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Renormalization Group Analysis of Coleman - Weinberg Symmetry Breaking in Two - Higgs Models

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Abstract

Using renormalization group techniques we investigate Coleman - Weinberg symmetry breaking in the standard model with two Higgs doublets. Bounds on the mass spectrum will be found from the requirement of vacuum stability and finiteness of the quartic couplings up to a unification scale.

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I. Introduction

Coleman-Weinberg symmetry breaking [1] has many attractive features. In the one-doublet standard model it sets a lower bound on the Higgs mass. As the scale of symmetry breaking is determined by the value of a dimensionless coupling constant at the unification scale, it also might give a hint how to solve the gauge hierarchy problem [2].

Our approach to the topic will be perturbative. We use the renormalization group equations (RGE's) to derive our results without using the explicit expression for the effective potential (EP), which can be found in ref. [3]. The course of this paper will be as follows : In chapter II we will review the use of RGE's in the effective potential formalism. In chapter III we discuss symmetry breaking in the two-doublet extension of the standard model, and in chapter IV we use the RGE to derive some bounds on the mass spectrum in this model.

II. Renormalization Group and the Effective Potential

Using the renormalization group one can investigate the behaviour of the EP under scaling the fields $\phi \rightarrow \sigma\phi$. The procedure is well known and can be found in refs. [1],[4]. The result is, with μ denoting the renormalization point,

$$V(\sigma\phi, \lambda, \mu) = \sigma^4 \exp \left(-4 \int_0^{\ln \sigma} dt \frac{\gamma(\bar{\lambda})}{\gamma(\bar{\lambda}) + 1} \right) V(\phi, \bar{\lambda}(\ln \sigma), \mu) \quad (II.1)$$

where $\bar{\lambda}(t)$ is a solution of

$$\frac{d}{dt} \bar{\lambda}(t) = \frac{\beta(\bar{\lambda})}{\gamma(\bar{\lambda}) + 1}, \quad \bar{\lambda}(0) = \lambda \quad (II.2)$$

Since we like to study Coleman-Weinberg symmetry breaking, we restrict ourselves to the massless case. Note the difference to the RGE's for 1-PI-Greens functions where the anomalous dimension in the denominators of eqs. (II.1-2) does not occur. In lowest order, however, the denominators can be neglected. Also the lowest order terms of the β -functions for the EP and the 1-PI-Greens functions coincide in massless theories. If there are several fields, we have one anomalous dimension for each field, and in eqn. (II.2) we have to set

$$\gamma = \frac{1}{n} \sum_j \gamma_j \quad (II.3)$$

to obtain the behaviour of V under scaling all fields simultaneously. β and γ -functions used in this paper are listed in the appendix.

III. Two - Higgs Model

We consider a massless potential of the form

$$\begin{aligned} V(\phi_1, \phi_2) = & \frac{\lambda_1}{2} (\phi_1^+ \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^+ \phi_2)^2 \\ & + \lambda_3 (\phi_1^+ \phi_1)(\phi_2^+ \phi_2) + \lambda_4 (\phi_1^+ \phi_2)(\phi_2^+ \phi_1) + \frac{\lambda_5}{2} ((\phi_1^+ \phi_2)^2 + (\phi_2^+ \phi_1)^2) \end{aligned} \quad (III.1)$$

The Yukawa couplings are chosen such that the up-like quarks couple to ϕ_1 and the down-like ones to ϕ_2 . This choice of potential and Yukawa-sector leads to a natural suppression of flavour changing neutral currents [5],[6] and is sufficiently general as long as only the top-quark contributions are taken into account.

At tree level we have the following stability bounds on the quartic couplings:

$$\begin{aligned}\lambda_1 &\geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2} \\ \lambda_3 + \lambda_4 - |\lambda_5| &\geq -\sqrt{\lambda_1 \lambda_2}\end{aligned}\tag{III.2 a-d}$$

For large values of the scale parameter t these bounds should also be respected by the effective couplings in the EP. At the symmetry-breaking scale however, some of these bounds might be violated (in fact some *must* be violated in order to have Coleman-Weinberg symmetry breaking). Naively we expect that this violation should not exceed orders of magnitude of λ_j^2, g_j^4 , because only in this case radiative corrections could stabilize the theory.

In our model we give both doublet hypercharge $Y = +1$, so our desired minimum of the EP should favour parallel alignment of the vacuum expectation values (VEV's) in order to have an unbroken electromagnetic gauge symmetry. Thus we like to have

$$\langle \phi_1 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}\tag{III.3}$$

For the tree potential this leads to the condition

$$\lambda_4 + \lambda_5 \leq 0\tag{III.4}$$

The sign of λ_5 determines the relative phase between the VEV's which we choose to be 0 leading to the tree level condition $\lambda_5 \leq 0$. Whether these bounds are necessary in the EP requires a case to case analysis.

We now proceed to the renormalization group analysis of symmetry breaking in this model. Unfortunately, the renormalization group allows only to study the behaviour of the EP under scaling both fields simultaneously. So we are only able to investigate the EP in one direction, which we choose to be the direction of symmetry breaking. Thus we write the fields as

$$\phi_1 = \frac{\rho}{\sqrt{2}} n_1, \quad \phi_2 = \frac{\rho}{\sqrt{2}} n_2\tag{III.5}$$

with

$$n_1^+ n_1 + n_2^+ n_2 = 1$$

The VEV's will be written as

$$\begin{aligned}\langle \rho^2 \rangle_0 &= v_1^2 + v_2^2 \equiv v^2 \\ \langle n_1^+ n_1 \rangle_0 &= \cos^2 \xi, \quad \langle n_2^+ n_2 \rangle_0 = \sin^2 \xi\end{aligned}\tag{III.6}$$

We are free to use the desired VEV's as renormalization point and choose renormalization conditions such that the EP coincides with the tree potential at the renormalization point. Thus we can write the EP in the direction of symmetry breaking as

$$V(\rho) = \frac{\rho^4}{4} \exp \left(-4 \int_0^t d\tau \gamma(\lambda) \right) \left(\frac{1}{2} \lambda_1(t) \cos^4 \xi + \frac{1}{2} \lambda_2(t) \sin^4 \xi + (\lambda_3(t) + \lambda_4(t) + \lambda_5(t)) \sin^2 \xi \cos^2 \xi \right) \quad (\text{III.7})$$

with $t = \ln \frac{\rho}{v}$. Extremalization of V leads to the symmetry - breaking condition (with an obvious notation for the various β -functions)

$$0 = \frac{d}{d\rho} V(\rho = v) = \frac{1}{2v} \left((\lambda_1 + \frac{\tilde{\beta}_1}{4}) v_1^4 + (\lambda_2 + \frac{\tilde{\beta}_2}{4}) v_2^4 + 2(\lambda_3 + \lambda_4 + \lambda_5 + \frac{\tilde{\beta}_3 + \tilde{\beta}_4 + \tilde{\beta}_5}{4}) v_1^2 v_2^2 \right) \quad (\text{III.8})$$

where $\tilde{\beta}_j$ is defined by

$$\tilde{\beta}_j = \beta_j - 4\gamma\lambda_j \quad (\text{III.9})$$

Symmetry breaking will take place at a scale, where (III.8) is satisfied.

We will concentrate on the case, where all λ_j 's are of order g_j^2 . Let us now form the second derivative of V in order to see if we really have a minimum. If there is one, we can give an expression for the mass of the particle corresponding to excitation in the symmetry-breaking direction. So, after using (III.8) in order to eliminate all first order terms we get

$$m_s^2 = \frac{d^2}{d\rho^2} V = \frac{1}{2v^2} \left(\frac{1}{2} (\tilde{\beta}_1 v_1^4 + \tilde{\beta}_2 v_2^4) + (\tilde{\beta}_3 + \tilde{\beta}_4 + \tilde{\beta}_5) v_1^2 v_2^2 \right) \quad (\text{III.10})$$

Before we check, whether this expression is really positive, we note that V also has to be minimized in all other directions. At tree level, this leads to a necessary condition for V being stationary

$$\lambda_1 v_1^4 - \lambda_2 v_2^4 = 0 \quad (\text{III.11})$$

We assume this to be true also for the EP, eventually up to corrections of order λ_j^2, g_j^4 . Using this, we obtain from eqn. (III.8)

$$\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 + \lambda_5 \approx O(\lambda_j^2, g_j^4) \quad (\text{III.12})$$

In order to get some information on m_s^2 in eqn. (III.10), it is sufficient to set the l.h.s.'s of eqns. (III.11/12) to zero. We can also get some information on the other masses in this model. In fact, all other excitations around the vacuum are very massive and we can rely on the tree - expressions

$$\begin{aligned} m_0^2 &= \frac{1}{2} \left(\lambda_1 v_1^2 + \lambda_2 v_2^2 + \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4(\lambda_3 + \lambda_4 + \lambda_5)^2 v_1^2 v_2^2} \right) \\ &= \sqrt{\lambda_1 \lambda_2} v^2 \\ m_\pm^2 &= -\frac{1}{2} (\lambda_4 + \lambda_5) v^2 = \frac{1}{2} (\lambda_3 + \sqrt{\lambda_1 \lambda_2}) v^2 \\ m_p^2 &= -\lambda_5 v^2 = (\lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2}) v^2 \end{aligned} \quad (\text{III.13})$$

where eqns. (III.12/13) have been used. Now we take the β and γ - functions from the appendix and use eqns. (III.11-13) to rewrite eqn. (III.10) as

$$m_s^2 = \frac{1}{8\pi^2 v^2} (2m_{\pm}^4 + m_0^4 + m_p^4 + 6m_W^4 + 3m_Z^4 - 12m_t^4) \quad (\text{III.14})$$

This result was previously derived in refs. [7] (see also [8],[9]). Thus our model consists of one light scalar and four heavy scalars. Note that m_p can consistently be set to zero to obtain the standard axion model.

IV. Bounds on Mass Spectrum

In the following we like to check, if there are bounds on the mass spectrum, enforced upon us by consistency requirements. Actually we are confronted with two classes of requirements. The first class is made up from

- (i) m_s^2 should be positive
- (ii) the stability constraints (III.2) should be respected above the symmetry breaking scale

These requirements lead to stringent bounds, necessary for consistency of this model. A violation of (i) means, that eqn. (III.8) does not describe a minimum of the potential, while a violation of (ii) implies a new minimum at some higher scale. The second class of requirements is enforced upon us by the limits of our perturbative approach. Yukawa and quartic couplings should not exceed certain limits in order to be in the perturbative domain. These bounds are much less stringent than the former ones (why should Nature do us the favour of allowing perturbative calculations?), but merely indicate a breakdown of our computational tools.

This pessimistic view might not be necessary, when we look for bounds on the quartic couplings. It is widely assumed and supported by nonperturbative calculations, that theories involving elementary scalars are trivial, which means that such models only make sense if all renormalized couplings are zero (for a recent review, see [10]). For the standard model this means that it can only be an effective theory, valid up to some unification scale. In perturbation theory triviality makes its presence felt by the appearance of poles in the flow of the quartic couplings. Although perturbation theory cannot prove triviality, there are arguments [11] that it correctly describes the energy scale where the model breaks down. Thus we propose a third stringent requirement

- (iii) The quartic couplings should remain finite up to some unification scale Λ .

As a cut-off we will take the Planck or GUT scale. Whether such a requirement can also be proposed for the Yukawa-couplings is an open question, since we don't have any nonperturbative results about what is going to happen, if these couplings get strong.

First we observe that the Yukawa couplings should be smaller than a certain value h_{max} , so that perturbation theory remains valid up to a given cut-off Λ . Furthermore it is easy to find an absolute upper bound on λ_2 by switching off all other quartic couplings. A look at the corresponding β -function reveals that the omitted terms always give a positive contribution and can only lower this limit. Bounds on λ_1 depend on the Yukawa-couplings. Strong Yukawa-couplings tend to destabilize the potential from below and they imply lower bounds on λ_1 . These bounds further depend on the choice of $m_0, m_{\pm}$ and m_p . As long as these masses are not too large (typically up to 150 GeV), however, these bounds are not very sensitive to

variation of these parameters. For a given m_0 we have an additional constraint. From eqn. (III.13) and the requirement $\lambda_2 \leq \lambda_2^{max}$ we have

$$\lambda_1 \geq \frac{m_0^4}{v^4 \lambda_2^{max}} \quad (IV.1)$$

These bounds, obtained from numerical integration of the RGE's, are plotted in fig.1 for various values of m_0 .

We would now like to translate these bounds into constraints on the mass spectrum. For a moment keep m_0 and m_t fixed. Then for a given h_t , the VEV's and thus λ_1 are fixed by

$$\lambda_1 = m_0^2 \left(\frac{h_t^2}{2m_t^2} - \frac{1}{v^2} \right) \quad (IV.2)$$

Eqn. (IV.2) describes a straight line in fig.1 which has to intersect the allowed region. For a given m_0 this leads to

$$m_t^2 \leq \max_{0 \leq h_t \leq h_{max}} \frac{h_t^2 m_0^2 v^2}{2(\lambda_1^{min}(m_0, h_t) v^2 + m_0^2)} \quad (IV.3)$$

where $\lambda_1^{min}(m_0, h_t)$ denotes the lower bound on λ_1 as a function of m_0 and h_t . An upper limit on m_0 is implied by

$$m_0^2 \leq \sqrt{\lambda_1^{max} \lambda_2^{max}} v^2 \quad (IV.4)$$

These bounds are plotted in fig. 2. Note, however, that there are some numerical uncertainties since we can scan the parameter range only in finite steps.

Let us now turn to $m_{\pm}$ and m_p . Upper bounds on these masses can be derived from condition (iii). We do this for small values of m_0 and h_t . In this case the bounds are again rather insensitive to variations of m_0 . Large values of m_0 will worsen the behaviour of the quartic couplings at large scales and thus lower these bounds. The result is plotted in fig. 3.

Finally let us take condition (i) to give a lower limit on the sum of heavy scalar masses. It is straightforward to see that

$$\begin{aligned} m_0 + 2m_{\pm} + m_p &\geq (m_0^4 + 2m_{\pm}^4 + m_p^4)^{\frac{1}{4}} \\ &\geq (12m_t^4 - 6m_W^4 - 3m_z^4)^{\frac{1}{4}} \end{aligned} \quad (IV.5)$$

This bound is plotted in fig. 4. Further it is straightforward to use the previously found upper bounds on the heavy scalar masses to give an upper bound on the mass of the light scalar. A lower bound on m_s is obviously given by setting all other scalar masses in eqn. (III.14) to zero. So the lower limit on m_s is the familiar Coleman-Weinberg mass of the one Higgs- model. These bounds are plotted in fig. 5 (see also ref. [12]). We see that the top-quark must be lighter than 130 GeV in order to have a positive m_s^2 .

V. Conclusions

We have seen from the previous chapter, that Coleman-Weinberg symmetry breaking in the two-Higgs-model is possible with top-masses up to 130 GeV. This is to be contrasted with the minimal model, where Coleman-Weinberg symmetry breaking is impossible for top-masses larger than appr. 80 GeV. For heavy top quarks we also have strong constraints on the ratio of the vacuum expectation values, which we have not explored in this paper. These limits have to be compared with existing phenomenological constraints from hadron physics (for a review see [13] and refs. therein).

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Appendix

Here we list the β and γ -functions taken from ref. [8],[14]. In the following g_1, g_2 and g_3 represent the $U(1), SU(2)$ and $SU(3)$ gauge coupling, and h_t the Yukawa-coupling of the top quark. Let N be the number of fermion generations and N_H the number of Higgs-doublets (in the case at hand we have $N = 3$ and $N_H = 2$)

$$\begin{aligned}
16\pi^2 \frac{d}{dt} g_1 &= \left(\frac{20}{9} N + \frac{1}{6} N_H \right) g_1^3 \\
16\pi^2 \frac{d}{dt} g_2 &= \left(-\frac{22}{3} + \frac{4}{3} N + \frac{1}{6} N_H \right) g_2^3 \\
16\pi^2 \frac{d}{dt} g_3 &= \left(-11 + \frac{4}{3} N \right) g_3^3 \\
16\pi^2 \frac{d}{dt} h_t &= \left(-8g_3^2 - \frac{9}{4} g_2^2 - \frac{5}{12} g_1^2 + \frac{9}{2} h_t^2 \right) h_t \\
16\pi^2 \frac{d}{dt} \lambda_1 &= 12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 \\
&\quad - 3\lambda_1(3g_2^2 + g_1^2 - 4h_t^2) + \frac{3}{2} g_2^4 + \frac{3}{4} (g_2^2 + g_1^2)^2 - 12h_t^4 \\
16\pi^2 \frac{d}{dt} \lambda_2 &= 12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 \\
&\quad - 3\lambda_2(3g_2^2 + g_1^2) + \frac{3}{2} g_2^4 + \frac{3}{4} (g_2^2 + g_1^2)^2 \\
16\pi^2 \frac{d}{dt} \lambda_3 &= (\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 \\
&\quad - 3\lambda_3(3g_2^2 + g_1^2) + \frac{9}{4} g_2^4 + \frac{3}{4} g_1^4 - \frac{3}{2} g_2^2 g_1^2 + 6\lambda_3 h_t^2
\end{aligned}$$

$$\begin{aligned}
16\pi^2 \frac{d}{dt} \lambda_4 &= 2(\lambda_1 + \lambda_2)\lambda_4 + 4(2\lambda_3 + \lambda_4)\lambda_4 + 8\lambda_5^2 \\
&\quad - 3\lambda_4(3g_2^2 + g_1^2) + 3g_2^2 g_1^2 + 6\lambda_4 h_t^2 \\
16\pi^2 \frac{d}{dt} \lambda_5 &= \lambda_5(2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 12\lambda_4 - 9g_2^2 - 3g_1^2 + 6h_t^2)
\end{aligned} \tag{A 1}$$

The γ -functions are (the subscript denotes the corresponding Higgs-doublet) defined by

$$\gamma_j = -\frac{1}{2} \frac{d}{dt} \ln Z_{\phi_j} \tag{A 2}$$

In our model we have

$$\begin{aligned}
\gamma_1 &= \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 3h_t^2 \\
\gamma_2 &= \frac{9}{4}g_2^2 + \frac{3}{4}g_1^2
\end{aligned} \tag{A 3}$$

As a numerical input we used

$$\begin{aligned}
g_1^2(M_Z) &= 0.127, \quad g_2^2(M_Z) = 0.425, \quad g_3^2(M_Z) = 1.27 \\
v &= \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}
\end{aligned} \tag{A 4}$$

In eqn. (IV.1) and (IV.4) we found from numerical integration

$$\lambda_2^{max} = 0.47, 0.57 \tag{A 5}$$

where the first value corresponds to $\Lambda = M_{Pl}$ and the latter one to $\Lambda = M_{GUT}$. h_{max} was found to be 1.23 and 1.3 for a cut-off at the Planck and GUT-scale, respectively.

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Figure Captions

Fig. 1: Bounds on λ_1 as a function of h_t .

Fig. 2: Allowed region for m_0 and m_t from eqn. (IV.3) and (IV.4).

Fig. 3: Allowed region for $m_{\pm}$ and m_p .

Fig. 4: Lower bound on the sum of the heavy scalar masses.

Fig. 5: Upper and lower bound on the light scalar mass.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5