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THE QCD PLASMA PHASE

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ABSTRACT

We discuss perturbative and non-perturbative properties of the QCD plasma phase. In particular we present recent Monte Carlo results for the equation of state, the heavy quark potential and screening lengths and give a comparison with expectations based on high temperature perturbation theory.

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1. INTRODUCTION

It is generally expected that we can learn a lot about fundamental non-perturbative aspects of QCD like confinement and chiral symmetry breaking by studying their temperature dependence. Quite early it has been speculated that QCD undergoes a phase transition from the low temperature confining phase to a new phase, the quark-gluon plasma, where asymptotic freedom is the dominant feature, confinement is lost and chiral symmetry is restored¹. Recently the transition to the quark-gluon plasma has been studied extensively by Monte Carlo simulations. In particular the order of the transition in the pure gauge sector, which is generally expected to be first order², has been examined on large lattices^{3,4}. This has made it clear that the transition is weaker than previously expected⁴, and it has even been questioned whether the transition is indeed first order³. The situation in the case of full QCD, i.e. in the presence of light dynamical fermions, is even more uncertain. The order of the chiral phase transition strongly depends on the number of colour and flavour degrees of freedom and also seems to be rather sensitive to the lattice size^{5,6}. The present status of the analysis of the QCD phase transition has been reviewed by M. Fukugita at this conference⁷. Here we want to leave aside a detailed discussion of the transition itself and concentrate on an analysis of the properties of the high temperature phase.

The high temperature phase of QCD is often characterized as a gas of asymptotically free quarks and gluons. From perturbative calculations at finite temperature⁸ we know that indeed the leading term in

the free energy is just the contribution of a free quark-gluon gas. This is modified in next-to-leading order by a small perturbative correction of $O(g^2)$. However, already in $O(g^4)$ infra-red divergencies occur and a summation of an infinite set of diagrams becomes necessary in order to eliminate them. The resulting finite contribution, the plasmon term, is of $O(g^3)$. This contribution to the thermodynamic potential is finite due to the generation of an electric mass term in the gluon propagator

$$m_E \sim g(T)T. \quad (1.1)$$

Further problems arise in higher orders from the magnetic sector in the gluon propagator. Linde⁹ has shown that a dynamical generation of a magnetic mass,

$$m_M \sim g^2(T)T, \quad (1.2)$$

is needed in order to get a finite contribution to the free energy of $O(g^6)$. Similar problems arise in higher orders and, assuming a magnetic mass of the form given in eq.(1.2), they all contribute to the $O(g^6)$ term. The coefficient of the g^6 -term thus is incalculable in perturbation theory.

Perturbation theory does not seem to provide us with a satisfactory description of the high temperature phase. Its validity regime is uncertain and the inherent infra-red problems indicate that a hierarchy of non-perturbative length scales

$$\begin{aligned} l_P &< l_E < l_M \\ 1/T &< 1/gT < 1/g^2T \end{aligned} \quad (1.3)$$

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related to the perturbative regime (l_P) as well as the generation of an electric (l_E) and magnetic (l_M) mass gap is still present in the plasma phase. Moreover, it has been argued that the perturbative expansion at finite temperature is set up in a "false vacuum"^{10,11,12} and thus is unreliable. If true, this would cast doubt even on the lowest order perturbative results at finite temperature.

Monte Carlo simulations provide a non-perturbative analysis of the high temperature phase. A comparison of numerical results with perturbation theory allows us to judge the validity regime of the perturbative approach and the importance of non-perturbative effects in the plasma phase. The above discussion suggests that this will depend on the observable under consideration. Different observables probe different length scales in the plasma. In particular, it is expected that bulk quantities such as the energy density are sensitive to the short-distance properties of the plasma, whereas the screening length clearly probes the long-distance properties of QCD in the high temperature phase.

In the next section we will discuss recent Monte Carlo results for the equation of state, i.e. the entropy, energy density and pressure. Section 3 is devoted to an analysis of short- and long-distance properties of the heavy quark potential. In each case a detailed comparison with perturbation theory is given. Some conclusions are drawn in section 4.

2. THE EQUATION OF STATE

Here we want to discuss Monte Carlo results for the equation of state (e.o.s.) in the high temperature phase. An e.o.s. widely used in the plasma phase is the phenomenological Bag e.o.s. for the energy density ϵ and the pressure P

$$\begin{aligned}\epsilon &= \epsilon_{SB} + B \\ P &= P_{SB} - B\end{aligned}\quad (2.1)$$

with $\epsilon_{SB} = 3P_{SB} = \frac{\pi^2}{15}(N^2 - 1 + \frac{7}{4}Nn_f)T^4$ denoting the energy density of a free gas of quarks and gluons in an $SU(N)$ gauge theory with n_f massless flavours. In particular this ansatz suggests that the entropy density, $s = (\epsilon + P)/T$, in the plasma phase agrees with that of a free gas. An inclusion of perturbative corrections, however, leads to a decrease of s/T^3 at finite temperature and the ideal gas behaviour is reached only at

Figure 1: Entropy density in units of T^3 versus β on lattices of size $N_\sigma^3 \times 4$, with $N_\sigma = 12$ (dots), 16 (triangles) and 24 (squares). The data on the $16^3 \times 4$ lattice have been shifted by $\Delta\beta = -0.02$ (see discussion in ref.4). The dashed and dashed-dotted curves give the perturbative results on a $12^3 \times 4$ lattice. Perturbative results for larger N_σ differ only little from this.

infinite temperature.

The entropy density can easily be calculated in Monte Carlo simulation. Let us start with a discussion of the pure gauge sector of QCD where the entropy density is proportional to the difference of space-space (P_σ) and space-time (P_τ) plaquettes. On a lattice of size $N_\sigma^3 \times N_\tau$ this is given by

$$\begin{aligned}\frac{s}{T^3} &= \frac{\epsilon + P}{T^4} \\ &= \frac{8N}{g^2} N_\tau^3 \left(1 - \frac{c'_\sigma - c'_\tau}{2} g^2\right) (\langle P_\sigma \rangle - \langle P_\tau \rangle)\end{aligned}\quad (2.2)$$

with $\beta = 6N/g^2$ being the bare coupling in a $SU(N)$ gauge theory and $c'_{\sigma(\tau)}$ derivatives of the bare coupling with respect to the temporal lattice spacing¹³. These derivatives have been computed perturbatively¹⁴ and for the case of $SU(3)$ they are given by

$$\begin{aligned}c'_\sigma &= 0.2016 + O(g^2) \\ c'_\tau &= -0.1319 + O(g^2)\end{aligned}\quad (2.3)$$

In fig.1 we show recent results for the entropy density obtained on lattices of size $N_\sigma^3 \times 4$, $N_\sigma = 12^{15}, 16$

Figure 2: Energy density and pressure in units of the ideal gas values on finite lattices versus T/T_c . Data on the $12^3 \times 4$ lattice (dots and triangles) are taken from ref.15 and include a subtraction of the vacuum contributions. The data on the $24^3 \times 4$ (squares) are taken from ref.4 and do not include a subtraction of the vacuum part.

and 24^4 . The numerical results are compared with perturbative behaviour up to $O(g^2)^{16}$. The perturbative results are obtained on lattices of the same size as those on which the Monte Carlo simulation have been performed and thus account for deviations from the perturbative continuum results due to finite lattice size effects. It can be seen that the entropy density approaches the ideal gas behaviour at temperatures $T \sim (2 - 3)T_c$. Close to T_c ($\beta \simeq 5.69$), however, the latent entropy is only about 30% of the corresponding ideal gas value. This result is quite different from what one would expect from a standard Bag e.o.s., even when perturbative corrections are included.

It is instructive to analyze what causes this large deviations from ideal gas behaviour close to T_c . To this extent we look separately at the contribution of the energy density and the pressure to s/T^3 . These quantities are displayed in fig.2. A comparison with the expected perturbative behaviour shows that the strong deviations from ideal gas behaviour are mainly due to the non-perturbative behaviour of the pressure. The energy density is quite close to the ideal gas value already at $T \simeq 1.2T_c$. A similar result for energy density and pressure has been obtained for the $SU(2)$ gauge theory^{17,18} and full QCD with light quarks¹⁹.

The different behaviour of energy density and pressure is not unexpected: in the case of a second or-

der phase transition the temperature dependence of the dominant contribution to ϵ and P close to T_c is expected to be given by

$$\begin{aligned} \epsilon &\sim (T - T_c)^{1-\alpha}, \\ P &\sim (T - T_c)^{2-\alpha}, \end{aligned} \quad (2.4)$$

where the critical exponent α characterizes the singular behaviour of the specific heat, $c_V = (\partial\epsilon/\partial T)_V$, at T_c . P thus is expected to drop faster close to T_c than ϵ ²⁰. For a weak first order phase transition a remnant of this behaviour may be visible.

The origin of the quasi ideal gas behaviour of the energy density and the non-perturbative behaviour of the pressure has been attributed to the presence of massive plasma excitations^{17,21,22}, which are expected to contribute to energy density as well as pressure in the plasma phase. The basic idea is the observation that in the plasma phase, heavy colour singlet excitations will contribute dominantly in the low momentum regime to the thermodynamic potential, while for large momenta a free massless gluon gas gives the dominant contribution. A simple model calculation based on this idea shows that the introduction of heavy low frequency modes in the gluon propagator affects predominantly the pressure whereas the energy density remains basically unaffected²³. An explicit comparison of the above ideas with the high statistics data available for the $SU(2)$ thermodynamics shows that a good quantitative description of the behaviour of energy density and pressure can be obtained this way¹⁸.

So far we have mainly discussed the e.o.s. in the pure gauge sector. In fig.3 we present results for energy density and pressure obtained in a simulation of QCD with two light quark flavours of mass $m_q/T = 0.1$ ¹⁹. They show a behaviour qualitatively similar to that for pure $SU(3)$ gauge theory, i.e. the energy density rises rapidly at T_c and stays nearly constant already close to T_c , whereas the pressure rises only slowly and the ideal gas relation $\epsilon = 3p$ is satisfied only for $T \sim 2T_c$. A comparison with perturbation theory, however, is more complicated in this case. Already a comparison of weak coupling perturbative results for ϵ obtained on finite lattices with the corresponding results of finite temperature perturbative calculations in the continuum shows that in the presence of fermions the finite size effects are more severe than in the pure gauge sector. In fact on

Figure 3: Energy density and pressure in units of T^4 versus β for QCD with two light flavours of mass $m/T = 0.1$. Data are taken from ref.19. Lines are drawn to guide the eye.

small lattices the $O(g^2)$ corrections to the leading ideal gas term of the energy density even have the "wrong" sign compared to the continuum calculations. For instance on a $12^3 \times 4$ lattice the perturbative lattice calculation for 2 light flavours of mass $m/T = 0.1$ yields for the fermionic and gluonic contributions to the energy density^{16,24}

$$\begin{aligned} \epsilon_G/T^4 &= 7.72 + (-0.842 + 0.859n_f)g^2, \\ \epsilon_F/T^4 &= 6.13n_f - 0.455n_fg^2. \end{aligned} \quad (2.5)$$

The $O(g^2)$ correction to the total energy density thus turns out to be positive on finite lattices of this size and the positive correction becomes larger with increasing number of flavours(n_f). A similar calculation in the continuum, on the other hand, gives a negative perturbative correction to the total energy density⁸

$$\begin{aligned} \epsilon/T^4 &= (\epsilon_G + \epsilon_F)/T^4 \\ &= \frac{8\pi^2}{15} + \frac{7\pi^2}{20}n_f - \left(\frac{1}{2} + \frac{5}{24}n_f\right)g^2. \end{aligned} \quad (2.6)$$

In fig.4 we show Monte Carlo results for the fermionic and gluonic contribution to the energy density²⁴ obtained on an $12^3 \times 4$ lattice for 2 flavour QCD with quarks of mass $m_q/T = 0.1$. It can be seen that the fermionic part of the energy density agrees quite well with the perturbative calculations, whereas the gluonic part overshoots the weak coupling result significantly and seems

Figure 4a

Figure 4b

Figure 4: The gluonic (a) and fermionic (b) parts of the energy density versus β measured on the $12^3 \times 4$ (dots) and $8^3 \times 4$ (triangles) lattice. The curves show the perturbative results on a $8^3 \times 4$ (full line) and $12^3 \times 4$ (dashed line) lattices. For the gluonic energy density these two lines overlap.

to approach it only for very large couplings. As the weak coupling perturbative calculation shows already the opposite trend to what one would expect on the basis of the continuum calculation it is likely that this large overshooting of the ideal gas limit is a lattice artifact. An increase of the spatial volume does not improve the situation much further. In fact the spatial dependence of the energy density is rather weak, once the spatial lattice extent is about twice as large as the thermal ex-

tent, i.e. $N_s/N_\tau \sim 2$. One can explicitly verify that even on lattices with temporal extent $N_\tau \sim 8$ the deviations from continuum perturbation theory are large for massless fermions.

A realistic simulation of the QCD phase diagram has to take into account the contribution of strange quarks to the partition function. Their mass is of the order of the phase transition temperature and thermal production of strange quarks is thus expected to be relevant above T_c . They may influence the chiral phase transition itself^{25,26} as well as the dynamics of the high temperature phase. As the strangeness content of the low temperature phase is small it has been argued that strangeness production in the plasma phase can yield genuine signals of plasma formation in heavy ion experiments^{27,28}.

On the basis of ideal gas relations for massive fermion gases one would expect a large contribution of the strange quark sector to the energy density. The energy density in a massive fermion gas is given by

$$\epsilon(m/T) = \alpha T^4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} \left[15 \left(\frac{mn}{T} \right)^2 K_2(mn/T) + 3 \left(\frac{mn}{T} \right)^3 K_1(mn/T) \right], \quad (2.7)$$

with α counting the number of degrees of freedom and K_i , $i = 1, 2$ denoting Bessel-functions of second kind. From eq.(2.7) one finds for the energy density of quarks with masses which are of the order of the temperature of the system

$$\frac{\epsilon(m/T=1)}{\epsilon(0)} = 0.88. \quad (2.8)$$

The contribution of strange quarks to the energy density of QCD has been studied recently²⁶ in a simulation with a light isodoublet of mass $m_u/T = m_d/T = 0.05$ and a heavy quark of mass $m_s/T = 1.0$. Results are shown in fig.5. They indicate that only at rather high temperatures is the perturbative result, eq.(2.8), approached. Close to T_c one finds $\epsilon(m_s/T)/\epsilon(m_u/T) \simeq 0.5$. In view of the large finite size effects discussed above this result has, of course, to be taken with caution. However, if verified on larger lattices it suggests that the thermal production of strange quarks close to T_c is rather small.

Figure 5: Fermionic energy density of light u- and d-quarks and heavy s-quark in units of T^4 versus β on a $12^3 \times 4$ lattice. Data are taken from ref.26. The dashed line gives the lowest order ideal gas behaviour on a finite lattice.

3. THE HEAVY QUARK POTENTIAL

The temperature dependence of the potential between a static quark and antiquark pair in a deconfining medium is of considerable interest for several reasons. The effect of such a medium on heavy quark bound states, in particular on the J/ψ , appears to provide an experimentally accessible signal for plasma formation in nuclear collisions²⁹; a quantitative understanding of colour screening is necessary for these considerations³⁰. In particular, the functional form of the potential at moderate distances $r \leq 1.0 fm$ and temperatures $T \simeq (200 - 300) MeV$ seems to be most relevant for the quantitative analysis of J/ψ bound state formation in the QCD plasma. In terms of the dimensionless quantity rT , one is thus mainly interested in the regime $0 \leq rT \leq 1.5$. A non-perturbative analysis is needed in this parameter range, as perturbative results for the potential^{31,32} are expected to become applicable only in the high temperature limit.

Besides the interest in the quantitative properties of the potential at intermediate distances an understanding of its asymptotic large distance behaviour is of fundamental importance. In this limit we can study the

temperature dependence of the Debye screening length. In lowest order perturbation theory one obtains for the screening mass (inverse screening length)

$$m_E(T)^2 = \left(\frac{N}{3} + \frac{n_f}{6} \right) g^2(T) T^2 . \quad (3.1)$$

However, higher order corrections are incalculable in a gauge invariant manner and it also has been argued that vacuum instabilities make a perturbative calculation of the screening length as well as the heavy quark potential impossible^{11,33}. It is thus of particular importance to compare non-perturbative calculations of the heavy quark potential with leading order perturbative results in order to understand to what extent this yields a good description of the heavy quark potential in the high temperature phase of QCD.

We will proceed discussing results of a detailed analysis of the influence of finite size effects on the $SU(2)$ heavy quark potential and the screening length extracted from the potential³⁴. We will then compare numerical results for the $SU(3)$ potential with finite temperature perturbation theory³⁵. Finally we discuss recent numerical results for the screening length close to T_c ^{3,4}, and present first results for QCD with dynamical fermions^{24,36}.

3.1 VOLUME DEPENDENCE OF THE POTENTIAL

The potential for a static $q\bar{q}$ pair separated by a distance r can be obtained from the correlation function $\langle \text{Tr} L(0) \text{Tr} L^+(R) \rangle$ for two Polyakov loops L , which are R lattice spacings a apart, i.e., $R = r/a$. We define the heavy quark potential by

$$\exp(-V_{q\bar{q}}(r, T)/T) = \frac{\langle \text{Tr} L(0) \text{Tr} L^+(R) \rangle}{\langle |L| \rangle^2} . \quad (3.2)$$

Here

$$L = \frac{1}{N_\sigma^3} \sum_{\vec{x}} \text{Tr} L(\vec{x}) \quad (3.3)$$

is the average Polyakov loop measured on a lattice of size $N_\sigma^3 \times N_\tau$. The normalization of the loop-loop correlation function in eq.(3.2) with $\langle |L| \rangle^2$ eliminates divergent self-energy terms in the potential and allows us to compare calculations on different size lattices performed at the same temperature $T = 1/N_\tau a$. On finite lattices of size $N_\sigma^3 \times N_\tau$ we can study the potential only

in a finite spatial interval

$$\frac{1}{N_\tau} \leq rT \leq \frac{1}{2} \frac{N_\sigma}{N_\tau} . \quad (3.4)$$

Due to the periodic spatial boundary conditions the potential is periodic around $N_\sigma/2$ and the quark-antiquark separation thus can only vary between $R = r/a = 1, 2, \dots, N_\sigma/2$. For instance, on a lattice with $N_\sigma/N_\tau = 2$ the potential will be periodic around $rT = 1$, no matter how large N_σ and N_τ are. One may thus expect that the potential will be strongly distorted for these values of rT and that this will also influence the determination of the screening length. In fig.6 we show results for the $SU(2)$ heavy quark potential at a high temperature, $T = 250\Lambda_L$ ($T \simeq 6T_c$ ³⁷), obtained on lattices of various sizes and with varying asymmetry between spatial and temporal lattice extent, $N_\sigma/N_\tau = 1, 2$ and 3 . We note that results from different size lattices follow a unique curve as long as the ratio N_σ/N_τ is the same. This indicates that we are indeed calculating the continuum heavy quark potential in a finite physical volume. In addition it is obvious that the potential becomes steeper with increasing N_σ/N_τ . A detailed analysis of the finite size dependence of the potential³⁴ shows that this can be parametrized as

$$\ln \left[\frac{V_\infty(r, T)}{T} \right] = \ln \left[\frac{V(r, T)}{T} \right] - \frac{c(r, T)}{V T^3} . \quad (3.5)$$

An extrapolation to infinite volume based on this ansatz is also shown in fig.6.

The functional form of the colour averaged potential, defined by eq.(3.2), is not well understood. A perturbative calculation shows that to lowest order only a 2-gluon exchange diagram contributes (the colour averaged potential is a thermal average of a colour singlet and octet potential, which to lowest order give opposite contributions. The 1-gluon exchange term is thus cancelled)¹². One therefore would expect that the potential behaves as $1/r^2 \exp(-2\mu r)$ at large T . However, the behaviour close to T_c is completely uncertain due to the apparent non-perturbative nature of the potential in this temperature regime. It thus seems to be appropriate to use a general ansatz of the form

$$\frac{V(r, T)}{T} = \frac{\alpha(T)}{(rT)^d} e^{-\mu r} \quad (3.6)$$

for the analysis of the temperature-dependent heavy quark potential. Parameters obtained from fits to the

Figure 6: The heavy quark potential $-V(r, T)/T$ versus rT at fixed temperature $T = 250\Lambda_L$. The couplings used in the simulation are $\beta = 2.988$ ($N_\tau = 4$), $\beta = 3.147$ ($N_\tau = 6$) and $\beta = 3.26$ ($N_\tau = 8$). The dashed line gives the potential extrapolated to infinite volume.

potentials shown in fig.6 using this ansatz (with periodic boundary conditions taken into account) are given in Table 1. One sees that finite size effects tend to lower the screening mass in a finite volume. For $N_\sigma/N_\tau = 3$ the screening mass is still about 20% lower than the asymptotic infinite volume result. This volume dependence of the screening mass is in qualitative agreement with perturbative calculations on finite lattices³⁸. Moreover, we see that the power of the subleading $1/rT$ -term is $d \simeq 1.6$, i.e. still smaller than the perturbative value, $d = 2$.

A similar analysis at $T = 55\Lambda_L$, i.e. $T/T_c \simeq 1.45$ is shown in fig.7. For the infinite volume extrapolated potential one obtains in this case $d = 1.2$ as a power of the $1/rT$ -term and $\mu/T = 2.83$ for the screening mass. As

N_σ/N_τ	α	d	μ/T
1	0.0386	1.30(1)	0.13(2)
2	0.0085	1.72(1)	0.81(3)
3	0.0127	1.58(1)	2.30(8)
∞	0.0119	1.61(1)	2.87(20)

Table 1: Parameters for the fits shown in fig.6.

expected the deviations from perturbative predictions are larger close to T_c . Here the potential seems to be well described by a simple Debye-screened Coulomb potential ($d = 1$). Similar results have been obtained for the functional form of the $SU(3)$ potential^{35,39}. Also in that case one finds indications that the perturbative form of the potential with $d = 2$ yields a better description of the Monte Carlo data at high temperature.

Figure 7: As in fig.6. but for fixed $N_\tau = 6$ and $T = 55\Lambda_L$.

3.2 COMPARISON WITH PERTURBATION THEORY

Further information about the validity regime of perturbative predictions for the heavy quark potential

can be obtained from a direct comparison of the functional form of the potentials in the singlet and octet channels³⁵. These potentials are given by³²

$$e^{-V_1(r,T)/T} = 3 \frac{\langle \text{Tr} L(0) L^\dagger(R) \rangle}{\langle |L| \rangle^2}, \quad (3.7)$$

$$e^{-V_8(r,T)/T} = \frac{9}{8} \frac{\langle \text{Tr} L(0) \text{Tr} L^\dagger(R) \rangle}{\langle |L| \rangle^2} - \frac{3}{8} \frac{\langle \text{Tr} L(0) L^\dagger(R) \rangle}{\langle |L| \rangle^2}. \quad (3.8)$$

The observables entering the definitions of V_1 and V_8 are not gauge invariant. One thus has to analyse them in a fixed gauge. A suitable choice is the Landau gauge⁴⁰ as this preserves the rotational symmetry of the potential. In perturbation theory the singlet and octet potentials are related by

$$\frac{V_1(r,T)}{V_8(r,T)} = -8 + O(g^4). \quad (3.9)$$

Note that the $O(g^2)$ corrections cancel here, while that is not the case for the relation between the singlet and colour-averaged potentials

$$\frac{V_1(r,T)^2}{TV(r,T)} = -16 + O(g^2). \quad (3.10)$$

These relations have been studied at three different couplings, $\beta = 5.75, 6.10$ and 8.0 on a $12^3 \times 4$ lattice. The β -values used correspond to temperatures $T/T_c = 1.16, 2.3$ and 13.7 ³⁵. Results are shown in fig.8. Clearly the agreement gets worse at large distances. We note, however, that the perturbative relations are approached with increasing temperature. For the largest β value one indeed finds the expected ratio between the singlet and octet potentials, while the relation between colour-averaged and singlet potentials is not quite fulfilled, even at this high temperature. This shows the importance of $O(g^2)$ corrections in these relations.

3.3 DEBYE SCREENING MASSES CLOSE TO T_c

From the experimental point of view the behaviour of the potential close to T_c , i.e. $T \simeq (1 - 1.5) \cdot T_c$ is most interesting as this is the temperature regime in which the melting of heavy quark bound states^{29,30} is expected to occur. This will, of course, depend on the

Figure 8: The ratios $-V_1(r,T)/V_8(r,T)$ (a) and $-V_1(r,T)^2/TV(r,T)$ (b) versus R for $\beta = 5.75$ (dots), 6.1 (squares) and 8.00 (triangles). The dashed lines correspond to the perturbative results.

actual form of the heavy quark potential in this temperature regime; in particular on its shape at intermediate distances, $rT \leq 1$.

The analysis of the $SU(2)$ as well as the $SU(3)$ potentials close to T_c has shown that the potential is well described by a simple Debye-screened Coulomb potential

$$V(r,T) = \frac{\alpha(T)}{r} e^{-\mu r}, \quad T \geq T_c. \quad (3.11)$$

The perturbative behaviour is found to set in only at

		μ/T	
β	ref.3	ref.4	ref.15
5.695	0.70(29)	0.66(16)	
5.70	1.36(8)	1.0(5)	
5.71	1.44(5)	1.7(3)	
5.72	1.68(5)	1.9(3)	
5.73		1.6(3)	
5.75	2.56(19)		2.44(28)
6.1			2.20(28)
8.0			2.32(24)

Table 2: Screening masses for the pure $SU(3)$ gauge theory obtained on lattices of size $N_\sigma^3 \times 4$, with $N_\sigma = 12$ (ref.15), 24 (ref.4). In ref.3 a cylindrical lattice of size $8^2 \times 32 \times 4$ has been used.

very high temperatures. The first numerical determinations of the screening mass^{41,42,43} have been based on the above ansatz. However, in these early studies the potential could only be analyzed at distances $rT \sim 1$. The recent high statistics simulations on large lattices^{3,4} allow a determination of the heavy quark potential up to distances $rT \sim 5$. They yield results for the asymptotic screening length in the plasma phase. Although both simulations have been performed with rather different techniques it is reassuring that the results obtained for the screening length close to T_c are in good agreement (Note that this is true only for the deconfined phase. In the confined phase there exist discrepancies which are not yet resolved). We display the results for μ/T in Table 2. It can be seen that the screening mass decreases strongly close to T_c ($\beta \simeq 5.69$). However, already at $T \simeq 1.2$ ($\beta = 5.75$) $\mu/T \simeq 2.5$, and the ratio stays constant for larger temperatures, i.e. $\mu \sim T$. The numerical value found for μ/T is close to the lowest order perturbative result, $\mu = 2m_D \sim 2T$, with m_D given by eq.(3.1) and $g(T) \sim 1$.

One has to emphasise that the screening masses extracted from the long-distance behaviour of the potential, using eq.(3.11)^{3,4}, or from the intermediate distance regime, using eq.(3.6) with $d > 1$ ¹⁵, are substantially smaller than those obtained in earlier simulations, where the analysis of the potential has been based on the form given in eq.(3.11) and where the potential could only be studied at distances up to

	T/T_c	α	μ_{eff}/T
SU(2)	1.45	0.10	2.8 ± 0.1
SU(3)	1.3	0.16	3.7 ± 0.1
QCD	1.4	0.10	4.06 ± 0.11

Table 3: Parameters for the heavy quark potential close to T_c . The values quoted for $SU(2)$ and $SU(3)$ are based on fits with the Debye-screened Coulomb potential, eq.(3.11), to the data presented in ref.34 and ref.35, respectively. The numbers quoted for QCD are based on data taken from ref.24 for a simulation on a $12^3 \times 4$ lattice with 2 light quarks of mass $m/T = 0.1$.

$rT \simeq 1$ ^{41,42}. Probably the situation will be similar with the potential calculations now performed with dynamical fermions^{24,36}. The statistics in these simulations is not sufficient to go to large distances or analyse the potential with an arbitrary power for the $1/rT$ -term. Nonetheless, as we have seen, eq.(3.11) gives a satisfactory description of the intermediate-distance part of the potential close to T_c , and can be used to analyse the Monte Carlo data in this regime. One should, however, interpret the screening length extracted from these data as an effective mass, μ_{eff} , that characterizes the behaviour of the potential for $rT \leq 1$. Results obtained this way for $SU(2)$, $SU(3)$ and QCD are compared in Table 3 for T close to T_c . Apparently the effective screening mass increases with increasing number of colours and/or flavours. This is in agreement with the general expectation that the screening is more efficient when more parton degrees of freedom are present at a given temperature. As mentioned above, the quantitative results obtained for the heavy quark potential close to T_c are of immediate interest for the discussion of bound state formation in the quark-gluon plasma. A screening mass of about $700 MeV$ seems to be large enough to prohibit bound state formation with charmed quarks in a Debye-screened Coulomb potential^{29,30}. The numerical simulations thus suggest that the screening mass at $T \sim (1.3 - 1.4)T_c$ is large enough to melt heavy quark bound states like the J/ψ in a thermalized quark-gluon plasma. Whether this is indeed the correct interpretation of the experimentally observed J/ψ suppression in heavy ion experiments⁴⁴ is still an open question and will require also a detailed analysis of pos-

sible other suppression mechanisms.

4. CONCLUSIONS

We have discussed recent Monte Carlo results for the equation of state and properties of the heavy quark potential in the QCD plasma phase. As expected we find that these quantities differ strongly from perturbative results close to T_c . However, quantities like the energy density that are only sensitive to the short distance properties in the plasma phase are in good agreement with perturbative predictions already at temperatures $T \simeq 1.2T_c$. Strong deviations from perturbative behaviour have been found for the pressure; the perturbative relation $\epsilon = 3P$ seems to hold only for $T \geq (2-3)T_c$.

At present the analysis of the equation of state for QCD with dynamical fermions suffers from large finite size effects in thermodynamic observables. Weak coupling perturbative calculations indicate that this situation will not improve soon, as much larger lattices are needed to eliminate the finite size effects. An alternative way might be to use an improved fermion action for these calculations.

The heavy quark potential seems to deviate clearly from perturbative predictions close to T_c . In particular the analysis of the singlet and octet potentials shows that perturbative predictions can be bad for certain observables even at temperatures as high as $T \sim 10T_c$. The comparison between Monte Carlo data and perturbation theory is worse at larger distances. This stresses the non-perturbative nature of the QCD plasma at large distances even at rather high temperature. There are, however, indications that the potential approaches the expected perturbative form for high temperatures and that the screening mass is close to the lowest order perturbative value already for $T \sim 2T_c$. Given the poor understanding of the screening mechanism gained from perturbative analyses so far, a further detailed numerical analysis of the large-distance part of the potential at high temperatures is certainly needed.

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