CERN-TH.5317/89

LAPP-TH-239/89

March 1989

INTERMITTENCY AND PARTICLE CORRELATIONS IN VERY SMALL RAPIDITY INTERVALS

J. Dias de Deus* and J.C. Seixas^{†+}

Theory Division, CERN,
CH-1211 Geneva 23, Switzerland.

ABSTRACT

We argue that the intermittency exponents f_i , characteristic of the dependence of the normalised factorial moments F_i on the bin rapidity δy , are δy and overall rapidity Y dependent and that, approximatively, they satisfy the scaling $f_i(\delta y, Y) \rightarrow \varphi_i(\eta \equiv \delta y Y)$ in the limits $\delta y \rightarrow 0$, $Y \rightarrow \infty$.

* Present address: CFMC/INIC, Av. Prof. Gama Pinto 2, P-1699 Lisboa Codex

[†] On leave of absence from CFMC/INIC Lisboa, Portugal

⁺ Address after March 1st 1989: LAPP/Annecy, France

There is now increasing evidence for intermittency in the rapidity distributions of particles produced in high energy interactions¹⁻³⁾. The observation in various experiments of large concentrations of particles in small rapidity bins^{4,5)} was a first indication for the possibility of intermittency in multiparticle production. Different recent analyses of particle correlations in small rapidity intervals^{6,7)}, making use of the factorial moments, have confirmed the presence of intermittency patterns suggestive of a fractal structure of the particle aggregates in rapidity.

It is clear that the observed intermittency, emphasizing the essential role of very short range rapidity correlations, is not related to the conventional, resonance dominated, short range correlations. The correlations measured in intermittency studies are in rapidity intervals orders of magnitude smaller than the resonance correlations. Most probably, intermittency is revealing the dynamics of collimated QCD evolution and branching with degradation of energy and momentum of off-shell partons^{8,9)}.

It has been argued¹⁾ that the convenient quantities to measure intermittency in rapidity distributions are the normalised factorial moments F_i , $i=1,2,\dots$,

$$F_i |_{\delta y} \equiv \frac{\langle n(n-1)\dots(n-i+1) \rangle}{\langle n \rangle^i} |_{\delta y} \quad , \quad (1)$$

where n is the number of particles in a rapidity bin δy and $\langle \dots \rangle$ means averaging over events. Here, following Ref.[10] and using high energy asymptotic translation invariance, we are simply looking at a central rapidity bin, for instance, at $y=0$. The factorial moments $F_i |_{\delta y}$ are non-negative quantities related to the inclusive probability of having i particles in δy , and are expected to be insensitive to statistical fluctuations. As a way of determining the strength of intermittency at a given overall rapidity $Y=\ln\sqrt{s}$, it was proposed¹⁾ to study the anticipated linear dependence of $\ln F_i$ on $\ln \delta y$,

$$\ln F_i \stackrel{\delta y \rightarrow 0}{\underset{Y=\ln\sqrt{s}}{\sim}} -f_i \ln \delta y \quad (2)$$

the f_i being constant and $f_i > 0$ in the case of intermittency.

If all that exists is a limiting rapidity scale Δy for short distance correlations, the one supplied by conventional resonances, then for studies with rapidity bins with $\delta y < \Delta y$ no dependence of the F_i on δy is to be observed and no intermittency occurs. However, correlations arise from evolution and branching, and these correlations have no small distance

limiting scale if infinite time is allowed for evolution, i.e., in the $Y \rightarrow \infty$ limit. Then a fully developed fractal structure may be revealed. At finite energy, branching and evolution have to stop and intermittency, with the definitions (2), strictly speaking cannot be true at finite overall rapidity Y .

In this note we thus question the validity of the fixed energy fits (2) and the attempts to extract δy independent exponent parameters f_i . At finite Y , as the branching evolution is cut-off, we claim that necessarily

$$\lim_{Y \xrightarrow[\delta y \rightarrow 0]{const.} \infty} f_i = 0 \quad (3).$$

The short distance finite energy cut-off is, most naturally, of the order of $1/Y$.

Let us introduce the inclusive probability $P_i(y_1, y_2, \dots, y_i; Y)$ of producing i particles with rapidities $y_1, y_2, \dots, y_i$ at the overall rapidity Y . For a rapidity bin δy , the factorial moments $\tilde{F}_i$ are defined by,

$$\tilde{F}_i |_{\delta y} = \int_{\delta y} P_i(y_1, y_2, \dots, y_i; Y) dy_1 \dots dy_i \quad (4)$$

where the $\tilde{F}_i$ are related to the F_i defined in (1) by

$$F_i \equiv \tilde{F}_i / F_1^i \quad (5)$$

Around $y=0$, the $P_i(y_1, y_2, \dots, y_i; Y)$ should only depend on $(i-1)$ y variables (asymptotic translation invariance). For instance, for $i=2$ the positive correlations generated by evolution and branching can be parametrized in the conventional way:

$$P_2(y_1, y_2; Y) = C_2(Y) \exp[-(y/\xi_2(Y))^2] \quad (6)$$

with $y = y_1 - y_2$ and $\xi_2(Y)$ denoting the correlation length. $P_1(y; Y)$ in the conditions we are considering is approximately constant. Using (6) and (5) we then obtain for the normalised factorial second moment,

$$F_2(\delta y, Y) = K_2(Y) \Phi(\delta y / \xi_2(Y)) / \delta y \quad (7)$$

where $\Phi(x)$ is the error function. Extending now the parametrization (6) to all P_i and introducing for each i an effective correlation length ξ_i controlling each of the $(i - 1)$ integrations we obtain, in general,

$$F_i(\delta y, Y) = K_i(Y) [\Phi(\delta y / \xi_i(Y)) / \delta y]^{(i-1)} \quad (8)$$

Our previous finite evolution and branching argument at finite Y now means that at finite Y the correlation lengths $\xi_i(Y)$ in (8) are finite.

From (8) we can compute the intermittency exponents f_i :

$$f_i \equiv -\frac{\partial \ln F_i}{\partial \ln \delta y} = -(i-1) \left[\frac{\partial \ln \Phi(\delta y / \xi_i)}{\partial \ln \delta y} - 1 \right] \quad (9)$$

The f_i from (9) are positive but δy dependent, and in the limit $\delta y \rightarrow 0$, since $\Phi(x) \sim x$ as $x \rightarrow 0$, $f_i \rightarrow 0$. Concerning the energy dependence of the f_i in (9), it is immediately seen that the f_i are scaling functions of the $\delta y / \xi_i$. Using the finite evolution and branching argument, one expects for the relation between ξ_i^0 and ξ_i corresponding to overall rapidity intervals Y^0 and Y

$$\xi_i = \xi_i^0 \frac{Y^0}{Y} \quad (10)$$

and

$$f_i(\delta y, Y) \xrightarrow[Y \rightarrow \infty]{\delta y \rightarrow 0} \varphi_i(\eta \equiv \delta y Y) \quad (11)$$

with $\varphi_i(0) = 0$.

In Fig. 1 we have compared the two parameter formula (8) with the NA22 $\sqrt{s}=22$ GeV data⁷⁾ in small rapidity intervals. The values of the parameters K_i and ξ_i , $i = 2, 3, 4, 5$ are given in the table. Our curves describe better the behaviour of the data than the two parameter linear fits, especially at larger values of δy . For $\delta y > 1$ the presence of resonances makes Eq. (8) not applicable. In Fig. 2 we plot the same moments for $\sqrt{s}=1$ TeV and $\sqrt{s}=22$ GeV. It is clear from the figure that the linear dependence of $\ln F_i$ on $-\ln \delta y$ extends to smaller rapidity bins for $\sqrt{s}=1$ TeV, in accordance with our previous argument. It would be of great interest to test the validity of (11).

We summarize our results. Intermittency, if due to evolution and branching, should have at finite energy a short distance cut-off of the order of $1/Y$, measured by the effective

correlation distances $\xi_i(Y) \sim 1/Y$. As a consequence, the intermittency exponents f_i , which in principle are functions of δy and Y , become scaling functions of $\eta \equiv \delta y Y$.

ACKNOWLEDGMENTS

Both authors would like to thank the Theory Division of CERN for the hospitality. One of us (J.D.D) also thanks GTAE for a travel grant.

References

1. A. Bialas and R. Peschanski, *Nucl. Phys.* B273 (1986) 703; *Phys. Lett.* B207 (1988) 59; *Nucl. Phys.* B308 (1988) 857.
2. I. M. Dremin, CERN preprint, CERN-TH 4693/87 (1987).
3. J. Dias de Deus, *Phys. Lett.* B 194 (1987) 297.
4. T. H. Burnett et al., *Phys. Rev. Lett.* 50 (1983) 2062.
5. M. Adamus et al., *Phys. Lett.* B 185 (1987) 200.
6. R. Holynski et al., IFJ Krakow preprint 1410/PII (1988);
I. Lund et al., talk presented at Hadron Structure '88, Piest'any, Czechoslovakia.
E. Gladysz-Dziadus, IFJ Krakow preprint 1432/PII (1988).
7. I. V. Ajinenko et al. (EHS/NA22 Coll.), HEN-309 (1989).
8. W. Ochs and J. Wosiek, *Phys. Lett.* B 214 (1988) 617.
9. M. Jedrzejczak, Warsaw University Institute of Physics preprint (1989).
10. B. Buschbeck, P. Lipa and R. Peschanski, *Phys. Lett.* B 215 (1988) 788.

Figure Captions

Figure 1. The curves correspond to formula (8) for the F_i , $i = 2, 3, 4, 5$, with the values of the parameters K_i and ξ_i given in the table. Data are from [7].

Figure 2. Behaviour of the factorial moments F_i , $i=2,3,4,5$ for $\sqrt{s}=22$ GeV (solid line) and 1 TeV (dotted line). The normalisation has been chosen simply for comparison.

TABLE

i	ξ_i	K_i
2	1.15	3.08
3	1.25	2.10
4	2.00	2.00
5	5.00	1.79

Table. Values of the parameters K_i and ξ_i of Eq. (8), $i=2,3,4,5$, used in the curves of Fig. 1. Note that the effective correlation lengths ξ_i increase with i .

Fig. 1

Fig. 2