
IN A CHIRAL MODEL OF QUARK AND GLUON CONFINEMENT

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ABSTRACT

The structure of non-strange baryons is investigated in a field-theoretical model which minimally incorporates soft confinement of quarks and gluons and approximate chiral symmetry. Baryonic states are recovered from the mean-field hedgehog solutions by the coherent cranking procedure, which generates mean chromomagnetic fields, modelling gluon exchange between quarks. The cranking method allows for a non-perturbative, self-consistent computation of gluonic effects on the nucleon and delta masses, corrected for spurious translational and rotational fluctuations, on the contribution $\Delta u + \Delta d$ of the non-strange quark helicities to the proton spin, and on various other baryonic observables. For the physical values of the pion parameters $m_\pi = 139.6$ MeV, $F_\pi = 93$ MeV, and the effective strong fine structure constant $\alpha_s \approx 0.5$, the results which we obtain for these observables, including $\Delta u + \Delta d \approx 0.26$, are in good agreement with experiment.

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1. - INTRODUCTION

The study of baryonic structure must take into account non-perturbative, low-energy properties of QCD such as chiral symmetry breaking and quark and gluon confinement. This is best done through the use of effective field theoretical models incorporating these features. In the Skyrme model [1], quarks and gluons are altogether absent, and baryons appear as topological solitons [1,2] of a non-linear σ -model, which is a truncation of an effective field theory of mesons, equivalent to QCD in the large N_c limit [3]. In the two-phase hybrid chiral bag model [4], an inner quark phase is separated from an outer meson phase by an MIT bag surface [5]. Gluons may also be introduced into this chiral bag [6], leading to a Lagrangian which is that of QCD inside the bag, and that of Skyrme outside. The disadvantage of this appealing and rather successful [7] picture lies in the assumption of a sharp bag boundary, which may not be realistic, and introduces into the field theory an extraneous object (the bag surface) with its own dynamics, which are not usually taken into account properly [5,8].

A softer interpolation between the quark and meson sectors is achieved in the so-called chiral soliton bag models. In these purely field-theoretical models, based on the Gell-Mann-Lévy [9] linear [10,11] or non-linear [12] σ -models, the local interaction of quark and meson fields ensures the existence, in the mean-field approximation, of non-topological [13] or topological soliton solutions representing hadrons. Gluon fields, confined through their interaction with a variable dielectric constant κ [14], may also be introduced in these models. In the minimal extension of the linear σ -model [15], κ depends only on the chiral meson fields; the resulting model interpolates smoothly between QCD and the non-linear σ -model [12].

The glueless chiral soliton bag model [10,11] has previously been used to compute, with various refinements, baryonic observables [10, 16-21]. Our aim in this paper is primarily to study gluon effects on these observables in the model introduced in Ref. [15]. The effect of gluon exchange on the nucleon and delta masses was recently studied perturbatively, to lowest order in the effective strong fine structure constant α_s [15,22,23]. However, such a perturbative treatment suffers from several drawbacks. The perturbative expansion may not be valid for realistic values of $\alpha_s \approx 0.5$. Furthermore, it cannot be used to study the effect of gluon exchange on observables other than masses (except at the price of higher order calculations, which have not been carried out). And, perhaps the most important, it does not properly take into account the confinement of the hadronic quark distribution induced by its coupling to the confined colour gauge field [14].

Non-perturbative, self-consistent computations of gluonic effects were first carried out by Haider and Liu [24] in the case of the Friedberg-Lee soliton bag

model [13,14], treated in the Abelian approximation, and improved by other authors [25,26]. It was found that, when the model parameters are adjusted so as to reproduce the observed nucleon and delta masses and mean square proton charge radius, the values for other observables were not significantly improved by taking into account gluon exchange. In the chiral soliton bag model, there is less freedom in the choice of model parameters, and it is essential to take into account gluon exchange in order to reproduce the observed nucleon-delta mass difference [16,27,28]. The effect of gluon exchange on the proton spin content, which is currently an outstanding theoretical problem [29-37], is also crucial; as suggested in Refs. [21] and [37], the mean chromomagnetic fields inside a hadron may polarize the valence quarks and thus reduce their contribution to the proton spin. As we shall see, the effect of gluon exchange on other observables, such as the isovector nucleon mean square charge radius and the isoscalar nucleon magnetic moment, is also important. In this paper we shall use a non-trivial generalization of the self-consistent method of Ref. [26] to compute these effects.

The chiral soliton bag model [10,11,15] is described in Section 2, and the mean-field equations of the model are solved using the hedgehog ansatz [38], leading to spherically symmetric, but glueless, soliton solutions. Extending previous work, we study the domain of existence of these solutions in parameter space. The mean-field approximation and the hedgehog assumption break both translational and rotational invariance which must be restored, at least approximately, in order to obtain baryonic states from these hedgehog solitons. The approximate correction of the soliton mass for spurious translational fluctuations is carried out in Section 3 by using coherent hedgehogs, defined as products of valence quarks and a coherent superposition of pions, as trial states; this correction leads to a reduction in the soliton mass. The approximate restoration of rotational and isorotational invariance is then carried out in Section 4 by cranking [39,17] the coherent hedgehog, thereby generating self-consistently non-vanishing chromomagnetic fields. The cranking method is applied in Section 5 to compute the nucleon and delta masses, taking care to correct for spurious rotational fluctuations; this leads to a further reduction in the mean nucleon-delta mass. Cranking is also used, in Section 6, to compute the contribution of the non-strange quark helicities to the proton spin. When the parameters of the model are adjusted to reproduce as far as possible the experimental mean nucleon-delta mass and the relative nucleon-delta mass splitting, the results obtained for this contribution with realistic values of α_s agree with present experimental knowledge; we show that this implies that all the proton spin is due to orbital angular momentum. The variation of other nucleon observables with the various model parameters is studied in Section 7. Our conclusions are presented briefly in Section 8.

2. - THE CHIRAL SOLITON BAG MODEL IN THE MEAN FIELD APPROXIMATION

The chiral soliton bag model, which incorporates confinement of quarks and gluons and $SU(2) \times SU(2)$ chiral symmetry breaking, both modelled by the interactions of a chiral multiplet $(\sigma, \vec{\phi})$, is defined by the Lagrangian [10,11,15]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} \kappa F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} [i\gamma^\mu D_\mu - f(\sigma - i\vec{\tau} \cdot \vec{\phi} \gamma^5)] \psi \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - U, \end{aligned} \quad (2.1)$$

where ψ is the two-flavour, three-colour quark field, $F_{\mu\nu}^a$ is the colour gauge field, D_μ is the gauge covariant derivative

$$D_\mu \psi = \partial_\mu \psi + i g_s \frac{\lambda_a}{2} A_\mu^a \psi, \quad (2.2)$$

U is the quartic potential

$$U = \frac{\lambda^2}{4} (\sigma^2 + \vec{\phi}^2 - F_\pi^2 + \frac{m_\pi^2}{\lambda^2})^2 - F_\pi m_\pi^2 \sigma - m_\pi^2 \left(\frac{m_\pi^2}{4\lambda^2} - F_\pi^2 \right), \quad (2.3)$$

($F_\pi = 93$ MeV being the pion decay constant), and the dielectric function κ [14] depends on the chiral invariant $\sigma^2 + \vec{\phi}^2$, and goes to zero as the chiral fields $(\sigma, \vec{\phi})$ go to the bottom $(F_\pi, 0)$ of the potential well, thereby confining the gluonic field $F_{\mu\nu}^a$. This model may be viewed as a smooth interpolation between QCD (in the limit $f \rightarrow 0$ and $\kappa \rightarrow 1$) and a non-linear σ -model (in the limit $\lambda^2 \rightarrow \infty$ and $g_s \rightarrow 0$) which admits topological soliton solutions [12] closely related [40] to those of the Skyrme model.

In this model, a hadron is defined in the mean-field approximation (MFA) as a state of N constituent quarks of Dirac wave functions q^i , surrounded by mean chiral fields σ , $\vec{\phi}$ and mean colour fields $\vec{E}_a^i$, $\vec{B}_a^i$, which satisfy the coupled stationary mean-field equations

$$-i\vec{\alpha} \cdot \vec{D} q^i + f\beta(\sigma - i\vec{\tau} \cdot \vec{\phi} \gamma^5) q^i = \epsilon_i q^i,$$

$$\Delta\sigma - \frac{\partial U}{\partial\sigma} - \frac{1}{2} \frac{\partial\kappa}{\partial\sigma} (\vec{B}_a \cdot \vec{B}_a - \vec{E}_a \cdot \vec{E}_a) = f \sum_{i=1}^N q^{i\dagger} \beta q^i,$$

$$\begin{aligned}
 \Delta\vec{\phi} - \frac{\partial U}{\partial\vec{\phi}} - \frac{1}{2} \frac{\partial K}{\partial\vec{\phi}} (\vec{B}_a \cdot \vec{B}_a - \vec{E}_a \cdot \vec{E}_a) &= -f \sum_{i=1}^N q^{i+} \beta i \vec{\tau} \gamma^5 q^i, \\
 \vec{\nabla} \cdot (\kappa \vec{E}_a) - g_s f_{abc} \kappa \vec{E}_b \cdot \vec{A}_c &= g_s \sum_{i=1}^N q^{i+} \frac{\lambda_a}{2} q^i, \\
 \vec{\nabla} \wedge (\kappa \vec{B}_a) - g_s f_{abc} \kappa \vec{E}_b V_c + g_s f_{abc} \kappa \vec{B}_b \wedge \vec{A}_c &= \\
 &= g_s \sum_{i=1}^N q^{i+} \frac{\lambda_a}{2} \vec{\alpha} q^i, \quad (2.4)
 \end{aligned}$$

with suitable boundary conditions.

A hadron in the ground state corresponds intuitively to the case where all the constituent quarks are in the same spatial, spherically symmetric, ground state. However, the only known way to obtain a spherically symmetric solution of the mean-field equations (2.4) is to assume further that the N quarks are in the same spatial and spin-isospin "hedgehog" state [38]

$$q_0 = \begin{pmatrix} u(r) \chi_R \\ i v(r) \vec{\sigma} \cdot \hat{x} \chi_R \end{pmatrix}, \quad (2.5)$$

with $(\vec{\sigma} + \vec{\tau}) \chi_h = 0$, leading to a spherically symmetric mean scalar field $\sigma_0 = \sigma_0(r)$ and to the hedgehog configuration for the mean pion field

$$\vec{\phi}_0 = \phi_0(r) \hat{x}.$$

If all the quarks are in the same spatial state, the total colour charges of hadrons (colour singlets) are zero, as well as the chromoelectric fields $\vec{E}_a$. Moreover, if the quarks are in the hedgehog state (2.5), which is such that

$$\chi_R^\dagger \vec{\sigma} \chi_R = 0, \quad (2.6)$$

the colour currents of the various quarks are (independently) zero, and so the chromomagnetic fields $\vec{B}_a$ are zero. So at this stage the localized solutions of the theory are identical to those which may be obtained in the case $g_s = 0$ [10,41]. We shall discuss them to some extent, as these solutions will serve as input for the

case $g_s \neq 0$.

The hedgehog ansatz reduces the system (2.4) to the following form

$$\begin{aligned} \frac{du}{dx} + f\phi_0 u &= -(\varepsilon + f\sigma_0)u , \\ \frac{dv}{dx} + \frac{2}{x}v - f\phi_0 v &= (\varepsilon - f\sigma_0)u , \\ \frac{1}{x} \frac{d^2}{dx^2} (x\sigma_0) - \frac{\partial U}{\partial \sigma}(\sigma_0, \phi_0) &= Nf(u^2 - v^2) , \\ \frac{1}{x} \frac{d^2}{dx^2} (x\phi_0) - \frac{2}{x^2}\phi_0 - \frac{\partial U}{\partial \phi}(\sigma_0, \phi_0) &= -2Nfuv . \end{aligned} \quad (2.7)$$

Regular localized solutions of this system may be obtained with the boundary conditions

$$\begin{aligned} v(0) &= 0 , \quad \frac{d\sigma_0}{dx}(0) = 0 , \quad \phi_0(0) = 0 , \\ u(\infty) &= 0 , \quad \sigma_0(\infty) \equiv \sigma_v = F_\pi , \quad \phi_0(\infty) = 0 , \end{aligned} \quad (2.8)$$

and the normalization

$$4\pi \int_0^\infty (u^2 + v^2) x^2 dx = 1 . \quad (2.9)$$

The asymptotic behaviours of these solutions have been discussed at length, for some parameter values, in Ref. [41]. More generally, the asymptotic behaviours of the quark wave functions u, v are

$$u(x) \sim \frac{1}{x} e^{-\beta x} , \quad \frac{v(x)}{u(x)} \simeq \frac{\beta}{f\sigma_v + \varepsilon} \left(1 + \frac{1}{\beta x}\right) , \quad (2.10)$$

where $\beta^2 \equiv f^2\sigma_v^2 - \varepsilon^2$. The pion field ϕ_0 has two possible asymptotic behaviours

$$\phi_0(x) \sim \frac{1}{x^2} e^{-2\beta x} \quad \text{if } \beta < \frac{m_\pi}{2} , \quad (2.11a)$$

$$\phi_0(x) \sim \frac{1}{x} e^{-m_\pi x} \quad \text{if } \frac{m_\pi}{2} < \beta , \quad (2.11b)$$

while the scalar field σ_0 has three possible asymptotic behaviours

$$\sigma_v - \sigma_0(x) \sim \frac{1}{x^2} e^{-2\beta x} \quad \text{if } \beta < \inf(m_\pi, \frac{m_\sigma}{2}) , \quad (2.12a)$$

$$\sigma_v - \sigma_o(x) \sim \frac{1}{x^2} e^{-2m_\pi x} \quad \text{if } m_\pi < \inf\left(\frac{m_\sigma}{2}, \beta\right), \quad (2.12b)$$

$$\sigma_v - \sigma_o(x) \sim \frac{1}{x} e^{-m_\sigma x} \quad \text{if } \frac{m_\sigma}{2} < \inf(m_\pi, \beta), \quad (2.12c)$$

where $m_\sigma^2 \equiv 2\lambda^2 F_\pi^2 + m_\pi^2$. Actually, the only case encountered in our calculations was case (2.12b).

Instead of solving the eigenvalue problem (2.7)-(2.9) directly, we perform a scale transformation [42,41] on the various fields so as to obtain dimensionless quantities:

$$\begin{aligned} x &= \frac{\bar{x}}{f\sigma_v}, \quad \varepsilon = f\sigma_v \bar{\varepsilon}, \quad \lambda = f\bar{\lambda}, \quad m_\pi = f\sigma_v \bar{m}_\pi, \\ u &= \left(\frac{f\sigma_v^3}{N}\right)^{1/2} \bar{u}, \quad v = \left(\frac{f\sigma_v^3}{N}\right)^{1/2} \bar{v}, \quad \sigma_o = \sigma_v \bar{\sigma}, \quad \phi_o = \sigma_v \bar{\phi} \end{aligned} \quad (2.13)$$

In the resulting differential equations and their boundary conditions, there is no longer any explicit dependence on σ_v , f or N . The eigenvalue ε , which in the original system is determined for each value of f , is here replaced by an input parameter $\bar{\varepsilon}$. For given values of $\bar{\varepsilon}$ and of the potential parameters $\bar{m}_\pi$ and $\bar{\lambda}$, or equivalently $\bar{m}_\pi$ and m_σ/m_π , we solve the system of coupled non-linear differential equations and their boundary conditions by using the COLSYS code [43]; actually, the boundary conditions at infinity (2.8) are replaced by boundary conditions at a large cut-off radius $\bar{R}$, which are easily derived [41] from the asymptotic behaviours (2.10)-(2.12). The precision of our computations may be evaluated either by the Rafelski energy test [44], which gives a precision of the order of 10^{-3} , or by the Goldberger-Treiman relation, which should be exact in this model, leading to a precision of the order of 10^{-6} (see Section 7). The resulting value of f , computed from the normalization condition

$$f^2 = \frac{4\pi}{N} \int_0^\infty (\bar{u}^2 + \bar{v}^2) \bar{x}^2 d\bar{x} \quad (2.14)$$

leads to a value of $m_\pi = f\sigma_v \bar{m}_\pi$ which in general is different from the physical value $m_\pi = 139.6$ MeV. This procedure is then repeated, adjusting iteratively the input parameter $\bar{m}_\pi$ with the shooting method [45], until the physical pion mass is reached.

As previously noticed [10,41,18], we observe that, for a given m_σ , soliton solutions only exist for values of the parameter f which are larger than some

critical value f_c (corresponding to a critical value $\bar{\epsilon}_c$). Actually, for each value of $f > f_c$, there exist two soliton solutions, one with $\bar{\epsilon} > \bar{\epsilon}_c$, corresponding to a classically unstable solution [18], the other with $\bar{\epsilon} < \bar{\epsilon}_c$, corresponding to the classically stable one. In this last case, when f decreases towards f_c , $\bar{\epsilon}$ monotonously increases towards $\bar{\epsilon}_c$. We have further investigated what happens when m_σ is varied. The behaviour of $\bar{\epsilon}$ as a function of f qualitatively remains unchanged. However, the values of f_c and the corresponding values of $\bar{\epsilon}_c$ depend rather strongly on m_σ , in particular for $m_\sigma < 800$ MeV. In Fig. 1, we show for $N = 3$ how, for decreasing m_σ , f_c increases slowly towards a maximum for $m_\sigma \approx 700$ MeV and then decreases rapidly. Figure 2 represents the values of $\bar{\epsilon}_c$ as a function of f_c ; when m_σ decreases $\bar{\epsilon}_c$ increases. As we shall see, the best mean nucleon-delta mass, when quantum corrections to the energy are included, will be obtained for $f = f_c(m_\sigma)$, m_σ being equal to 600 MeV, i.e., f_c being close to its maximum value.

3. - CENTRE-OF-ENERGY CORRECTION

The mean-field approximation breaks translational invariance, while the hedgehog assumption breaks rotational invariance, so that the mean-field hedgehog hadrons of the preceding section are neither eigenstates of total momentum $\vec{P}$ nor of total angular momentum $\vec{J}$. The broken symmetries may in principle be restored [46] by Peierls-Yoccoz projection [47] of these unphysical states onto eigenstates of momentum and angular momentum, interpreted as physical hadrons. As rotations do not commute with translations, we must proceed in two stages; first, correct for translational fluctuations by projecting the mean-field hedgehog hadrons onto hedgehog hadrons in the centre-of-energy frame; then, restore rotational invariance by projecting these hedgehog hadrons on eigenstates of $\vec{J}^2$. Actually, this two-stage projection is far too complex to be carried out exactly, and we shall resort to approximate projection techniques to correct for translational (in this section) and rotational (in the next two sections) symmetry breaking [20].

In the Peierls-Yoccoz method, momentum eigenstates are obtained from a localized state $|h\rangle$ by the projection

$$|\vec{p}\rangle = \int d^3x e^{i(\vec{P}-\vec{p})\cdot\vec{x}} |h\rangle \quad (3.1)$$

The quantum state $|h\rangle$ is chosen, in the "variation before projection" approximation, to minimize the expectation value of the Hamiltonian when variation is restricted to a certain set of trial states. As in previous approximate constructions of hadronic energy-momentum eigenstates in non-topological soliton bag models

[48,49], we shall choose these trial states to be products of valence quarks and a coherent superposition [50] of mesons. In our case, such states, which we shall term "coherent hedgehogs" to recall the fact that the valence quarks are in hedgehog states, are defined in the Schrödinger picture by [49,16]

$$|h(t)\rangle = e^{-iHt} e^{A^+(\mathbf{f})} b_1^+ \dots b_N^+ |0\rangle, \quad (3.2)$$

where the b_i^+ create N quarks in the same spin-isospin state of wave function $q_0(\vec{r})$, and the operator

$$A^+(\mathbf{f}) = \frac{i}{2} \int d^3k \vec{f}(\vec{k}) \cdot [\omega(\vec{k}) \vec{\phi}(\vec{k}) - i\vec{\pi}(\vec{k})] \quad (3.3)$$

[where $\omega(\vec{k}) = (\vec{k}^2 + m_\pi^2)^{\frac{1}{2}}$, and $\vec{\phi}(\vec{k})$, $\vec{\pi}(\vec{k})$ are the Fourier transforms of the pion field operators and their conjugate momenta] creates pions of wave function $\vec{f}(\vec{r})$ [the inverse Fourier transform of $\vec{f}(\vec{k})$]. As we shall presently recall, the energy of these quantum states is precisely that obtained in the mean-field approximation. Note that we have not included in (3.2) a coherent superposition of the "massive" σ -particles because, owing to their large mass m_σ , the σ -fluctuations are badly described by a coherent state. In what follows, we shall treat the σ -field as classical; an improvement over this would be to treat the σ -fluctuations in the random phase approximation [18]. On the other hand, we expect the fluctuations of the quasi-Goldstone field $\vec{\phi}$ to be well described in a coherent state approach.

Computation of the expectation values

$$\langle \vec{\phi}(\vec{x}) \rangle \equiv \vec{\phi}_0(\vec{x}), \quad \langle \vec{\pi}(\vec{x}) \rangle \equiv \vec{\pi}_0(\vec{x}) \quad (3.4)$$

in the coherent hedgehog state shows that they are related to the coherent wave function $\vec{f}$ by

$$\vec{f}(\vec{k}) = -i\vec{\phi}_0(\vec{k}) + \frac{\vec{\pi}_0(\vec{k})}{\omega(\vec{k})}. \quad (3.5)$$

This may be generalized straightforwardly to expectation values of normal-ordered products of $\vec{\phi}$ and $\vec{\pi}$ fields. We can then compute the expectation value of the Hamiltonian, which is equal to its classical expression

$$\langle H \rangle = H_{cl}(q_0, \sigma_0, \vec{\phi}_0, \vec{\pi}_0), \quad (3.6)$$

so that extremization of (3.6) leads to the result that $\vec{\pi}_0 = 0$, while q_0 , σ_0 and $\vec{\phi}_0 = \phi_0 \hat{r}$ solve the coupled stationary mean-field equations of the previous section, as anticipated in the notation. The advantage of the coherent hedgehog approach is that some quantum information can now be extracted from these mean fields. For instance, the mean number of pions in the coherent hedgehog (3.2) is

$$N_\pi = \frac{1}{2} \int d^3k \, \omega(\vec{k}) |\vec{f}(\vec{k})|^2 . \quad (3.7)$$

For the state which minimizes $\langle H \rangle$, this reduces to

$$N_\pi = N_0 \equiv \frac{1}{2} \int d^3k \, \omega(\vec{k}) |\vec{\phi}_0(\vec{k})|^2 . \quad (3.8)$$

In Fig. 3 we have represented N_0 as a function of f for the value $m_\sigma = 1200$ MeV used by many authors [10,16,17,19]. We see that, as f increases, i.e., as the pions get more strongly coupled to the quarks, their average number also increases. The same feature has been observed by other authors [27,51].

Knowing the state $|h\rangle$, one may determine the Peierls-Yoccoz mass of the hadron, given by the expectation value of the Hamiltonian $\langle H \rangle_{\vec{p}=0}$ in the state (3.1) with $\vec{p} = 0$. We shall not compute this exactly but, following Refs. [52] and [49], use the inequality (valid for any trial state)

$$M^2 \leq \langle H^2 \rangle - \langle \vec{P}^2 \rangle , \quad (3.9)$$

and the equality $\langle H^2 \rangle = \langle H \rangle^2$ (the coherent hedgehog is a minimum uncertainty state for the energy, if loop corrections are ignored), to obtain the estimate (actually an upper bound)

$$M^2 \simeq \langle H \rangle^2 - \langle \vec{P}_q^2 \rangle - \langle \vec{P}_\pi^2 \rangle \quad (3.10)$$

where the expectation values, computed in the coherent hedgehog state $|h\rangle$, are

$$\langle H \rangle \equiv E_0 = N\epsilon + \int d^3x \left[\frac{1}{2} (\vec{\nabla} \sigma_0)^2 + \frac{1}{2} (\vec{\nabla} \vec{\phi}_0)^2 + U(\sigma_0, \vec{\phi}_0) \right] , \quad (3.11)$$

the hedgehog energy, and

$$\begin{aligned}\langle \vec{P}_q^2 \rangle &\equiv P_q^2 = N \int d^3x \vec{\nabla} q_0^+ \cdot \vec{\nabla} q_0 , \\ \langle \vec{P}_\pi^2 \rangle &\equiv P_\pi^2 = \frac{1}{2} \int d^3k \omega(\vec{k}) \vec{k}^2 |\tilde{\phi}_0(\vec{k})|^2 ,\end{aligned}\tag{3.12}$$

the corrections for quark and coherent pion translational fluctuations.

Figure 4 shows the behaviours of E_0 , P_q , P_π and M as functions of f for the same value $m_\sigma = 1200$ MeV. Clearly, P^2/E_0^2 is not much smaller than 1, as was often suggested [10,53]. The centre-of-energy corrections are important and increase quickly with f , the resulting hedgehog mass decreasing from 1045 MeV at $f_c = 3.95$ to zero at $f \approx 5.8$. Because correcting for rotational fluctuations, as we shall see in Section 5, will further decrease this hedgehog mass, from now on we choose f to be equal to f_c , leading for a given value of m_σ to a value of M which is as close as possible to the experimental mean nucleon-delta mass. The same quantities as in Fig. 4 are shown in Fig. 5 as functions of m_σ [when $f = f_c(m_\sigma)$]. Although both the uncorrected hedgehog energy E_0 and the corrections P_q and P_π decrease rapidly with decreasing m_σ , the resulting hedgehog mass remains very nearly constant. The mean pion number N_0 (Fig. 6) also decreases when m_σ is decreased, being approximately 0.3 for $m_\sigma \approx 600$ MeV.

4. - CRANKING THE COHERENT HEDGEHOG AND THE SELF-CONSISTENT CHROMOMAGNETIC FIELD

In the remainder of this paper, we assume that the main effect of the centre-of-energy correction is taken into account in (3.10), and that the hedgehog hadrons projected in the centre-of-energy frame can still be approximated by the coherent hedgehog states (3.2). These states, which are neither eigenstates of angular momentum $\vec{J}$ nor of isospin $\vec{I}$, are however eigenstates of grand spin $\vec{J}+\vec{I}$:

$$(\vec{J} + \vec{I}) |h\rangle = 0 .\tag{4.1}$$

In this model, the isospin operator may be decomposed as

$$\vec{I} = \vec{I}_q + \vec{I}_\pi ,\tag{4.2}$$

where, in the Schrödinger picture,

$$\vec{I}_q = \int d^3x : \psi^\dagger \frac{\vec{\tau}}{2} \psi : , \quad \vec{I}_\pi = \int d^3x : \vec{\phi} \wedge \vec{\pi} : .\tag{4.3}$$

The expectation values of both these operators in the coherent hedgehog state are zero, the first one because of the hedgehog property (2.6), the second because

$\vec{\pi}_0 = 0$. It then follows from (4.1) that the coherent hedgehog hadrons have zero mean spin and isospin,

$$\langle \vec{J} \rangle = -\langle \vec{I} \rangle = 0, \quad (4.4)$$

while the expectation values $\langle \vec{J}^2 \rangle = \langle \vec{I}^2 \rangle$ are non-zero [16] and in fact, as we shall see, rather large.

It is therefore necessary to project the hedgehog hadrons on (approximate) eigenstates of spin and isospin. Actually, because of the grand-spin symmetry, states with equal spin and isospin may be obtained from the coherent hedgehog by a single Peierls-Yoccoz angular momentum projection, which has been carried out previously both in the case of the cloudy bag [54] model [55] and of the chiral quark-meson model [27,16,19]. This could in principle be applied to the computation of gluonic effects in the chiral soliton bag model, if variation after projection is used, at the cost of intricate numerical computations. An approximate projection method was used in Ref. [15] to compute the gluonic contribution to the nucleon-delta mass difference; however, this method cannot be used to compute other gluonic effects.

Rotational invariance may also be restored by the semi-classical cranking method [56,46]. In this method, which was previously applied to the Skyrme model [39], to the chiral bag model [57] and to chiral soliton bag models [17,28], the hadronic spin and isospin are generated by time-dependent rotations (in space or isospace); the hadronic energy is therefore modified by a spin-isospin-dependent centrifugal term, leading to a mass splitting between the nucleon and delta. Cranking also affects various other mean field quantities, including the colour currents, which now generate non-vanishing chromomagnetic fields; this was applied in Refs. [6] and [22] to compute the gluonic contribution to the nucleon-delta mass difference in the one-gluon exchange approximation. In the present section, we shall apply the cranking method to the chiral soliton bag model by taking into account self-consistently the mean chromomagnetic field generated by cranking.

It was proposed in Ref. [20] to improve on the mean field cranking method by cranking the coherent hedgehog, thus combining the simplicity of the cranking method with the richer quantum content of the coherent state description, which includes the effect of rotational fluctuations. It was shown that, in the case $\alpha_s = 0$, the values obtained by the coherent cranking method for the nucleon and delta energies were in quite good agreement with those obtained by coherent state projection [16], showing that this method is a good approximation to the full angular momentum projection.

In the coherent cranking method, the chiral soliton bag model is cranked by isorotating a coherent hedgehog state $|h'(t)\rangle$ of the form (3.2) to

$$|h'_\omega(t)\rangle = B e^{-i\vec{\omega} \cdot \vec{I} t} |h'(t)\rangle , \quad (4.5)$$

where B is a time-independent $SU(2)$ matrix, $\vec{\omega}$ is a time-independent vector, and $\vec{I}$ is the isospin operator (4.2), and choosing the cranked hedgehog $|h'_\omega\rangle$ so as to minimize the cranked energy $\langle H + \vec{\omega} \cdot \vec{I} \rangle'_\omega = \langle H + \vec{\omega} \cdot \vec{I} \rangle'$. The coherent hedgehog $|h'\rangle$ no longer extremizes the energy $\langle H \rangle'$, so that the quark and pion wave functions $q_0, \vec{\phi}_0, \vec{\pi}_0$ of the previous section are replaced by cranked wave functions $q', \vec{\phi}', \vec{\pi}'$ (as a matter of fact, the term "coherent hedgehog" is no longer appropriate, as the cranked wave functions are only approximately spherically symmetric, but we shall stick to it anyway). The resulting cranked mean spin and isospin are, for slow rotations, linear in $\vec{\omega}$,

$$\langle \vec{J} \rangle' = - \langle \vec{I} \rangle' = \mathcal{J} \vec{\omega} + \mathcal{O}(\vec{\omega}^2) , \quad (4.6)$$

thus defining the hadronic moment of inertia $\mathcal{J}$. The various observables of a hadron of spin-isospin j depend on the cranking parameter $\omega = |\vec{\omega}|$, which is obtained by solving the physical constraint [20]

$$\langle \vec{J}^2 \rangle' = \langle \vec{I}^2 \rangle' = (\Delta \vec{I})^2(\omega) + \mathcal{J}^2 \omega^2 = j(j+1) , \quad (4.7)$$

where $(\Delta \vec{I})^2$ is the mean square isospin fluctuation.

Cranking does not directly affect the (flavourless) mean colour fields, but gives rise to non-vanishing currents which generate non-vanishing chromomagnetic fields $\vec{B}_a$ (the chromoelectric fields are, of course, still zero). The extremization of the cranked energy thus involves self-consistently the cranked mean fields $q', \sigma', \vec{\phi}', \vec{\pi}', \vec{B}'_a$. To carry out this extremization, we expand the cranked mean fields in powers of ω [17]:

$$\begin{aligned} q' &= q_0 + \omega q_1 + \dots , \quad \sigma' = \sigma_0 + \omega \sigma_1 + \dots , \\ \vec{\phi}' &= \vec{\phi}_0 + \omega \vec{\phi}_1 + \dots , \quad \vec{\pi}' = \omega \vec{\pi}_1 + \dots , \quad \vec{B}'_a = \omega \vec{B}_{1a} + \dots . \end{aligned} \quad (4.8)$$

The corresponding expansion of the cranked energy is

$$E' \equiv \langle H + \vec{\omega} \cdot \vec{I} \rangle' = E_0 + \omega^2 E_2 + \dots . \quad (4.9)$$

The zeroth-order energy is the hedgehog energy (3.11). The second-order energy may be decomposed as

$$E_2 = E_{2q} + E_{2m} + E_{2g} , \quad (4.10)$$

where, after using the zeroth-order equations [Eqs. (2.4) with $g_s = 0$], the normalization constraint

$$\int d^3x q'^{\dagger} q' = 1 , \quad (4.11)$$

and taking into account the fact that the first-order meson fields σ_1 and $\vec{\phi}_1$ are identically zero [17], we obtain

$$E_{2q} = \int d^3x N \left[q_1^{\dagger} (\hbar_0 - \epsilon) q_1 + q_1^{\dagger} \hat{\omega} \cdot \frac{\vec{\tau}}{2} q_0 + q_0^{\dagger} \hat{\omega} \cdot \frac{\vec{\tau}}{2} q_1 \right] \quad (4.12)$$

[$\hbar_0 = \vec{\alpha} \cdot \vec{p} + f\beta(\sigma_0 - i\vec{\tau} \cdot \vec{\phi}_0 \gamma^5)$ is the zeroth-order Dirac Hamiltonian],

$$E_{2m} = \int d^3x \left[\frac{1}{2} \vec{\pi}_1^2 + \hat{\omega} \cdot (\vec{\phi}_0 \wedge \vec{\pi}_1) \right] , \quad (4.13)$$

and

$$E_{2g} = \int d^3x \sum_a \sum_{i \neq j} \left[\frac{\kappa_0}{2} \vec{B}_{ii}^a \cdot \vec{B}_{jj}^a - g_s \vec{A}_{ii}^a \langle \frac{\lambda^a}{2} \rangle_j \cdot (q_1^{\dagger} \vec{\alpha} q_0 + q_0^{\dagger} \vec{\alpha} q_1) \right] . \quad (4.14)$$

In writing (4.14) we have excluded from the sum the chromomagnetic self-energy ($i = j$) terms, as in previous bag or soliton bag treatments [58,6,24-26].

The extremization of E_{2m} can be carried out independently and leads to

$$\vec{\pi}_1 = - \hat{\omega} \wedge \vec{\phi}_0 . \quad (4.15)$$

To carry out the extremization of the remainder $E_{2q} + E_{2g}$ of the second-order energy, we parametrize the first-order mean fields q_1 , $\vec{A}_{11}^a$ by [17,22]

$$q_1 = \hat{\omega} \cdot \vec{\xi} \quad , \quad \vec{\xi} = \begin{pmatrix} [A(x)\vec{\sigma} + B(x)(\vec{\sigma}/3 - \hat{x}(\vec{\sigma} \cdot \hat{x}))] \chi_h \\ [iC(x)\hat{x} - D(x)\hat{x} \wedge \vec{\sigma}] \chi_h \end{pmatrix} \quad ,$$

$$\vec{A}_{ii}^a = g_s \langle \frac{\lambda^a}{2} \rangle_i Q(x) \hat{\omega} \wedge \hat{x} \quad . \quad (4.16)$$

Inserting these into (4.10), integrating over the angles, and varying the resulting functional form of the second-order energy leads to the system of differential equations, written for a baryon ($N = 3$):

$$\begin{aligned} \frac{d}{dx} (A - \frac{2B}{3}) - \frac{2B}{x} &= -(\frac{1}{2}\sigma_0 + \epsilon)C - \frac{1}{2}\phi_0 (A - \frac{2B}{3}) - \frac{v}{2} \quad , \\ \frac{d}{dx} (A + \frac{B}{3}) + \frac{B}{x} &= (\frac{1}{2}\sigma_0 + \epsilon)D + \frac{1}{2}\phi_0 (A + \frac{B}{3}) - \frac{v}{2} + \frac{16\pi}{3} \alpha_s Q u \quad , \\ \frac{dC}{dx} - \frac{2D}{x} &= -(\frac{1}{2}\sigma_0 - \epsilon)(A - \frac{2B}{3}) + \frac{1}{2}\phi_0 C + \frac{u}{2} \quad , \\ \frac{dD}{dx} - \frac{C-D}{x} &= (\frac{1}{2}\sigma_0 - \epsilon)(A + \frac{B}{3}) - \frac{1}{2}\phi_0 D - \frac{u}{2} + \frac{16\pi}{3} \alpha_s Q v \quad , \\ -\frac{1}{x} \frac{d}{dx} (\kappa_0 \frac{d}{dx} (xQ)) + \frac{2}{x^2} \kappa_0 Q &= 2 \left[(A + \frac{B}{3})v - Du \right] \quad , \end{aligned} \quad (4.17)$$

where we have used $\sum_a \sum_{i \neq j} \langle \lambda^a/2 \rangle_i \langle \lambda^a/2 \rangle_j = -4$, and $\alpha_s = g_s^2/4\pi$. An interesting point is that the chromomagnetic potential Q solves linearized equations, not, as in previous self-consistent treatments of the Friedberg-Lee model [24-26], because of restriction to one-gluon exchange, but because the expansion in powers of ω is, as far as the chromomagnetic fields are concerned, an expansions in powers of $g_s \omega$ [57].

We solved the system (4.17), with the natural boundary conditions

$$\begin{aligned} C(0) &= D(0) = Q(0) = 0 \quad , \\ A(\infty) &= B(\infty) = \kappa_0 Q(\infty) = 0 \quad , \end{aligned} \quad (4.18)$$

for a family of dielectric functions of the form [59]

$$\kappa = \left(1 - \frac{\sigma^2 + \vec{\Phi}^2}{F_\pi^2} \right)^n . \quad (4.19)$$

The arbitrariness in the choice (4.19) is balanced by the fact that the confining exponent $n > 0$ is a free parameter in our model. The value $n = 0$ corresponds to unconfined gluons; the requirement that the string tension, which may also be computed in such a model, be finite, leads to the restriction $n > 1$ [59]. For any m_σ , and not too large values of f , the mean-field solution $\kappa(r)$ then interpolates monotonously between a value $\kappa(0) \simeq 1$ and the vacuum value $\kappa(\infty) = 0$. An example of the radial dependence of the dielectric function $\kappa(r)$ is shown in Fig. 7, for the parameter values $m_\sigma = 600$ MeV, $f = f_c(m_\sigma)$, $n = 2$.

The asymptotic behaviours of the solutions depend on the value of n ; we only consider here the case $m_\pi < \inf[(m_\sigma/2), \beta]$. In this case, it follows from Eqs. (2.11b) and (2.12b) that, asymptotically,

$$R_0(\kappa) \sim \kappa^{-2n} e^{-2nm_\pi \kappa} . \quad (4.20)$$

For $n < \beta/m_\pi$, the chromomagnetic potential $A(r)$ vanishes at infinity, and so can be neglected in the asymptotic study of the first four equations (4.17), which leads to the asymptotic behaviours

$$\begin{aligned} A(\kappa) &\simeq \frac{\varepsilon}{2\beta} \kappa u(\kappa) , & B(\kappa) &\sim u(\kappa) , \\ C(\kappa) &\simeq D(\kappa) \simeq \frac{\varepsilon}{2\beta} \kappa v(\kappa) . \end{aligned} \quad (4.21)$$

(see Appendix A for more details). It then follows from the last of Eqs. (4.17), and Eqs. (2.10), (4.20) and (4.21), that, asymptotically,

$$Q(\kappa) \sim \kappa^{2n-1} e^{2(hm_\pi - \beta)\kappa} . \quad (4.22)$$

For $n > \beta/m_\pi$, this does not vanish asymptotically, so that the above derivation no longer holds. We show in Appendix A that the self-consistent asymptotic behaviours are, in this case,

$$\begin{aligned} A(\kappa) &\simeq -\frac{\varepsilon}{2\beta} \kappa u(\kappa) , & B(\kappa) &\sim u(\kappa) , \\ C(\kappa) &\simeq D(\kappa) \simeq -\frac{\varepsilon}{2\beta} \kappa v(\kappa) , \\ \frac{16\pi}{3} \alpha_s Q(\kappa) &\simeq \varepsilon \left(\kappa - \frac{1}{\beta} \right) . \end{aligned} \quad (4.23)$$

The chromomagnetic potential rises linearly, so that the chromomagnetic field $\vec{B}_{li}^a$ generated by each quark tends to a constant vector collinear to $\vec{\omega}$; a similar behaviour was previously found to occur in the Friedberg-Lee model, treated perturbatively in the strong coupling constant g_s [60].

The functions u , v , σ_0 , ϕ_0 , determined according to the procedure described in Section 2, served as input for the solution of the system of coupled linear differential equations (4.17) with the appropriate boundary conditions, again by using the COLSYS code [43]. The resulting values of the first-order quark wave functions may be used to compute the baryonic moment of inertia, which may be written as the sum of quark and meson contributions [17]

$$\mathcal{I} = \mathcal{I}_q + \mathcal{I}_m , \quad (4.24)$$

where, according to Eqs. (4.3), (4.6), (4.15) and (4.16),

$$\begin{aligned} \mathcal{I}_q &= -3 \int d^3x (q_1^+ \hat{\omega} \cdot \vec{\tau}_2 q_0 + q_0^+ \hat{\omega} \cdot \vec{\tau}_2 q_1) = 3 \int d^3x (Au + \frac{C-2D}{3} v) , \\ \mathcal{I}_m &= \int d^3x (\hat{\omega} \wedge \vec{\phi}_0)^2 = \int d^3x \frac{2}{3} \phi_0^2 . \end{aligned} \quad (4.25)$$

In Appendix B we show that, for small α_s , this form of $\mathcal{I}$ linearizes to

$$\mathcal{I} = \mathcal{I}_0 + \alpha_s \mathcal{I}_2 + \dots , \quad (4.26)$$

where $\mathcal{I}_0$ is the moment of inertia to zeroth order in α_s , and $\mathcal{I}_2$ is the perturbative gluonic contribution to the moment of inertia [22]

$$\mathcal{I}_2 = - \frac{32\pi}{3} \int d^3x \, 2 \left[(A + \frac{B}{3}) v - Du \right] \alpha . \quad (4.27)$$

The values we obtain for the moment of inertia depend both on the exponent n of the confining function (4.19) and on the effective strong fine structure constant α_s . In Fig. 8, we show how $\mathcal{I}$ behaves as a function of α_s for different values of n , m_σ being equal to 600 MeV and $f = f_c(m_\sigma)$. We see that $\mathcal{I}$ decreases rapidly when either n or α_s increases. This reflects the fact that more strongly confined gluons or a stronger quark-gluon coupling constant α_s lead to a more compact quark distribution, classically correlated with a smaller moment of inertia. However, we should take care that, for too small values of the moment of inertia, the cranking procedure breaks down, the corresponding values of ω^2 being indeed too large to allow a power expansion. The dashed line in Fig. 8 corresponds to the linearized moment of inertia for the value $n = 1$. Clearly, perturbative

cranking may be used only for very small values of α_s . Also shown in this Figure is the small α_s -independent meson contribution $\mathcal{J}_m$ to the moment of inertia.

These behaviours hold for all values of m_σ . Figure 9 represents the variation of $\mathcal{J}$ and $\mathcal{J}_m$ with m_σ , for $f = f_c(m_\sigma)$, when gluon exchange is not taken into account ($\alpha_s = 0$, so that $\mathcal{J} = \mathcal{J}_0$). $\mathcal{J}_0$ is very large for small values of m_σ , and decreases rapidly with increasing m_σ , apparently reaching a limiting value $\mathcal{J}_0 \approx 2 \text{ fm}$ for $m_\sigma > 1000 \text{ MeV}$. On the contrary, $\mathcal{J}_m$ increases slightly over the same range of m_σ values. The variations with f of $\mathcal{J}_0$ and $\mathcal{J}_m$ are shown in Fig. 10 for $m_\sigma = 1200 \text{ MeV}$. Curiously, the moment of inertia per pion $\mathcal{J}_m/N_0$ stays remarkably constant. Of course, the total baryonic moment of inertia $\mathcal{J}_0$ does vary with f , although weakly.

5. - THE NUCLEON AND DELTA MASSES

The cranking equations (4.15) and (B.1) may be used to relate the second-order energy (4.10) to the moment of inertia, leading to the value of the cranked energy

$$E' \equiv \langle H + \vec{\omega} \cdot \vec{I} \rangle = E_0 - \frac{1}{2} \mathcal{J} \omega^2 \quad (5.1)$$

for slow rotations. The corresponding energy $E \equiv \langle H \rangle'$ of the cranked hedgehog is

$$E = E_0 + \frac{1}{2} \mathcal{J} \omega^2, \quad (5.2)$$

which is the familiar cranking result [46].

The squared angular velocity ω^2 is now fixed so that the mean square isospin attains its physical value

$$\langle \vec{I}^2 \rangle = (\Delta \vec{I})^2 + \mathcal{J}^2 \omega^2 = j(j+1). \quad (5.3)$$

The mean square isospin fluctuation $(\Delta \vec{I})^2$ in the coherent hedgehog state $|h'(t)\rangle$ may be computed by Wick's theorem, with the obvious result

$$(\Delta \vec{I})^2 = \frac{3}{4} N + 2 N_\pi(\omega^2), \quad (5.4)$$

N being the number of constituent quarks, and

$$N_\pi(\omega^2) = \frac{1}{2} \int d^3k \, \omega(\vec{k}) |\vec{f}'(\vec{k})|^2 \quad (5.5)$$

the cranked mean pion number. The resulting value of the baryonic energy ($N = 3$) is

$$E = E_0 + \frac{1}{2J} \left[j(j+1) - \frac{9}{4} - 2N_\pi(\omega^2) \right] . \quad (5.6)$$

The first correction to the mean-field hedgehog energy E_0 is the semiclassical cranking term $j(j+1)/2J$. The corrections for quark rotational fluctuations $[-(9/4)/2J]$ and for pion rotational fluctuations $(-2N_\pi/2J)$ were first given in Refs. [17] and [20] respectively.

The mean pion number depends on the squared angular velocity ω^2 , and so on the spin-isospin quantum number j , as first observed in Ref. [16]. This dependence may be parametrized by

$$N_\pi(\omega^2) = N_0 + \omega^2 N_2 + \dots , \quad (5.7)$$

where N_0 is given in (3.8), and

$$N_2 = \int d^3k \, \omega(\vec{k}) \operatorname{Re} [\tilde{\phi}_0^*(\vec{k}) \cdot \tilde{\phi}_2(\vec{k})] + \frac{1}{2} \int \frac{d^3k}{\omega(\vec{k})} |\tilde{\pi}_1(\vec{k})|^2 . \quad (5.8)$$

The computation of $\tilde{\phi}_2(\vec{k})$ would require solving a new system of coupled differential equations giving the second-order mean fields q_2 , σ_2 and $\vec{\phi}_2$ in terms of q_0 , σ_0 , $\vec{\phi}_0$ and q_1 . We have not carried this out, and shall assume that N_2 may be approximated by its second term

$$N_2 \approx \frac{1}{2} \int \frac{d^3k}{\omega(\vec{k})} |\tilde{\pi}_1(\vec{k})|^2 = \frac{1}{3} \int \frac{d^3k}{\omega(\vec{k})} |\tilde{\phi}_0(\vec{k})|^2 . \quad (5.9)$$

which varies between 0.07 fm^2 for $m_\sigma = 600 \text{ MeV}$ and 0.13 fm^2 for $m_\sigma = 2000 \text{ MeV}$. Eliminating ω^2 between Eqs. (5.2) and (5.6)-(5.7), we obtain

$$E = E_0 + \frac{1}{2J'} \left[j(j+1) - \frac{9}{4} - 2N_0 \right] , \quad (5.10)$$

where

$$J' = J + \frac{2N_2}{J} \quad (5.11)$$

acts as an effective moment of inertia. Our derivation of Eq. (5.10) is valid only if the perturbative correction to the moment of inertia is relatively small,

$2N_2/\mathfrak{J} \ll \mathfrak{J}$, which again implies that the moment of inertia should not be too small; in practice, we chose $\mathfrak{J} \gtrsim 0.6$ fm as the criterion for the validity of Eq. (5.10).

The values given by this equation for the nucleon ($j = 1/2$) and delta ($j = 3/2$) energies were compared in Ref. [20] with those obtained by coherent state projection [16]; it was found that for $m_\sigma = 1200$ MeV, $\alpha_s = 0$, and various values of f , the nucleon energies computed by the two methods differ by less than 1%, while the delta energies differ by less than 4%; the values given by Eqs. (5.7) and (5.9) for the mean pion number in the nucleon and the delta also agree rather well with those of Ref. [16].

A somewhat unsatisfactory feature of our cranking procedure is that the coherent hedgehog $|h'\rangle$ which minimizes the cranked energy E' is now only approximately annihilated by the grand spin operator $\vec{J} + \vec{I}$, so that the mean square angular momentum $\langle \vec{J}^2 \rangle'$, computed to order ω^2 , is no longer equal to $j(j+1)$ when ω^2 is fixed by (5.3). This point is further elaborated in Appendix C, where the difference $(\Delta \vec{J})^2 - (\Delta \vec{I})^2$ is analytically computed to order ω^2 . This difference turns out to be small with respect to the second-order contribution $2N_2\omega^2$ to $(\Delta \vec{I})^2$ so that, to a good approximation, we also have, besides (5.3),

$$\langle \vec{J}^2 \rangle' \approx j(j+1) . \quad (5.12)$$

As previously discussed, we must also take into account the centre-of-energy correction by replacing in (5.10) the (uncranked) hedgehog energy E_0 by the hedgehog mass M given by (3.10), leading to

$$E = M + \frac{1}{2\mathfrak{J}'} \left[j(j+1) - \frac{9}{4} - 2N_0 \right] , \quad (5.13)$$

which is the final expression for the fully quantum-corrected baryonic energy. The resulting mean nucleon-delta mass $\bar{E}$ and nucleon-delta mass difference ΔE are given by

$$\begin{aligned} \bar{E} &= M - \frac{N_0}{\mathfrak{J}'} , \\ \Delta E &= \frac{3}{2\mathfrak{J}'} , \end{aligned} \quad (5.14)$$

so that the main effect of our correction for rotational fluctuations is to lower further the mean nucleon-delta mass. A secondary effect is a reduction in the N - Δ mass difference from the naive value $3/2 \mathfrak{J}$, as $\mathfrak{J}' > \mathfrak{J}$ from Eq. (5.11), thereby improving the agreement with the projection calculations [16,19], as shown in

Table 1; the remaining discrepancy may be due partly to the fact that our approximation (5.9) underestimates N_2 .

Let us first consider the case $\alpha_s = 0$, gluon exchange effects not being taken into account. The nucleon-delta mass difference ΔE_π (due in this case only to pion exchange) increases only slightly with increasing f (as shown in Fig. 11 for $m_\sigma = 1200$ MeV). For $f = f_c(m_\sigma)$, ΔE_π increases rather rapidly with increasing m_σ , until it reaches a value which remains approximately unchanged for $m_\sigma > 1000$ MeV, as may be seen in Fig. 12. Including now gluon exchange effects self-consistently, we show in Fig. 13 [for $m_\sigma = 600$ MeV and $f = f_c(m_\sigma)$] how the nucleon-delta mass difference ΔE increases both with the strong fine structure constant α_s and with the confining exponent n . For the realistic cases, $n > 1$, the gluonic contribution to ΔE appears to be important compared to the pionic contribution ΔE_π . A similar strong α_s - and n -dependence of ΔE is observed for all values of m_σ . Of course, the mean nucleon-delta mass $\bar{E}$ also depends on n and α_s , albeit very weakly.

Having chosen in Section 3 $f = f_c(m_\sigma)$ to optimize the value of $\bar{E}$ leaves us with the three free parameters m_σ , α_s , and the confining exponent n . We shall first constrain these parameters by assuming (as in our previous treatments of gluonic effects [15,20,22]) that the dimensionless ratio $\Delta E/\bar{E}$ is equal to its experimental value

$$\frac{\Delta E}{\bar{E}}(m_\sigma, \alpha_s, n) = 0.27, \quad (5.15)$$

which gives, for the effective moment of inertia,

$$\mathcal{I}' = \frac{N_0 + 5.56}{M}. \quad (5.16)$$

In Fig. 14, we give the resulting values of α_s as functions of n for $m_\sigma = 600, 800$ and 1200 MeV and $f = f_c(m_\sigma)$. For a given n , these values decrease as m_σ increases. They also decrease by several orders of magnitude when n is varied from 0 to 3. For $n \gtrsim 3$, the effective strong fine structure constant becomes much too small to be realistic. Indeed, recent experiments [61] seem to rule out values of α_s which at $q \approx 1000$ MeV are smaller than 0.25. If we assume, for example, $m_\sigma \approx 600$ MeV and n integer, then the choice $n = 2$ leads to a realistic value of $\alpha_s \approx 0.4$.

When $f = f_c(m_\sigma)$ and the constraint (5.15) or (5.16) is enforced, the baryonic moment of inertia $\mathcal{I}$, related to $\mathcal{I}'$ by (5.11), or baryonic masses, given by (5.13), depend only on m_σ . The moment of inertia $\mathcal{I}(m_\sigma)$ roughly behaves like $1/M$, fluctuating around 0.9 fm [well above the limit of validity 0.6 fm of Eq. (5.10)]. The nucleon and delta masses $E_N(m_\sigma)$ and $E_\Delta(m_\sigma)$ are plotted in Fig. 15; whatever the value of m_σ is, these values are always somewhat smaller than the experimental

ones, the best value for the mean nucleon-delta mass being $\bar{E} = 1020$ MeV for $m_\sigma = 450$ MeV, to be compared with the experimental value $\bar{E} = 1085$ MeV. The inclusion of a bare quark mass term in the Lagrangian (2.1) should improve this value.

Likewise, the mean number of pions in the nucleon $N_\pi(\omega_N^2)$ given by (5.7) and (5.3) depends only on m_σ when $f = f_c$ and the relation (5.15) is imposed. This number, which is roughly 50% of N_0 , varies from 0.10 for $m_\sigma = 450$ MeV to 0.45 for $m_\sigma = 2000$ MeV, to be compared with the cloudy bag model estimate [62] $N_\pi \approx 0.5$.

Bounds on the possible values of m_σ now follow from the study of the stability of the baryonic states. The physical nucleon and delta do not decay into three free quarks, which is ensured in the model if

$$E_\Delta = M + \frac{1}{25} \left[\frac{3}{2} - 2N_\pi(\omega_\Delta^2) \right] < 3f\sigma_v, \quad (5.17)$$

leading to the lower bound for the mean pion number

$$N_\pi(\omega_\Delta^2) > 0.75 - (3f\sigma_v - M)5, \quad (5.18)$$

or equivalently, using (5.16), to

$$N_0 > 6.31 \frac{M}{3f\sigma_v} - 5.56. \quad (5.19)$$

When the constraint (5.15) is satisfied, the resulting lower bound on m_σ is thus [for $f = f_c(m_\sigma)$] $m_\sigma > 600$ MeV (see Fig. 15). On the other hand, unwanted higher spin-isospin states ($j > 5/2$) disappear from the baryonic spectrum if they are unstable against decay into three free quarks:

$$N_\pi(\omega_{5/2}^2) < 3.25 - (3f\sigma_v - M)5, \quad (5.20)$$

or, using (5.16),

$$N_0 < 8.81 \frac{M}{3f\sigma_v} - 5.56. \quad (5.21)$$

Our results show that this is always satisfied for the choice $f = f_c(m_\sigma)$, while if m_σ is fixed for instance at 1200 MeV, and f varies, the unwanted excited states appear for $f > 4.30$, thus suggesting that the allowed values of f should not lie too far from f_c .

Finally, pions may be emitted by the baryon if the squared cranking frequency ω^2 exceeds the squared pion mass m_π^2 [63] (in our treatment, this follows from the fact that the second-order field equations for ϕ_2 do not admit localized solutions if $\omega^2 > m_\pi^2$). Computing the effective ω^2 from Eqs. (5.2) and (5.6), we see that $\omega_N^2 < 0$, so that the nucleon is stable, while the delta is unstable by pion emission, as it should be, if

$$N_\pi(\omega_\Delta^2) < 0.75 - \frac{1}{2} m_\pi^2 \eta^2 . \quad (5.22)$$

When $f = f_c(m_\sigma)$ and the constraint (5.15) is satisfied, the resulting upper bound on m_σ is $m_\sigma < 800$ MeV. So, to summarize, the constraints on m_σ which follow from the existence of the delta resonance are [for $f = f_c(m_\sigma)$]

$$600 \text{ MeV} \leq m_\sigma \leq 800 \text{ MeV} , \quad (5.23)$$

the lower bound giving the best mean nucleon-delta mass $\bar{E} \approx 1000$ MeV.

6. - THE SPIN OF THE PROTON

A straightforward analysis of the recent EMC measurements of the polarized proton structure function [29], combined with information from neutron and hyperon decay, leads to the conclusion [30] that the flavour-singlet axial current matrix element is compatible with zero:

$$\langle p | A_\mu^0 | p \rangle \simeq 0 . \quad (6.1)$$

In three-flavour QCD, the current operator

$$\vec{A}^0 = \frac{1}{2} \sqrt{\frac{2}{3}} \psi^\dagger \vec{\sigma} \psi \quad (6.2)$$

is proportional to the quark-spin operator, so that Eq. (6.1) implies that very little of the proton spin is carried by quark helicities, the "experimental" value [32]

$$\Delta q = \Delta u + \Delta d + \Delta s = \langle p | \psi^\dagger \sigma_3 \psi | p \rangle = -0.03 \pm 0.21 \quad (6.3)$$

resulting from a compensation between the positive contribution of the non-strange quarks

$$\Delta u + \Delta d = 0.21 \pm 0.14 , \quad (6.4)$$

and a negative strange quark contribution Δs .

This result, in apparent contradiction to the naive quark model, can be understood [35] if one considers the nucleon, at relatively large distances, as a loosely-bound system of massive (non-relativistic) constituent quarks which, at short distances, can be resolved in nearly massless (extreme relativistic) bare quarks surrounded by a cloud of $q\bar{q}$ and $g\bar{g}$ pairs. Confinement of quarks and gluons plays an essential rôle in this dual picture, and so it was natural that the first attempts [31,32] to explain the result (6.1) were made in the limiting case of the Skyrme model, where quarks (and gluons) are absent, but the matrix element (6.1) can nevertheless, in principle, be different from zero. It was shown [31] that, to zeroth order in the $1/N_c$ expansion, $\Delta q = 0$ in the chiral symmetric Skyrme model. The term of order $1/N_c$ may be computed by cranking techniques, leading to a result of second order in the cranking parameter ω [33]; numerical values for Δq vary between 0.1 and 0.3, depending on the choice of parameters.

In models with constituent quarks, the cranking technique allows for a direct estimation of Δq or, in the case of our two-flavour model, of $\Delta u + \Delta d$, as first observed in Ref. [21]. The mean angular momentum $\langle \vec{J} \rangle = \vec{J} \vec{\omega}$ generated by cranking may be broken down into quark spin, quark orbital and meson orbital components

$$\vec{J} \vec{\omega} = (\vec{J}_{qs} + \vec{J}_{qe} + \vec{J}_m) \vec{\omega} , \quad (6.5)$$

where the quark spin component is

$$\vec{J}_{qs} \vec{\omega} \equiv N \int d^3x \, q'^{\dagger} \frac{\vec{\sigma}}{2} q' . \quad (6.6)$$

Computing the right-hand side to first order in $\vec{\omega}$ with the ansatz (4.16), we obtain for a baryon

$$\begin{aligned} \vec{J}_{qs} &= 3 \int d^3x \left(Au + \frac{C+2D}{3} v \right) , \\ \vec{J}_{qe} &= 3 \int d^3x \left(-\frac{4}{3} D v \right) \end{aligned} \quad (6.7)$$

[$\vec{J}_m$ has been given in (4.25)].

As shown in Fig. 16, for $\alpha_s = 0$, the non-strange quark contribution to the proton spin

$$\Delta u + \Delta d = \frac{\vec{J}_{qs}}{\vec{J}} \quad (6.8)$$

increases with decreasing f , our preferred choice $f = f_c(m_\sigma)$ leading for

$m_\sigma = 1200$ MeV to a value $\Delta u + \Delta d = 0.60$, in poor agreement with (6.1). However, as shown in Fig. 17, for $m_\sigma = 600$ MeV, $f = f_c(m_\sigma)$ and $n = 0, 1, 2$ and 3 , $\Delta u + \Delta d$ decreases quickly with increasing n and α_s , demonstrating that the effect of exchanged gluons is crucial. The same is true for all values of m_σ , the $\alpha_s = 0$ value $(\Delta u + \Delta d)_0$ decreasing when m_σ increases (Fig. 18). If we now assume that the parameters α_s and n are constrained by (5.15) so that the relative nucleon-delta mass splitting attains its experimental value, we obtain, as shown in Fig. 19, a non-strange quark contribution which decreases not only with decreasing α_s (i.e., increasing n), but also with increasing m_σ . For $600 \text{ MeV} < m_\sigma < 800 \text{ MeV}$ and values of α_s which lie between 0.25 and 0.85 , as suggested by experiment (for $q \approx 1000 \text{ MeV}$) [61], we then obtain

$$0.21 \leq \Delta u + \Delta d \leq 0.32 \quad (6.9)$$

corresponding to the shaded area in Fig. 19. These values are in good agreement with the "experimental" value (6.4).

As discussed in Refs. [21] and [64], these results may actually be used to estimate the total quark helicity contribution $\Delta q = \Delta u + \Delta d + \Delta s$ and the gluonic helicity contribution Δg to the proton spin. First, it was shown [34] that, when the anomalous gluon contribution [65] to the flavour-singlet axial current matrix element is taken into account in the interpretation of the EMC experiment, Eq. (6.4) should be replaced by

$$\Delta u + \Delta d = 0.21 + 2 \frac{\alpha_s}{2\pi} \Delta g \pm 0.14 \quad (6.10)$$

Our results (6.9) may then be used to derive

$$-0.07 \leq \frac{\alpha_s}{2\pi} \Delta g \leq 0.13 \quad , \quad (6.11)$$

consistent with $\Delta g = 0$. Then, going over to the three-flavour world, we assume that, because $SU(2) \times SU(2)$ is a better symmetry than $SU(3) \times SU(3)$ [66], our two-flavour computations give substantially correct values for the non-strange contribution $\Delta u + \Delta d$ (as has been checked, for other observables, in Ref. [67]), while $SU(3) \times SU(3)$ symmetry breaking generates a non-negligible strange contribution Δs , for which the experimental value [32] is

$$\Delta s = -0.24 + \frac{\alpha_s}{2\pi} \Delta g \pm 0.07 \quad (6.12)$$

Combining this with the previous results, we obtain

$$-0.03 \leq \Delta q = \Delta u + \Delta d + \Delta s \leq 0.13, \quad (6.13)$$

consistent with $\Delta q = 0$. We conclude that all the proton spin is in fact due to orbital angular momentum (as speculated in Ref. [36]). To arrive at this conclusion, it was essential to take into account the effect of gluon exchange between quarks [37], which does not contribute to Δg [64], but strongly enhances the relative non-strange quark orbital contribution.

It would be worthwhile complementing this investigation by a more direct computation of Δs and Δg . This should involve the effect of polarized $s\bar{s}$ and $g\bar{g}$ pairs, which could be taken care of by incorporating vector mesons in an $SU(3) \times SU(3)$ extended version of our model.

7. - OTHER NUCLEON OBSERVABLES

To compute nucleon observables in the model, we should in principle project a suitably chosen state $|h\rangle$ [such as the coherent hedgehog (3.2)] on eigenstates $|N\rangle$ of spin-isospin with $j = \frac{1}{2}$, and calculate the expectation values $\langle N|\theta|N\rangle$ of the relevant field-theoretical operators θ . The first computations [10] used an approximate projection method, which amounts to assuming that the nucleon is a product of three quarks in spin-isospin eigenstates with radial wave functions u, v , while the mean pion field is the hedgehog field $\vec{\phi}_0$. The full projection was carried out in Ref. [16], using coherent hedgehogs, and in Ref. [19] using generalized (deformed) hedgehogs, but the results were given only for one set of parameter values. A more systematic survey was undertaken in Ref. [51] where, however, the coherent hedgehog ansatz was replaced by the coherent pair approximation, leading to quite different values for the nucleon observables. The application of these methods to the computation of gluonic effects on the nucleon observables does not seem feasible.

In the cranking approach [39,17], the quantum projection is replaced by a quantization of the collective rotations (4.5) in isospace. The expectation values $\langle \theta \rangle'_\omega$ in the cranked coherent hedgehog $|h'_\omega(t)\rangle$ are first computed in terms of the collective variables $B, \vec{\omega}$; these variables are then quantized so that the "requantized" $\hat{I} = \langle \vec{I} \rangle'_\omega$ and $\hat{J} = \langle \vec{J} \rangle'_\omega$ satisfy the commutation relations for isospin and angular momentum; collective eigenstates $|N\rangle$ of spin-isospin are then constructed in the resulting Hilbert space, and the expectation values $\langle N|\hat{\theta}|N\rangle$ of the requantized $\hat{\theta} = \langle \theta \rangle'_\omega$ are evaluated. The effect of gluon exchange on the nucleon observables may be investigated by this method. Those observables which are of zeroth order in ω , such as the isoscalar nucleon mean square charge radius $\langle r^2 \rangle_{I=0}$, the isovector nucleon magnetic moment $\mu_{I=1}$, the nucleon axial-vector coupling constant

constant $g_A(0)$, the pion-nucleon coupling constant $g_{\pi NN}(0)$ and the pion-nucleon sigma term $\sigma_{\pi N}(0)$, are unaffected by gluon exchange. Those observables which are of first order in ω , such as the isovector nucleon mean square charge radius $\langle r^2 \rangle_{I=1}$ and the isoscalar nucleon magnetic moment $\mu_{I=0}$, are affected by gluon exchange.

The values obtained for these observables by the three methods just described: approximate projection [10], coherent projection [19] and cranking [17] are compared in Table 2 for the set of parameters [19] $m_\sigma = 1200$ MeV, $f = 5.00$, $\alpha_s = 0$. The values for the quark contributions to $\mu_{I=1}$, $g_A(0)$ and $g_{\pi NN}(0)$ given by the cranking method differ by a factor 3/5 from those given by the approximate projection method; as shown in Ref. [68], this factor may be written $N_c/(N_c+2)$, where N_c is the number of colours, and goes over to 1 in the limit $N_c \rightarrow \infty$. The disagreement between the values given for these isospin-dependent observables by the cranking and the projection method implies that, in this case, the quantization of the collective variables is a poor substitute for the full projection. The cranking expressions for the other observables ($\langle r^2 \rangle_{I=0}$, $\langle r^2 \rangle_{I=1}^m$, $\mu_{I=0}$) do not depend on this requantization, and the resulting values are consistent with those given by coherent projection, except for $\langle r^2 \rangle_{I=1}^m$. In this last case, we think that the coherent projection computation, which involves a convolution transform (Eqs. (5.5) and (A.32) of Ref. [16]) and thus depends critically on the behaviour of the pion trial wave function, is unreliable; indeed, the value (0.42 fm for $m_\sigma = 1200$ MeV, $f = 5.00$, $\alpha_s = 0$) given for $\langle r^2 \rangle_{I=1}^m$ by the approximate generalized hedgehog projection [19] is 30% larger than the value (0.32 fm) given by the hedgehog projection [16,19], and it may be expected that a yet more general ansatz would further increase this value. As already mentioned in Section 5, the values given by the coherent projection and coherent cranking methods for the nucleon-delta mass difference are also in fairly good agreement (Table 1). We expect that coherent projection would also give for the quark helicity contribution $\Delta u + \Delta d$ to the proton spin values consistent with our cranking results.

Let us now discuss the variation with the model parameters of these various observables, as computed in the cranking approach. In Ref. [17], the $I = 0$ and $I = 1$ contributions to $\langle r^2 \rangle$ were given in the MFA as functions of q_0 (2.5) and q_1 (4.16). The resulting expressions are, in terms of the radial functions,

$$\langle r^2 \rangle_{I=0}^q = 4\pi \int_0^\infty dx x^4 (u^2 + v^2) \quad (7.1)$$

for the (cranking-independent) isoscalar quark part, and

$$\langle r^2 \rangle_{I=1} = \langle r^2 \rangle_{I=1}^q + \langle r^2 \rangle_{I=1}^m \quad (7.2)$$

for the (cranking-dependent) isovector part, where

$$\langle r^2 \rangle_{I=1}^q = \frac{12\pi}{J} \int_0^\infty dx \, x^4 [Au + (C-2D)\frac{v}{3}] \quad (7.3)$$

is the quark contribution and

$$\langle r^2 \rangle_{I=1}^m = \frac{8\pi}{3J} \int_0^\infty dx \, x^4 \phi_0^2 \quad (7.4)$$

is the meson contribution, J being the moment of inertia given by (4.24)-(4.25). The proton and neutron mean square charge radii are then

$$\langle r^2 \rangle_p^n = \frac{1}{2} (\langle r^2 \rangle_{I=0}^q \pm \langle r^2 \rangle_{I=1}^q) \quad (7.5)$$

These static values should be corrected for recoil. We use, for the quark contribution to the recoil-corrected mean square charge radius, the approximate formula, originally derived in a quantum-mechanical framework [52], and recovered in a projection-and-boost approach by neglecting quantum correlations and fluctuations [69,70],

$$\langle r^2 \rangle_c^q = \left(1 - \frac{2\varepsilon}{E_0} + \frac{3\varepsilon^2}{E_0^2} \right) \langle r^2 \rangle^q + \frac{3}{2E_0^2} \quad (7.6)$$

where ε is the quark eigenvalue and E_0 the uncorrected hedgehog energy given by (3.11); we assume that in the same approximation the meson contribution (7.4) is not recoil-corrected.

The resulting corrected isoscalar quark contribution to $\langle r^2 \rangle$ is plotted in Fig. 20 as a function of m_σ for $f = f_c(m_\sigma)$. Being rather large at small values of m_σ , it decreases when m_σ increases towards a value which lies a little above the experimental one. We note that the recoil corrections, which are generally rather important, reduce the mean square charge radius by an amount which increases with decreasing m_σ .

The dependence on the fine structure constant α_s and on the confining exponent n of the isovector parts of $\langle r^2 \rangle$ is displayed in Figs. 21 and 22 for $m_\sigma = 600$ MeV and $f = f_c(m_\sigma)$. The first one of these figures, which concerns the corrected quark contribution, shows that when gluons are unconfined ($n = 0$), the quark mean square charge radius increases with increasing α_s . More naturally, for confined gluons

($n \gtrsim 1$), $\langle r^2 \rangle_{I=1}^q$ shrinks when either α_s or the gluon confining exponent n are increased; it may even become negative for sufficiently large α_s , which is not a problem (only the sum of the quark and meson contributions is observable), but indicates a breakdown of the naive interpretation in terms of constituent quarks surrounded by a meson cloud. The meson contribution to $\langle r^2 \rangle_{I=1}$, displayed in Fig. 22, depends on cranking only through the factor $1/\mathfrak{J}$ so that, when $\mathfrak{J}$ decreases, i.e., when the quark distribution contracts, the meson charge distribution, on the contrary, spreads out. Because of this competition between the quark core contraction and the meson cloud expansion, the total recoil-corrected $I = 1$ contribution to the nucleon mean square charge radius, $\langle r^2 \rangle_{I=1}^c$, considered as a function of α_s , behaves differently for different values of n , as shown in Fig. 23. When $n = 0$ or 1 , that is when gluons are unconfined or only weakly confined, $\langle r^2 \rangle_{I=1}^c$ increases with increasing quark-gluon coupling α_s . For $n \gtrsim 2$, more realistically, $\langle r^2 \rangle_{I=1}^c$ is strongly reduced by gluon exchange; a similar decrease of the proton mean square charge radius with increasing α_s , when $n = 2$, was previously observed in a self-consistent treatment of gluon exchange in the Friedberg-Lee model [25]. In the case $n = 2$, for $\alpha_s = 0.4$, which is the value obtained by imposing (5.15), we obtain for the recoil-corrected proton mean square radius $\langle r^2 \rangle_{pc} = (0.84 \text{ fm})^2$ [the uncorrected value is $(0.91 \text{ fm})^2$], in good agreement with the experimental value $(0.80 \text{ fm})^2$; 58% of this value is due to quarks, and the rest to pions. All this holds qualitatively for all values of m_σ , the overall magnitudes being slightly reduced when m_σ is increased.

Let us now consider the magnetic moment. In Ref. [17] one may also find the derivation of the $I = 0$ and $I = 1$ contributions to μ in the MFA. The expressions obtained in terms of the radial functions are

$$\mu_{I=0}^q = \frac{4\pi}{3\mathfrak{J}} \int_0^\infty dx x^3 \left[(A + \frac{B}{3})v - Du \right] \quad (7.7)$$

for the cranking-dependent isoscalar quark part, and

$$\mu_{I=1} = \mu_{I=1}^q + \mu_{I=1}^m \quad (7.8)$$

for the cranking-independent isovector part, where

$$\mu_{I=1}^q = \frac{8\pi}{3} \int_0^\infty dx x^3 uv \quad (7.9)$$

is the quark contribution, and

$$\mu_{I=1}^m = \frac{8\pi}{9} \int_0^\infty dx x^2 \phi_o^2 \quad (7.10)$$

is the meson contribution. The proton and neutron magnetic moments are then given by

$$\mu_p = \frac{1}{2} (\mu_{I=0}^q \pm \mu_{I=1}) . \quad (7.11)$$

Here too the static quark contribution to the magnetic moment should in principle be recoil-corrected. Neglecting centre-of-mass effects, the recoil-corrected proton magnetic moment is given, in the case of a bag [71] or Friedberg-Lee soliton bag [69], by

$$\mu_{pc} = \left(1 - \frac{\varepsilon}{E_0}\right) \mu_p + \frac{3}{10E_0} g_A(0) , \quad (7.12)$$

where the first correction is the recoil correction proper, and the second correction is due to the spin precession. However, as made clear in Ref. [70], centre-of-mass effects cannot be neglected; moreover, it is not obvious that the precession term as given above is valid for cranked chiral soliton bags. Lacking a proper treatment of recoil corrections, we shall consider only uncorrected magnetic moments.

We show in Fig. 24 how the quark and meson contributions to the isovector part of μ behave with m_σ for $f = f_c(m_\sigma)$; the total $\mu_{I=1}$ increases only slightly, remaining far below its experimental value $4.70\mu_N$. In Fig. 25, we represent the dependence of the isoscalar quark contribution to the magnetic moment on n and α_s for $m_\sigma = 600$ MeV and $f = f_c(m_\sigma)$. We see that $\mu_{I=0}^q$ increases when either α_s or n is increased; this increase is similar to that observed for the proton magnetic moment in the Friedberg-Lee model treated self-consistently [25]. This behaviour holds for all values of m_σ , the overall magnitude of $\mu_{I=0}^q$ increasing slightly with increasing m_σ . If now the constraint (5.15) is imposed, surprisingly the values obtained for $\mu_{I=0}^q$ do not depend on n (or α_s), which is due to the fact that $\mu_{I=0}^q$ varies almost linearly with the nucleon-delta mass difference ΔE (compare Figs. 13 and 25). So for $m_\sigma = 600$ MeV we obtain (for any n) $\mu_p = 1.96 \mu_N$ and $\mu_n = -0.77 \mu_N$, the quark contribution being the most important (81% of μ_p and 52% of μ_n). We attribute the bad agreement of these values with the experimental ones ($2.79 \mu_N$ and $-1.91 \mu_N$ respectively) to the previously discussed fact that the collective co-ordinate quantization tends to underestimate $\mu_{I=1}$. For comparison, the approximate projection scheme [10], which amounts to multiplying the cranking value for $\mu_{I=1}^q$ by a factor of 5/3 (see also Ref. [7], where a similar correction is advocated in the case of the chiral bag model), would give the values $2.62 \mu_N$ for μ_p and $-1.43 \mu_N$ for μ_n .

Let us now discuss the observables which do not depend on cranking. In terms of the radial functions, the axial-vector coupling constant $g_A(0)$ is given in the MFA by [17]

$$g_A(0) = g_A^q + g_A^m, \quad (7.13)$$

where

$$g_A^q = 4\pi \int_0^\infty dx \, x^2 \left(u^2 - \frac{v^2}{3} \right) \quad (7.14)$$

is the quark contribution, and

$$g_A^m = \frac{8\pi}{9} \int_0^\infty dx \, x \left[2\sigma_0 \phi_0 + x \left(\sigma_0 \frac{d\phi_0}{dx} - \phi_0 \frac{d\sigma_0}{dx} \right) \right] \quad (7.15)$$

is the meson contribution (let us point out that commonly-quoted values of g_A^m are found to be somewhat reduced when one takes into account properly the negative asymptotic contribution to (7.15) [23]). We find that, while increasing with increasing f for a given m_σ , g_A also increases very slowly when m_σ is increased and $f = f_c(m_\sigma)$. This nearly constant behaviour of g_A as a function of m_σ (which is related to the effective quark confinement radius, see Fig. 20) is a "Cheshire cat"-like feature of our model [72]. For $m_\sigma = 600$ MeV, we obtain $g_A(0) = 1.09$ (the quarks contributing for 78%). A more precise projection calculation of $g_A(0)$, which is somewhat underestimated by the cranking computation, should improve the agreement with the experimental value 1.25.

As for the pion-nucleon coupling constant $g_{\pi NN}(0)$, it is given in terms of the MFA radial functions by [17]

$$g_{\pi NN}(0) = g_{\pi NN}^q + g_{\pi NN}^m, \quad (7.16)$$

where

$$\frac{g_{\pi NN}^q}{2m_N} = \frac{8\pi}{3} f \int_0^\infty dx \, x^3 uv \quad (7.17)$$

is the quark contribution, and

$$\frac{g_{\pi NN}^m}{2m_N} = -\frac{4\pi}{9} \lambda^2 \int_0^\infty dx \, x^3 \phi_0 (\sigma_0^2 + \phi_0^2 - F_\pi^2) \quad (7.18)$$

is the meson contribution.

The Goldberger-Treiman relation

$$g_{\pi NN}(0) \frac{m_\pi}{2m_N} = g_A(0) \frac{m_\pi}{2F_\pi} \quad (7.19)$$

is exact for the linear σ -model in the MFA, so that the value of the quantity $(g_{\pi NN}(0)m_\pi/2m_N)/(g_A(0)m_\pi/2F_\pi) - 1$ computed with formulae (7.13)-(7.18) may be taken as a measure of the precision of our numerical computations. The values we obtain range from $2 \cdot 10^{-6}$ for $m_\sigma = 600$ MeV to $-5 \cdot 10^{-6}$ for $m_\sigma = 2000$ MeV. This is very satisfactory and shows that our numerical errors really are negligible.

If we now want to compare $g_{\pi NN}$ with experiment, we need to compute it at $q^2 = -m_\pi^2$. This we do [39,17] by using the large-distance behaviour of the ϕ_0 field

$$\phi_0(R) = a(m_\pi + \frac{1}{R}) \frac{e^{-m_\pi R}}{R}, \quad (7.20)$$

R being some cut-off radius and a a constant which can be numerically determined and in terms of which we may write

$$g_{\pi NN}(-m_\pi^2) = \frac{8\pi}{3} a m_N. \quad (7.21)$$

The value of $\Delta = 1 - g_{\pi NN}(0)/g_{\pi NN}(-m_\pi^2)$ provides us with a measure of the observed violation of the Goldberger-Treiman relation. Experimentally, $\Delta = 0.08 \pm 0.02$ [66]. We find that, for $m_\sigma = 600$ MeV, Δ is equal to 0.09, a value which slightly decreases when m_σ increases, being equal to 0.06 for $m_\sigma = 2000$ MeV, still in good agreement with experiment. The values we obtain for the pion-nucleon coupling constant on mass-shell for $f = f_c(m_\sigma)$ increase slightly with increasing m_σ , the value $g_{\pi NN}(-m_\pi^2) = 12.1$ for $m_\sigma = 600$ MeV being rather good compared to the experimental value 13.5.

Finally, we compute the pion-nucleon sigma term $\sigma_{\pi N}(0)$. This term, which is a measure of the nucleon mass shift following from dynamical breaking of chiral symmetry, is defined by [73]

$$\sigma_{\pi N}(0) = \langle N | \Sigma_{\pi N} | N \rangle - \langle 0 | \Sigma_{\pi N} | 0 \rangle, \quad (7.22)$$

where

$$\Sigma_{\pi N} = \frac{1}{3} \sum_{\alpha=1}^3 [Q_5^\alpha, [Q_5^\alpha, H_{SB}]] \quad (7.23)$$

is the isospin-averaged double commutator of the axial charge Q_5^α with the symmetry-breaking term of the Hamiltonian

$$H_{SB} = -F_\pi m_\pi^2 \int d^3x \sigma \quad (7.24)$$

In terms of the radial functions, we thus have in the MFA

$$\sigma_{\pi N}(0) = 4\pi F_\pi m_\pi^2 \int_0^\infty dx \, x^2 (F_\pi - \sigma) \quad (7.25)$$

We find that, for a given value of m_σ , $\sigma_{\pi N}(0)$ increases quickly with increasing f , and we obtain, for $f = f_c(m_\sigma)$, $\sigma_{\pi N}(0) = 92$ MeV for $m_\sigma = 600$ MeV, decreasing to 72 MeV for $m_\sigma = 800$ MeV and 51 MeV for $m_\sigma = 2000$ MeV. Data from πN scattering amplitudes, analytically continued to the Cheng-Dashen point ($q^2 = 2m_\pi^2$) [74] and thereafter extrapolated to the unphysical point ($q^2 = 0$) using chiral perturbation theory [75], suggest that [76] $\sigma_{\pi N}(0) \sim (60 \pm 10)$ MeV. This value, which does not agree with the value $\sigma_{\pi N}(0) \approx 35$ MeV obtained in perturbative QCD by giving masses to the quarks [76], is in good agreement with our results, showing the importance of taking into account non-perturbative effects in the computation of the pion-nucleon sigma term.

8. - CONCLUSION

In this paper, we have studied the effect of gluon exchange in the chiral soliton bag model, treated self-consistently by the cranking method, and approximately corrected for spurious translational and rotational fluctuations. While the variation of the baryonic moment of inertia $\mathcal{J}$ with α_s is definitely non-linear (Fig. 8), the other observables we have studied - with the notable exception of the isovector nucleon mean square charge radius (Fig. 23) - vary quasi-linearly with α_s . For realistic values of the effective strong fine structure constant α_s (≈ 0.5) and of the confining exponent n (≈ 2), the effect of gluon exchange is strong. For instance, for $m_\sigma = 600$ MeV, $f = 4.05$, $n = 2$, $\alpha_s = 0.4$, gluon exchange, which accounts for 70% of the nucleon-delta mass difference, reduces the non-strange quark helicity contribution to the proton spin to 33% of the $\alpha_s = 0$ value, the isovector nucleon mean square charge radius to 50% of the $\alpha_s = 0$ value, and increases the isoscalar nucleon magnetic moment by 50%. The resulting values of the baryonic observables we computed are summarized in Table 3 for this parameter set, as well as for the parameter set $m_\sigma = 800$ MeV, $f = 4.05$, $n = 3/2$, $\alpha_s = 0.56$,

which leads to better values for the neutron mean square charge radius, the isoscalar nucleon magnetic moment and the pion-nucleon sigma term. All in all, our results are in fairly good agreement with experiment, except for the (α_s -independent) isovector nucleon magnetic moment, which for the second parameter set is only 61% of its experimental value (but this failure is essentially due to the use of collective co-ordinate quantization). These two parameter sets have been given only as examples. Indeed, when α_s and n are constrained so that the relative nucleon-delta mass splitting attains its experimental value, the various other baryonic observables do not depend much on the other model parameters, except for the mean nucleon-delta mass, which decreases quickly with increasing f (Fig. 4), thus favouring low values of $f \approx f_c(m_\sigma)$, and the isoscalar nucleon mean square charge radius (Fig. 20) and the pion-nucleon sigma term, which both vary quickly with m_σ , favouring values of $m_\sigma \gtrsim 800$ MeV (however, within our approximations, the delta is unstable for $m_\sigma < 800$ MeV, so that the value of 800 MeV is a compromise).

Such a self-consistent treatment of gluon exchange has not yet been carried out for other chiral models of confinement. The effect of perturbative gluon exchange on baryonic observables in the MIT bag model was studied recently [77,37]; however, the value of $\alpha_s \approx 2.2$ used in these studies is both unrealistic and too large to allow a perturbative treatment. In the hybrid chiral bag model, gluon exchange effects are expected to be important at large bag radii [7], but to date only the effect of gluon exchange on the nucleon-delta mass difference has been treated perturbatively [6]. It should also be stressed that another important feature of our study, the (approximate) correction of baryonic energies for translational and rotational fluctuations, has yet to be carried out in the case of the Skyrme or hybrid models (the necessity of correcting for spurious rotational fluctuations in the Skyrme model was recently pointed out by other authors [19,78]).

A crucial prediction of our study is the value of the contribution of the non-strange quark helicities to the spin of the proton, for which we obtained the mean value $\Delta u + \Delta d \approx 0.26$, consistent with experimental data, and we concluded that this meant that the contributions Δq and Δg of the quark and gluon helicities to the proton spin vanish. After this [64] and previously-mentioned model investigations [31-33,37], the size of these contributions was also investigated very recently [79-81], using chiral bag models. In Ref. [79], the flavour singlet axial matrix element (6.1) was saturated by the axial anomaly, leading for $\alpha_s \approx 1.5$ to a small, but non-zero, value $\Delta q \approx 0.29$. In another recent work [80,81], $\Delta u + \Delta d$ was estimated in the hybrid chiral bag model to be of the order of 1/4 for the special value of the chiral angle $\theta = \pi/2$, corresponding to a vanishing orbital contribution from the valence quarks; it would be worthwhile to extend this study to the case of

other values of the chiral angle θ . In our model, more realistically, the orbital angular momentum of the valence quarks accounts for roughly one half of the proton's spin.

Our successful prediction $\Delta q \approx 0$, together with the good results we have obtained for other baryonic observables, taking carefully into account corrections for translational and rotational fluctuations and gluon exchange, and using realistic values of the model parameters m_π , F_π and α_s , show that the chiral soliton bag model of quark and gluon confinement indeed qualifies as a viable effective model of baryonic structure.

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APPENDIX A

ASYMPTOTIC BEHAVIOURS OF THE FIRST-ORDER MEAN FIELDS

To derive these, we neglect in the system (4.17) the functions $\sigma_v - \sigma_o(r)$ and $\phi_o(r)$, which vanish exponentially [Eqs. (2.11) and (2.12)], and use the asymptotic behaviours (2.10) for $u(r)$ and $v(r)$, and those of the dielectric function

$$K_o(x) \sim x^{-2n} e^{-2\gamma x}, \quad \gamma = n\beta, \quad \text{if } \beta < \inf(m_\pi, \frac{m_\sigma}{2}), \quad (\text{A.1a})$$

$$\gamma = nm_\pi, \quad \text{if } m_\pi < \inf(\frac{m_\sigma}{2}, \beta), \quad (\text{A.1b})$$

$$K_o(x) \sim x^{-n} e^{-2\gamma x}, \quad \gamma = n\frac{m_\sigma}{2}, \quad \text{if } \frac{m_\sigma}{2} < \inf(m_\pi, \beta). \quad (\text{A.1c})$$

For $\gamma < \beta$, assuming that the chromomagnetic potential $A(r)$ vanishes at infinity, we obtain from the first four equations (4.17) the asymptotic behaviours

$$\begin{aligned} A(x) - \frac{2B(x)}{3} &\simeq \left[\frac{\epsilon}{2\beta} x + a + O\left(\frac{1}{x}\right) \right] u(x), \\ A(x) + \frac{B(x)}{3} &\simeq \left[\frac{\epsilon}{2\beta} x + b + O\left(\frac{1}{x}\right) \right] u(x), \\ C(x) &\simeq \left[\frac{\epsilon}{2\beta} x + c + O\left(\frac{1}{x}\right) \right] v(x), \\ D(x) &\simeq \left[-\frac{\epsilon}{2\beta} x + d + O\left(\frac{1}{x}\right) \right] v(x), \end{aligned} \quad (\text{A.2})$$

where the constants a, b, c, d are related by

$$a - c = b + d = \frac{f\sigma_v}{2\beta^2}, \quad (\text{A.3})$$

leading to the asymptotic relations (which were used in the numerical calculations as boundary conditions for some large cut-off radius)

$$\begin{aligned} \frac{C-D}{2A-B/3} &\simeq \frac{f\sigma_v - \epsilon}{\beta} \left(1 - \frac{f\sigma_v - \epsilon}{\beta\epsilon x} \right), \\ \frac{C+D}{B} &\simeq -\frac{f\sigma_v - \epsilon}{\beta} \left(1 - \frac{2}{\beta x} \right) \end{aligned} \quad (\text{A.4})$$

[this last equation incorporates a relation between the terms in $1/r$ in Eqs. (A.2)]. Feeding these asymptotic behaviours in the last equation (4.17), we obtain the asymptotic behaviours of the chromomagnetic potential

$$\begin{aligned} A(x) &\simeq x^{2n-1} e^{2(\gamma-\beta)x} && \text{in cases a or b,} \\ A(x) &\simeq x^{n-1} e^{2(\gamma-\beta)x} && \text{in case c.} \end{aligned} \quad (A.5)$$

For $\gamma > \beta$ the above derivation leads to an inconsistency [$A(r)$ does not vanish at infinity]. However, the asymptotic study of the first four coupled equations (4.17) indicates that, at any rate, the first-order wave functions A, B, C, D cannot decrease faster at infinity than $ru(r)$; this does not lead to a self-consistent asymptotic solution of the full system unless the dominant asymptotic behaviour of the chromomagnetic current $(A+B/3)v - Du$ is suppressed. Making this hypothesis, we obtain from the first four equations (4.17) the asymptotic behaviours

$$\begin{aligned} A(x) - \frac{2B(x)}{3} &\simeq \left[-\frac{\varepsilon}{2\beta} x + a + O\left(\frac{1}{x}\right) \right] u(x), \\ A(x) + \frac{B(x)}{3} &\simeq \left[-\frac{\varepsilon}{2\beta} x + \frac{f\sigma_v + 2\varepsilon}{4\beta^2} + O\left(\frac{1}{x}\right) \right] u(x), \\ C(x) &\simeq \left[-\frac{\varepsilon}{2\beta} x + a - \frac{f\sigma_v - 2\varepsilon}{2\beta^2} + O\left(\frac{1}{x}\right) \right] v(x), \\ D(x) &\simeq \left[-\frac{\varepsilon}{2\beta} x + \frac{f\sigma_v + 2\varepsilon}{4\beta^2} + O\left(\frac{1}{x}\right) \right] v(x), \\ \frac{16\pi}{3} \alpha_s A(x) &\simeq \varepsilon \left[x - \frac{1}{\beta} + O\left(\frac{1}{x}\right) \right], \end{aligned} \quad (A.6)$$

where a is an unknown constant. No asymptotic information can be extracted from the last equation (4.17), where the left-hand side is of the order of $e^{-2\gamma r}$, while it is only known that, to any order p of the expansion (A.6) in powers of $1/r$, the right-hand side vanishes faster than $r^{-p-2} e^{-2\beta r}$. The asymptotic relations which were used in the numerical calculations are

$$\frac{D}{A+B/3} \simeq \frac{v}{u}, \quad \frac{C}{A-2B/3} \simeq \frac{f\sigma_v - \varepsilon}{\beta} \left(1 + \frac{f\sigma_v - \varepsilon}{\beta \varepsilon x} \right). \quad (A.7)$$

APPENDIX B

LINEARIZATION IN α_s OF THE MOMENT OF INERTIA

The Euler equations for the extremization of $E_{2q} + E_{2g}$ [Eqs. (4.12) and (4.14)] may be written

$$\begin{aligned} (\hbar_0 - \varepsilon) q_1 + \hat{\omega} \cdot \frac{\vec{\tau}}{2} q_0 - \alpha_s \eta \vec{\alpha} \cdot \vec{A} q_0 &= 0, \\ \vec{\nabla}_\Lambda (\kappa_0 \vec{\nabla}_\Lambda \vec{A}) &= q_1^+ \vec{\alpha} \frac{q_0}{2} + q_0^+ \vec{\alpha} \frac{q_1}{2}, \end{aligned} \quad (B.1)$$

where we have put

$$\begin{aligned} \vec{A} &\equiv \alpha \hat{\omega}_\Lambda \hat{\kappa}, \\ \eta &\equiv \frac{4\pi}{N} \sum_a \sum_{i \neq j} \langle \frac{\lambda^a}{2} \rangle_i \langle \frac{\lambda^a}{2} \rangle_j \end{aligned} \quad (B.2)$$

[$\eta = -(16\pi/3)$ for a baryon]. The system (B.1) may be treated perturbatively by expanding the mean fields $q_1, \vec{A}$ in powers of $\alpha_s^{\frac{1}{2}}$:

$$\begin{aligned} q_1 &= q_{10} + \alpha_s q_{12} + \dots, \\ \vec{A} &= \vec{A}_0 + \alpha_s \vec{A}_2 + \dots, \end{aligned} \quad (B.3)$$

leading to the sequence of equations

$$\begin{aligned} (\hbar_0 - \varepsilon) q_{10} + \hat{\omega} \cdot \frac{\vec{\tau}}{2} q_0 &= 0, \\ (\hbar_0 - \varepsilon) q_{12} - \eta \vec{\alpha} \cdot \vec{A}_0 q_0 &= 0, \end{aligned} \quad (B.4)$$

etc., and

$$\vec{\nabla}_\Lambda (\kappa_0 \vec{\nabla}_\Lambda \vec{A}_0) = q_{10}^+ \vec{\alpha} q_0 + q_0^+ \vec{\alpha} q_{10}, \quad (B.5)$$

etc. The moment of inertia (4.24) may similarly be expanded:

$$J = J_0 + \alpha_s J_2 + \dots, \quad (B.6)$$

where

$$\mathcal{J}_0 = -N \int d^3x (q_{10}^+ \hat{\omega} \cdot \vec{\mathcal{E}}_2 q_0 + q_0^+ \hat{\omega} \cdot \vec{\mathcal{E}}_2 q_{10}) + \int d^3x \frac{2}{3} \phi_0^2 \quad (\text{B.7})$$

is the zeroth-order moment of inertia, and

$$\begin{aligned} \mathcal{J}_2 &= -N \int d^3x (q_{12}^+ \hat{\omega} \cdot \vec{\mathcal{E}}_2 q_0 + q_0^+ \hat{\omega} \cdot \vec{\mathcal{E}}_2 q_{12}) \\ &= N\eta \int d^3x (q_0^+ \vec{\alpha} q_{10} + q_{10}^+ \vec{\alpha} q_0) \cdot \vec{A}_0, \end{aligned} \quad (\text{B.8})$$

after using Eqs. (B.4) and the hermiticity of the operator $(h_0 - \epsilon)$. This result may be compared with that obtained in Ref. [22] by inserting the expansion (B.6) in the Eq. (5.2) for the energy

$$E = E_0 + \frac{\langle \vec{\mathcal{I}} \rangle^2}{2\mathcal{J}_0} (1 - \alpha_s \frac{\mathcal{J}_2}{\mathcal{J}_0} + \dots), \quad (\text{B.9})$$

leading to the identification

$$E_{2g_0} = -\frac{1}{2} \alpha_s \mathcal{J}_2, \quad (\text{B.10})$$

which follows here from Eqs. (4.14), (B.5) and (B.8).

APPENDIX C

SECOND-ORDER MEAN SQUARE ANGULAR MOMENTUM FLUCTUATIONS

As mentioned in Section 5, cranking destroys the precise hedgehog correlation between spin and isospin, so that the cranked hedgehog $|h'\rangle$ is no longer annihilated by the grand spin operator $\vec{J} + \vec{I}$. To show this, it is sufficient to compute the action of the corresponding Dirac operator on the first-order quark wave function $q_1 = \hat{\omega} \cdot \vec{\xi}$:

$$(\vec{L} + \frac{\vec{\sigma}}{2} + \frac{\vec{\tau}}{2}) q_1 = i \hat{\omega} \cdot \vec{\xi} , \quad (C.1)$$

which is orthogonal to q_1 , but non-zero. It follows that the cranked mean square angular momentum $\langle \vec{J}^2 \rangle$ and mean square isospin $\langle \vec{I}^2 \rangle$ are no longer equal. To order ω^2 , their difference comes only from the quark contribution

$$\langle \vec{J}^2 - \vec{I}^2 \rangle = N \omega^2 \int d^3x q_1^\dagger [(\vec{\sigma} \cdot \vec{L} + 1)^2 - 1] q_1 . \quad (C.2)$$

Using

$$\int d^3x q_1^\dagger q_1 = \int d^3x \left[A^2 + \frac{2}{9} B^2 + \frac{1}{3} C^2 + \frac{2}{3} D^2 \right] , \quad (C.3)$$

and

$$(\vec{\sigma} \cdot \vec{L} + 1) q_1 (A, B, C, D) = q_1 (A, -2B, C+2D, C) , \quad (C.4)$$

we obtain

$$\langle \vec{J}^2 - \vec{I}^2 \rangle = \frac{2}{3} N \omega^2 \int d^3x \left[B^2 + (C+D)^2 \right] . \quad (C.5)$$

Table 1

The nucleon-delta mass difference (in MeV) as obtained by cranking, coherent cranking, generalized hedgehog projection and hedgehog projection for $m_\sigma = 1200$ MeV, $f = 5.38$ and $\alpha_s = 0$.

	Cranking [17]	Coh. Cranking	GH Proj.[19]	HH Proj. [16]
ΔE	252	198	160	152

Table 2

The quark isoscalar and isovector and the meson isovector contributions to the nucleon mean square charge radius (in fm^2), the quark and meson isovector and the quark isoscalar contributions to the nucleon magnetic moment (in μ_N), the quark and meson contributions to the nucleon axial-vector coupling constant, and the pion-nucleon coupling constant, as obtained by approximate projection, generalized hedgehog projection and cranking for $m_\sigma = 1200$ MeV, $f = 5$ and $\alpha_s = 0$.

	Appr. Proj. [10]	GH Proj. [19]	Cranking
$\langle r^2 \rangle_{I=0}^q$	0.51	0.54	0.51
$\langle r^2 \rangle_{I=1}^q$	0.51	0.32	0.30
$\langle r^2 \rangle_{I=1}^m$	0.90	0.42	0.90
$\mu_{I=1}^q$	2.92	2.51	1.75
$\mu_{I=1}^m$	2.13	2.64	2.13
$\mu_{I=0}^q$	0.58	0.36	0.43
$g_A^q(0)$	1.15	0.96	0.69
$g_A^m(0)$	0.64	0.79	0.64
$g_{\pi NN}(0) \frac{m_\pi}{2m_N}$	1.48	1.30	1.04

Table 3

Theoretical versus experimental values of various baryonic observables for the parameter values $m_\sigma = 600$ MeV, $f = 4.05$, $n = 2$, $\alpha_s = 0.4$ and $m_\sigma = 800$ MeV, $f = 4.05$, $n = 3/2$, $\alpha_s = 0.56$.

	$m_\sigma = 600$ MeV; $f = 4.05$ $n = 2$; $\alpha_s = 0.4$	$m_\sigma = 800$ MeV; $f = 4.05$ $n = 3/2$; $\alpha_s = 0.56$	Experiment
E_N	867 MeV	834 MeV	939 MeV
E_Δ	1137 MeV	1095 MeV	1232 MeV
$\Delta u + \Delta d$	0.26	0.26	0.21 ± 0.14
$\langle r^2 \rangle_{pc}$	0.71 fm^2	0.83 fm^2	0.64 fm^2
$\langle r^2 \rangle_{nc}$	0.13 fm^2	-0.16 fm^2	-0.13 fm^2
μ_p	$1.96 \mu_N$	$1.91 \mu_N$	$2.79 \mu_N$
μ_n	$-0.77 \mu_N$	$-0.99 \mu_N$	$-1.91 \mu_N$
$g_A(0)$	1.09	1.13	1.25
$g_{\pi NN}(0)$	12.11	12.24	13.45
$\sigma_{\pi N}(0)$	92 MeV	72 MeV	$(60 \pm 10) \text{ MeV}$

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FIGURE CAPTIONS

- Fig. 1 : The critical value f_c of the quark-meson coupling constant as a function of m_σ .
- Fig. 2 : The critical value $\bar{\epsilon}_c$ of the scaled quark energy as a function of f_c ($\bar{\epsilon}_c$ increases with decreasing m_σ).
- Fig. 3 : The average number of pions N_0 in the coherent hedgehog as a function of the quark-meson coupling constant f for $m_\sigma = 1200$ MeV.
- Fig. 4 : The hedgehog energy E_0 , the quark and coherent pion r.m.s. momenta P_q and P_π , and the resulting corrected hedgehog mass M as functions of f for $m_\sigma = 1200$ MeV.
- Fig. 5 : The same quantities as in Fig. 4 as functions of m_σ for $f = f_c(m_\sigma)$.
- Fig. 6 : The average number of pions N_0 in the coherent hedgehog as a function of m_σ for $f = f_c(m_\sigma)$.
- Fig. 7 : The dielectric function $\kappa(r)$ for $m_\sigma = 600$ MeV, $f = 4.05$ and the confining exponent $n = 2$.
- Fig. 8 : The moment of inertia $\mathcal{J}$ as a function of the effective strong fine structure constant α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n ; $\mathcal{J}_m$ is the meson contribution to $\mathcal{J}$.
- Fig. 9 : The $\alpha_s = 0$ moment of inertia $\mathcal{J}_0$ and its meson component $\mathcal{J}_m$ as functions of m_σ for $f = f_c(m_\sigma)$.
- Fig. 10 : The $\alpha_s = 0$ moment of inertia $\mathcal{J}_0$ and its meson component $\mathcal{J}_m$ as functions of f for $m_\sigma = 1200$ MeV.
- Fig. 11 : The $\alpha_s = 0$ nucleon-delta mass difference ΔE_π as a function of f for $m_\sigma = 1200$ MeV.
- Fig. 12 : The $\alpha_s = 0$ nucleon-delta mass difference ΔE_π as a function of m_σ for $f = f_c(m_\sigma)$.
- Fig. 13 : The nucleon-delta mass difference ΔE as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n ; ΔE_π is the $\alpha_s = 0$ contribution to ΔE .

- Fig. 14 : The effective strong fine structure constant α_s resulting from Eq. (5.15) as a function of the confining exponent n for various values of m_σ and $f = f_c(m_\sigma)$.
- Fig. 15 : The (fully quantum-corrected) nucleon and delta masses E_N and E_Δ as functions of m_σ for $f = f_c(m_\sigma)$, α_s and n being constrained by Eq. (5.15); $f\sigma_v$ is the free quark mass.
- Fig. 16 : The $\alpha_s = 0$ non-strange quark helicity contribution to the proton spin $(\Delta u + \Delta d)_0$ as a function of f for $m_\sigma = 1200$ MeV.
- Fig. 17 : The non-strange quark helicity contribution to the proton spin $\Delta u + \Delta d$ as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n .
- Fig. 18 : The $\alpha_s = 0$ non-strange quark helicity contribution to the proton spin $(\Delta u + \Delta d)_0$ as a function of m_σ for $f = f_c(m_\sigma)$.
- Fig. 19 : The non-strange quark helicity contribution to the proton spin $\Delta u + \Delta d$ as a function of α_s for various values of m_σ and $f = f_c(m_\sigma)$ (solid lines) and for various values of n (dashed lines), α_s , n and m_σ being related by Eq. (5.15); the shaded area corresponds to $600 \text{ MeV} < m_\sigma < 800 \text{ MeV}$ and $0.25 < \alpha_s < 0.85$.
- Fig. 20 : The recoil-corrected isoscalar quark contribution to the nucleon mean square charge radius $\langle r^2 \rangle_{I=0}^q$ as a function of m_σ for $f = f_c(m_\sigma)$.
- Fig. 21 : The recoil-corrected isovector quark contribution to the nucleon mean square charge radius $\langle r^2 \rangle_{I=1}^q$ as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n .
- Fig. 22 : The isovector meson contribution to the nucleon mean square charge radius $\langle r^2 \rangle_{I=1}^m$ as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n .
- Fig. 23 : The recoil-corrected isovector part of the nucleon mean square charge radius $\langle r^2 \rangle_{I=1}$ as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n .
- Fig. 24 : The isovector part of the nucleon magnetic moment $\mu_{I=1}$ as a function of m_σ for $f = f_c(m_\sigma)$; $\mu_{I=1}^q$ and $\mu_{I=1}^m$ are the quark and meson contributions to $\mu_{I=1}$.

Fig. 25 : The isoscalar quark contribution to the nucleon magnetic moment $\mu_{I=0}^q$ as a function of α_s for $m_\sigma = 600$ MeV, $f = 4.05$ and various values of n .

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