CERN-TH.5269/88

B-MESON MASS AND DECAY CONSTANT
FROM LATTICE QCD: A PRELIMINARY STUDY

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ABSTRACT

We present the results of the first computation on large lattices of the B-meson binding energy and decay constant following the method proposed by Eichten. The results obtained at $\beta = 6$ on a $20 \times 10^2 \times 40$ lattice and at $\beta = 6.2$ on a $16^3 \times 48$ lattice show that it is necessary to have a statistical sample which is much larger than in similar computations with light mesons in order to obtain results of comparable precision.

The study of the weak properties of B-mesons is one of the prominent sources of information on flavour dynamics.

The determination of the as yet unknown Cabibbo-Kobayashi-Maskawa (C-K-M) matrix element $|V_{ub}|$ [1], the effects of CP-violation and attempts to determine the possible existence of new interactions beyond the Standard Model[2] all rely up to now on phenomenological models for the decay of B-mesons. This is due to the fact that B-physics is at the interface of the perturbative and non-perturbative QCD regimes. The light quark inside the meson is in fact subject to QCD non-perturbative effects because its typical momentum is of order $\mu \ll m_Q$.

Lattice QCD offers in principle the possibility of computing the spectrum and the decay matrix elements of B-mesons from first principles and without any free parameter. In practice however we cannot put a b-quark directly on the lattices which are currently used. This is because the typical inverse lattice spacing $a^{-1} \sim 2 - 3 \text{ GeV}$ and the condition that the ultraviolet cut-off is much larger than the characteristic scale (i.e. $m_Q a \ll 1$) cannot be satisfied. Fortunately when m_Q is very large we do not need to put a dynamical heavy quark on the lattice. It has been shown by E.Eichten[3] that, in the limit where the heavy quark Q has a very large mass the long distance physics (i.e. physics at distances which are large compared with $1/m_Q$) can be described by an effective field theory with a static colour source. All the relevant physical quantities (masses, decay constants, weak matrix elements) can be then systematically expanded in powers of $1/m_Q$.

In this paper we present a numerical calculation on the lattice of the B-meson lattice binding energy and pseudo-scalar decay constant at the lowest order in $1/m_Q$. As will be explained below we only have been able so far to put an upper bound on the B-meson binding energy and decay constant. The results are encouraging however and we believe that this paper can provide useful information for further studies of heavy quark physics.

We have computed the zero momentum correlation function of the axial current $A_\mu = \bar{Q} \gamma_\mu \gamma_5 q$, where $Q (q)$ is the heavy (light) quark field :

$$G(t) = \sum_{\mathbf{x}} \langle A_0(\mathbf{x}, t) A_0^\dagger(0, 0) \rangle \quad (1)$$

At large time distances $G(t)$ is dominated by the lightest intermediate state, which, since we use the axial current as the source, is expected to be the lightest pseudoscalar

particle (whose mass and decay constant we denote as M and f_M respectively) so that:

$$G(t) \rightarrow \frac{f_M^2 M}{2} e^{-Mt} \quad (2)$$

The normalization of the decay constant is fixed by the relation:

$$\langle M | A_\mu | 0 \rangle = i f_M p_\mu \quad (3)$$

(with this normalization $f_\pi = 132 \text{ MeV}$).

On the lattice, for $m_Q a \gg 1$, the correlation function in eq.(1) can be written, at lowest order in $1/m_Q$, as:

$$G(t) = \langle \text{Tr} \left[\frac{(1 + \gamma_0)}{2} S(0, t | 0, 0) P_{0,t}^{0,0} \right] \rangle e^{-m_Q^{Latt} t} = \bar{G}(t) e^{-m_Q^{Latt} t} \quad (4)$$

where $S(0, t | 0, 0)$ is the propagator of the light quark between the origin and the point of coordinates $(x = 0, t)$, $P_{0,t}^{0,0}$ is the ordered product of links in the time direction at $x = 0$ and m_Q^{Latt} is an external parameter to be adjusted to fix the meson mass to its experimental value. Its role and renormalization will be discussed below. In eq.(4) $\langle \dots \rangle$ indicates the average over the gauge field configurations generated with some standard Monte Carlo technique.

At large time distances $\bar{G}(t)$ is expected to behave as:

$$\bar{G}(t) = K e^{-\Delta E t} \quad (5)$$

Eqs.(2),(4) and (5) thus imply:

$$K = \frac{(f_M^{Latt})^2 M a^3}{2} \quad (6)$$

$$M = \Delta E + m_Q^{Latt} \quad (7)$$

Thus the study of $\bar{G}(t)$ allows to evaluate (up to terms of order $1/m_Q$) the "binding energy" ΔE and the lattice decay constant f_M^{Latt} . m_Q^{Latt} is fixed by the condition that its value, added with the binding energy ΔE , measured from the slope of

$\ln[\bar{G}(t)] = \text{const.} - \Delta Et$, gives the experimental value of the meson mass M [eq.(7)]. It can be easily shown that the binding energy ΔE diverges linearly in a^{-1} due to the linear singularity $\simeq \int d^3q/q^2$ induced by one-gluon exchange in the heavy quark self-energy. Since M is a physical, finite and measurable quantity, the "bare" quark mass m_Q^{Latt} must be linearly divergent too in order to cancel the corresponding term in ΔE . The determination of m_Q^{Latt} from the B-meson propagator is important in order to define the heavy quark mass which has to be used in the calculations of the spectrum of $\bar{Q}Q$ states using the static QCD potential derived from the lattice calculation of the Wilson loops[4].

We have computed $\bar{G}(t)$ by numerical simulation of QCD on a $20 \times 10 \times 2 \times 40$ lattice at $\beta = 6/g^2 = 6$ and on a $16 \times 3 \times 48$ lattice at $\beta = 6.2$. The corresponding values of the lattice spacing are $a^{-1} \simeq 2.2 \text{ GeV}$ and $a^{-1} \simeq 3.0 \text{ GeV}$ respectively. A description of the light quark spectroscopy and of the calibration of the lattice spacing can be found in ref.[5]. The light quark propagators were computed using the standard Gauss-Seidel method for two values of the Wilson parameter K_{IV} in both cases ($K_{IV} = 0.1515$ and 0.1530 at $\beta = 6$ and $K_{IV} = 0.1500$ and 0.1510 at $\beta = 6.2$). $\bar{G}(t)$ was averaged over 30 configurations at $\beta = 6$ and over 15 configurations at $\beta = 6.2$. As an example we report in fig.(1) $\bar{G}(t)$ as a function of t for 30 configurations, at $\beta = 6$ and $K_{IV} = 0.1515$. The statistical errors are also shown in the figure. They become larger than 100% for $t \geq 15$. In fig.(2) the function $R(t) = \ln[\bar{G}(t)/\bar{G}(t+1)]$ at the same value of β and K_{IV} is also reported. For large time distances, when $\bar{G}(t)$ is dominated by the lightest intermediate state, $R(t)$ should become a constant, $R(t) \rightarrow \Delta Ea$. This is never the case in the range of t where the statistical error on $\bar{G}(t)$ is reasonably small and in the range $t \leq 13$ $R(t)$ is always decreasing in time. This means that highest excited states still give a sizeable contribution to the correlation function. A two-exponential fit to $\bar{G}(t)$, of the form $\bar{G}(t) = Ke^{-\Delta Et} + K'e^{-\Delta E't}$, shows that for $6 \leq t \leq 13$ the state with energy $\Delta E'$ still contributes to about 50% of the correlation. We cannot include points with $t \geq 13$ since there the statistical error on $\bar{G}(t)$ becomes too large. The relative statistical error at a certain distance appears to be significantly worse in the present case than in the case of mesons made of two light quarks. The reason is probably due to the fact that the propagator of the heavy quark is proportional to $\delta(x)$ and thus the sum over all space points, which for light quarks helps to reduce the statistical noise, is absent in the present case.

For the reasons discussed above we are not able to give a precise number for ΔE or f_M with the statistics at our disposal. It is however possible to put an upper bound on both these quantities since all the excited states have, by definition, an energy larger than ΔE and give a positive contribution to $\bar{G}(t)$. The best result has been obtained by using the correlation function at $\beta = 6$ and $K_W = 0.1515$ where we have found:

$$\Delta E a \leq 0.50$$

$$f_B^{Latt} \leq 260 MeV \quad (8)$$

at the renormalization scale of $2.2 GeV$.

To obtain the result in eq.(8) we have used the relation [see eq.(6)]:

$$f_B^{Latt} = \sqrt{2 * K * a^{-3} / M_B} \quad (9)$$

with $a^{-1} = 2.2 GeV$, which corresponds to the larger determination of a^{-1} given by the rho mass at the chiral limit[5], and $M_B = 5.27 GeV$. K has been obtained by fixing the binding energy to its upper bound, $\Delta E a = 0.50$ [see fig.(2)], and fitting $\bar{G}(t)$, shown in fig.(1) to eq.(5) in the interval $10 \leq t \leq 13$.

At the two values of β considered here, using a two exponential fit to the correlation function, in the interval $t = 6 - 13$, we found the following values for ΔE and f_B^{Latt} :

$$\Delta E a = 0.46$$

$$f_B^{Latt} = 172 MeV \quad (10)$$

using $a^{-1} = 2.2 GeV$ at $\beta = 6$ with $K_W = 0.1515$ and:

$$\Delta E a = 0.31$$

$$f_B^{Latt} = 131 MeV \quad (11)$$

using $a^{-1} = 3.0 GeV$ at $\beta = 6.2$ with $K_W = 0.1500$. The values for f_B^{Latt} reported in eqs.(10) and (11) are quite unstable due to the large fluctuations in the correlation

functions. The result in fact oscillate between 50 and $260 MeV$ when we change the time interval of the two exponential fit. For this reason we believe that our most reliable estimate is the bound given in eq.(8).

To obtain the physical value of the meson decay constant we use the relation:

$$f_M = \left(\frac{\alpha_s(M)}{\alpha_s(1/a)} \right)^\gamma \left(1 + \frac{\alpha_s}{2\pi} C \right) f_M^{latt} \quad (12)$$

where the first factor is due to the anomalous dimension of the axial current present when the ultra-violet cut-off $1/a$ is smaller than the heavy meson mass[6]. The second factor connects the lattice axial current operator to the corresponding operator renormalized in the continuum. It has been computed in ref.[7] where they show that the effect of the two factors in eq.(12) for the B-meson decreases the lattice result by about 20%. Taking the largest acceptable value of the inverse lattice spacing at $\beta = 6$, $a^{-1} = 2.2 GeV$, using the figure reported in eq.(8) and the relation $f_B \simeq 0.8 f_B^{latt}$ we obtain:

$$f_B \leq 210 MeV \quad (13)$$

Given the uncertainties of this determination we do not try to give any estimate of the statistical error on the number reported in eq.(13). A real upper limit at the 90% confidence level would require a much larger statistical sample.

The exercise done in this work shows that for systems made of a light quark and a static colour source it is necessary to study correlation functions at large distances in order to isolate the lightest state. For this reason a larger statistics than in the case of mesons composed of light quarks is needed. The results reported in ref.[3] were obtained at $\beta = 5.7$ on a $8^2 * 16 * 24$ lattice, which was certainly too short in time to isolate the lightest pseudoscalar particle and thus it was not possible to extract any value for f_B .

From this study on larger lattices we are encouraged to continue the program of the calculation of the B-meson properties following the suggestion by Eichten and we are planning to improve our results on a larger sample of gauge field configurations on the APE machine[8].

ACKNOWLEDGEMENTS

We thank M.B.Gavela, L.Maiani, G.C. Rossi and M.Testa for continuous and fruitful discussions on the subject of this paper and on the renormalization of the relevant operators. We thank all the members of the Computer Center of RAL, of CCVR at Ecole Polytechnique and of CINECA for help and assistance in running the programs.

Figure captions

Fig.(1) The axial current-axial current zero momentum correlation function of eq.(1), computed using eq.(4), is reported as a function of the dimensionless time distance ta . The correlation has been obtained from 30 configurations on a $20 * 10^2 * 40$ lattice, at $\beta = 6$ and $K_{1V} = 0.1515$.

Fig.(2) $R(t) = \ln[\bar{G}(t)/\bar{G}(t+1)]$ as a function of ta from the correlation given in fig.(1).

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- Figure 1 -

$R(t)$ K -0.1515

- Figure 2 -