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TOP QUARK MASS AND MIXINGS IN THE PRESENCE OF A HEAVY QUARK FAMILY

Carl H. ALBRIGHT*

CERN - Geneva

Abstract

The top quark mass and mixings are studied in the framework of an extended Fritzsch model for purposes of illustration, where the fourth-family of heavy quarks either form a standard doublet or exist simply as isosinglets. We have fixed the masses of the heavy quarks near the infrared fixed point, unlike previous studies which select the top quark mass in a narrow range and then try to predict the heavy quark masses. The models are satisfactory with a top mass near 70 GeV and suggest that the appearance of a top quark mass significantly lower than 90 GeV, as favored by three quark families in previous studies, can serve as a strong hint of new physics beyond the standard model.

*Permanent address: Department of Physics, Northern Illinois University, DeKalb, Illinois 60115

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In a recent series of papers,¹⁻⁴ the author, in collaboration early on with Jarlskog and Lindholm and more recently with Lindner, has explored in some detail three family quark mass matrices in the weak basis and their predictions for the KM mixing matrix, $B - \bar{B}$ mixing, $|V_{ub}|/|V_{cb}|$, the bag parameter in K decay and other accessible experimental quantities of interest. The mass matrices which appear to give the best fits to all the known data are those of Fritzsch⁵ based on hierarchical chiral symmetry breaking and modifications thereof suggested by Lindner and the author.⁴ The standard Fritzsch model is satisfactory² only in the presence of two Higgs doublets, while the model suggested in ref. 4 also works with the minimal Higgs structure. Corrections have also been given^{3,4} to account for mass matrix evolution from a high chiral symmetry-breaking scale ~ 100 TeV, for example, down to the 1 GeV scale, due to nonlinear effects introduced by Higgs exchange graphs. The outcome of this study is that the present known data can be fit in the three quark framework, provided the top quark mass is approximately 90 GeV. Although many other authors⁶ have also looked into this problem, our approach through the invariant operator trace technique of Jarlskog,⁷ along with inclusion of nonlinear evolution effects, has enabled us to present rather precise results for each model in a graphically clear fashion.

In this letter we proceed to study the case of four quark families. Although the literature on four standard doublet families is also rather extensive,⁸ we adopt a different approach. In place of setting the top quark mass in some range like 30 - 60 GeV to find the masses of the fourth-family members, we shall fix the heavy fourth-family masses near the infrared fixed point to study the restrictions on the top quark mass and mixings. Of even more interest are the recent suggestions⁹ that, in the framework of an $SU(2)_L \times SU(2)_R \times U(1)_Y$ theory, a family of isoscalar quarks or mirror quarks can become massive at some higher scale and, in turn, generate

masses for the standard three quark families by a seesaw mechanism and radiative corrections. These models lead naturally to flavor-changing neutral currents in tree level. Given the apparently excellent results previously obtained for three quark families and the possibility that a fourth quark family actually exists, the question arises whether the new family effectively decouples from the other three, or whether suitable mixings can be generated also in accord with present data. If the latter is the case, we would naturally guess that the predictions for the top quark mass would be substantially altered. We shall show that satisfactory mixing results can be obtained, now with the somewhat lower value of $m_t \simeq 70$ GeV. This present study remains incomplete in that the rather complex evolution effects have not yet been calculated, but we have tried to minimize those effects for purposes of illustration by restricting the choice of fourth-family masses so as to present these very timely results as soon as possible. The evolution details will be presented elsewhere, along with a more comprehensive selection of fourth family masses.

For the purposes of this initial study and to reduce the complexity as much as possible, we shall assume that mixings with the fourth family do occur but adopt the simple nearest-neighbor Fritzsche form of the mass matrices in the weak basis as extended to four families:

$$\mathbf{M}^U = \begin{pmatrix} 0 & A & 0 & 0 \\ A & 0 & B & 0 \\ 0 & B & 0 & D \\ 0 & 0 & D^* & E \end{pmatrix}, \quad \mathbf{M}^D = \begin{pmatrix} 0 & A' & 0 & 0 \\ A'^* & 0 & B' & 0 \\ 0 & B'^* & 0 & D' \\ 0 & 0 & D'^* & E' \end{pmatrix} \quad (1a)$$

This implies that the t and b quarks get massive at the second stage of chiral symmetry breaking via a seesaw mechanism and/or radiative corrections. We have eliminated the unphysical phases by a diagonal phase rotation, so that 12 parameters are available to predict the 17 independent physical parameters (8 masses + 6

mixing angles and 3 phases of the extended KM matrix). For these Hermitian mass matrices, the 8 magnitude parameters can be evaluated from the invariant traces $Tr \mathbf{M}, Tr \mathbf{M}^2, Tr \mathbf{M}^3$ and determinant, $Det \mathbf{M}$, where we identify the diagonal mass matrices in the mass eigenbases as

$$\mathbf{D}^U = diag(m_u, -m_c, m_t, -m_X), \quad \mathbf{D}^D = diag(m_d, -m_s, m_b, -m_Y) \quad (1b)$$

with the signs chosen to satisfy positivity requirements for $|D|^2$, etc. We are then left with just $\phi_D, \phi_{A'}, \phi_{B'}$ and $\phi_{D'}$ to explain the 9 extended KM mixing angles and phases. Of course, m_t, m_X and m_Y are unknown; we fix $m_X^{phys} = 250$ GeV and $m_Y^{phys} = 240$ GeV, which are values associated with the infrared fixed points, a theoretically appealing choice and one which minimizes the nonlinear effects of evolution¹⁰ from the $\Lambda_{SB} \sim 100$ TeV scale down to 1 GeV and minimizes, as well, the radiative corrections to the ρ parameter.¹¹

The 4×4 unitary matrices U and U' which diagonalize the Hermitian matrices in (1a) satisfy

$$U \mathbf{M}^U U^\dagger = \mathbf{D}^U, \quad U' \mathbf{M}^D U'^\dagger = \mathbf{D}^D \quad (2)$$

and relate the weak and mass eigenstates with four standard doublet families by

$$\begin{aligned} \psi^U &= (u', c', t', X')^T = U^\dagger \Psi^U = U^\dagger (u, c, t, X)^T \\ \psi^D &= (d', s', b', Y')^T = U'^\dagger \Psi^D = U'^\dagger (d, s, b, Y)^T \end{aligned} \quad (3)$$

so the extended 4×4 unitary KM matrix is just the matrix product

$$(\mathbf{V}_{KM})_{\alpha\beta} = U_{\alpha\gamma} (U'^\dagger)_{\gamma\beta}; \quad \alpha, \beta, \gamma = 1, 2, 3, 4 \quad (4a)$$

However, in the case where the fourth family is isosinglet and only three quark families participate in the $SU(2)_L$ charged current weak interactions, the three pairs of weak eigenstates are given by $\psi_{1,2,3}^U = u', c', t'$ and $\psi_{1,2,3}^D = d', s', b'$,

respectively. The fourth combinations, $\psi_4^U = X'$ and $\psi_4^D = Y'$, are the irrelevant weak isoscalars. In this latter case, the extended 4×4 KM matrix is nonunitary and given by

$$(\mathcal{V}_{KM})_{\alpha\beta} = U_{\alpha i}(U'^{\dagger})_{i\beta}; \quad \alpha, \beta = 1, 2, 3, 4; \quad i = 1, 2, 3 \quad (4b)$$

in terms of the product of 4×3 and 3×4 matrices. In either case, the “observed” KM matrix is

$$(V_{KM}^{obs})_{ij} = (\mathbf{V}_{KM})_{ij}; \quad i, j = 1, 2, 3 \quad (5a)$$

or

$$(V_{KM}^{obs})_{ij} = (\mathcal{V}_{KM})_{ij}; \quad i, j = 1, 2, 3 \quad (5b)$$

respectively.

For comparison with the experimental information on the mixing matrix, we use the recent analysis of Schubert¹² for three quark families described by the mixing matrix

$$(|\mathbf{V}_{ij}|) = \begin{pmatrix} 0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0.0000 \pm 0.0123 \\ 0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0460 \pm 0.0060 \\ 0.0101 \pm 0.0122 & 0.0449 \pm 0.0065 & 0.9989 \pm 0.0003 \end{pmatrix} \quad (6)$$

but impose only the constraints on V_{ud}, V_{us}, V_{cd} and V_{cb} in our extension to four quark families. The other entries have been considerably restricted by three-family unitarity.*

In the case of four standard families, the absolute square of the 4×4 KM matrix can be calculated directly with the help of Jarlskog's projection operator trace technique⁷

$$|(\mathbf{V}_{KM})_{\alpha\beta}|^2 = Tr (P_{\alpha} P'_{\beta}) \quad (7a)$$

*For another somewhat less restrictive determination of the extended KM matrix, see the analysis of Gilman, Kleinknecht and Renk in ref. 13.

where the α th projection operator for the up quark sector is

$$P_\alpha = \frac{(\mathbf{M}^U - \lambda_\beta I)(\mathbf{M}^U - \lambda_\gamma I)(\mathbf{M}^U - \lambda_\delta I)}{(\lambda_\alpha - \lambda_\beta)(\lambda_\alpha - \lambda_\gamma)(\lambda_\alpha - \lambda_\delta)} \quad (7b)$$

and λ_α , etc., are the mass eigenvalues of the up quark matrix given in (1b) with the appropriate signs; the subscripts are permutations of 1,2,3,4. In the case of an isosinglet fourth family, we have the identity

$$(\mathcal{V}_{KM})_{\alpha\beta} = (\mathbf{V}_{KM})_{\alpha\beta} - U_{\alpha 4} U'_{4\beta}^\dagger \quad (8a)$$

From this it follows that the square of this mixing matrix element can be written as a generalization of the Jarlskog trace technique

$$|(\mathcal{V}_{KM})_{\alpha\beta}|^2 = \text{Tr} \left(P_\alpha P'_\beta - E_4 P_\alpha P'_\beta E_4 - E_4 P'_\beta P_\alpha E_4 + E_4 P_\alpha E_4 P'_\beta E_4 \right) \quad (8b)$$

which takes into account the nonunitary nature of $\mathcal{V}_{KM}$. Here $E_4 = \text{diag} (0, 0, 0, 1)$.

As was previously done¹⁻⁴ in the case of three-family mass matrices, the squares of the KM matrix elements are fitted to within one standard deviation, but now only for V_{ud} , V_{us} , V_{cd} and V_{cb} by varying m_t , $\phi_{A'}$, $\phi_{B'}$, ϕ_D and $\phi_{D'}$. The results are insensitive to the two phase angles ϕ_D and $\phi_{D'}$, and we can set them equal to zero. With the lighter quark masses chosen following the Gasser-Leutwyler prescription¹⁴ as in ref. 4, the results are shown in Fig. 1a and 1b for the fourth-family doublet quarks and isoscalar quarks, respectively, where the physical regions for the KM matrix appear as annular rings with the phase angle $\phi_{A'}$ constrained to $85^\circ \pm 2^\circ$.

The box diagrams for the $B_d - \bar{B}_d$ mixing parameter x_d in the three family case are proportional to the combinations $m_t^2 |V_{td}^* V_{tb}|^2 R$, where R is a correction factor involving logarithms of quark masses and the charged Higgs mass in the case of two doublet Higgs models.² From the ARGUS and CLEO data,¹⁵ one finds that this

combination is approximately equal to²

$$m_t^2 |V_{td}^* V_{tb}|^2 R = (2.0 \pm 0.5) \frac{(0.140)^2}{B_B f_B^2} \quad (9a)$$

All three factors on the lefthand side of (9a) vary as one sweeps through the $\phi_{B'}$ vs. m_t plane. With the four-family models of interest in this letter, major contributions to x_d arise from the X quark, as well as crossed terms involving t and X exchange. Inspection of the KM matrices derived for each case by fitting the Schubert data according to the prescription described above, however, reveals that the X quark exchange dominates the one-loop graphs, especially in the isosinglet case. For simplicity, we replace (9a) by the approximation

$$m_X^2 |V_{Xd}^* V_{Xb}|^2 R_X = (2.0 \pm 0.5) \frac{(0.140)^2}{B_B f_B^2} \quad (9b)$$

where R_X is evaluated for the X quark rather than the t quark. With m_X^{phys} fixed at 250 GeV, we have $m_X = m_X(m_X) = 241$ GeV, $R_X = 0.47$ for the minimal Higgs model and $R_X = 1.33$ for the two-doublet Higgs model, where we have chosen 50 GeV for the charged Higgs scalar mass and have set the two vacuum expectation values equal. Now only the square of the KM matrix elements on the lefthand side of (9b) varies with $\phi_{B'}$ and m_t .

The physically-allowed regions for the $B_d - \bar{B}_d$ mixing results appear as bands, with the single-hatched regions referring to the standard Higgs model and the double-hatched regions to the two-doublet Higgs model. Overlap of the ring and band represents the true physical region for the model at hand. With the assignments of $m_X^{phys} = 250$ GeV and $m_Y^{phys} = 240$ GeV, we see that the preferred top quark mass is about 70 GeV for four standard quark families and two Higgs doublets; with two isosinglet quarks, the preferred top mass is again about 70 GeV but with minimal Higgs structure, while the top mass drops to about 46 GeV with

two Higgs doublets. In fact, this latter possibility is ruled out by the recent lower bounds determined by the CDF, UA1 and UA2 groups.¹⁶

It is of interest to compare the predictions of other physical quantities for four families with those derived earlier for three families. This is done in Table I, where we have included only the three-family results without evolution. As shown in ref. 3, evolution from a 100 TeV scale down to 1 GeV changes the three family results by 5 - 10%. The effective J value associated with CP violation can again be defined by¹⁷

$$\begin{aligned} J_{eff} &\equiv \text{Im} (V_{us} V_{cb} V_{ub}^* V_{cs}^*) \\ &\simeq |V_{us}| |V_{cb}| |V_{ub}| |V_{cs}| \sin \delta \end{aligned} \quad (10)$$

from which the CP phase angle δ can be determined, where we have chosen the new standard parametrization of the KM matrix in ref. 18. The x_s parameter for $B_s - \bar{B}_s$ mixing is approximately $x_d |V_{Xs}|^2 / |V_{Xd}|^2$, while the $b \rightarrow u/b \rightarrow c$ ratio again involves $|V_{ub}|/|V_{cb}|$. The bag parameter B_K in K decay can again be determined from ϵ_K , and to a good approximation is given by¹⁹

$$B_K \simeq |\epsilon_K| \left[\frac{G_F^2}{12\sqrt{2}\pi^2} \frac{m_K}{\Delta m_K} f_K^2 \eta_2 m_X^2 R_X \text{Im} (V_{Xs}^* V_{Xd})^2 \right]^{-1} \quad (11)$$

Unfortunately, this result is not invariant, unlike J in Eq. (10), but depends on the parametrization adopted for the phases in the KM mixing matrix. We choose $\eta_2 = 0.6$ and evaluate B_K for the convention of ref. 16 in Table I. The results for ϵ'/ϵ as determined from penguin diagrams are even more complex with four families present and have not been evaluated.

As seen from Table I, most of the results are in reasonable agreement with the presently available information with the exception of the isosinglet case with two Higgs doublets, where a value of $m_t = 46$ GeV has already been excluded.¹⁶ The bag parameter in K decay, as calculated in the parametrization of ref. 18, tends

to be low with four families, as already noted by Gronau, Johnson and Schechter²⁰ on the basis of a model-independent argument. We do not feel, however, that these results are sufficient to exclude the four-family models at this time, given the parametrization noninvariance and phase uncertainty mentioned above. In fact, a relative phase rotation of the first and second columns by just 12° is sufficient to restore B_K to the preferred value near 0.66 quoted by Bardeen, Buras and Gérard in ref. 21.

As a further comparison of the two four-family cases, we present the extended KM matrices for the special points in the central overlap regions of Fig. 1 that have been singled out in Table I. Although, in principle, one should be able to calculate all the matrix elements from their absolute values and unitarity,²² when it applies, these evaluations were carried out by exactly computing the unitary matrices U and U' of (2) and then making use of Eqs. (4a,b). Finally, phase rotations of the quark fields then enable us to write V_{KM} in the phase convention of ref. 18 for the fourth-doublet case

$$V_{KM}^{4th DOUBLET} = \begin{pmatrix} 0.9753 & 0.2211 & -0.0004 & -0.0015 \\ -0.2205 & 0.9728 & 0.0467 & -0.0482 \\ 0.0069 & -0.0265 & 0.9383 & 0.3445 \\ -0.0123 & 0.0601 & -0.3425 & 0.9373 \end{pmatrix} \pm i \begin{pmatrix} 0 & 0 & 0.0027 & -0.0023 \\ 0 & 0.0012 & 0 & 0.0228 \\ 0 & 0.0079 & 0.0023 & 0 \\ -0.0078 & 0.0203 & 0.0003 & 0 \end{pmatrix} \quad (12a)$$

and for the isosinglet case

$$V_{KM}^{2SINGLETs} = \begin{pmatrix} 0.9753 & 0.2210 & -0.0004 & -0.0001 \\ -0.2202 & 0.9719 & 0.0508 & -0.0109 \\ 0.0039 & -0.0148 & 0.8889 & 0.0996 \\ 0.0174 & -0.0778 & 0.4416 & 0.0507 \end{pmatrix} \pm i \begin{pmatrix} 0 & 0 & 0.0027 & -0.0006 \\ 0 & 0.0016 & 0 & 0 \\ 0.0008 & 0.0001 & 0 & 0 \\ 0.0040 & 0.0008 & 0 & 0 \end{pmatrix} \quad (12b)$$

It is clear from these matrices that, while V_{KM} is unitary in the former case and nonunitary in the latter case, in both cases some elements in the fourth row are larger than their counterparts in the third row. This plays a major role in the successful fit to the $B - \bar{B}$ mixing data given in Eq. (9b) but negates the simplifying procedure of Bjorken²³ to extract the phases of the KM matrix elements from their absolute values. In both cases, we find that the ratio $|V_{ub}|/|V_{cb}| = 0.053$, as in the three family Fritzsch model with two Higgs doublets.² It is also clear that the 3×3 submatrices for the three observed families are not greatly different from that of (6), deduced by Schubert from present data and three family unitarity; however, we note that the fourth row and fourth column are considerably more asymmetric than the first three, especially with isosinglets present.

Of special interest in the isoscalar model is the appearance of tree-level flavor-changing neutral currents (FCNC's). The neutral current in this case is given by

$$J_\mu^{(0)} = \bar{\psi}_L^U \gamma_\mu \psi_L^U - \bar{\psi}_L^D \gamma_\mu \psi_L^D - 2 \sin^2 \theta_W J_\mu^{em} \quad (13a)$$

$$= \bar{\Psi}_L^U \gamma_\mu V_{NC}^U \Psi_L^U - \bar{\Psi}_L^D \gamma_\mu V_{NC}^D \Psi_L^D - 2 \sin^2 \theta_W J_\mu^{em} \quad (13b)$$

where in (13a) only the three weak doublet fields enter, while the four up and

down mass eigenfields appear in (13b). The neutral current mixing matrices for the lefthanded currents are related to the 4×4 unitary matrices U and U' in (2) by

$$(V_{NC}^U)_{\alpha\beta} = U_{\alpha i}(U^\dagger)_{i\beta} = \delta_{\alpha\beta} - U_{\alpha 4}(U^\dagger)_{4\beta} \quad (14a)$$

$$(V_{NC}^D)_{\alpha\beta} = U'_{\alpha i}(U'^\dagger)_{i\beta} = \delta_{\alpha\beta} - U'_{\alpha 4}(U'^\dagger)_{4\beta} \quad (14b)$$

In the phase convention where V_{NC}^U is taken to be real and for the parameters of the isoscalar model in the center of the physical region in Fig. 1b where $m_t = 70.5$ GeV, the explicit forms of these two FCNC matrices, are calculated to be

$$V_{NC}^U = \begin{pmatrix} 1.0000 & -0.0002 & 0.0015 & 0.0030 \\ -0.0002 & 0.9958 & 0.0289 & 0.0576 \\ 0.0015 & 0.0289 & 0.8002 & -0.3988 \\ 0.0030 & 0.0576 & -0.3988 & 0.2040 \end{pmatrix} \quad (15a)$$

$$V_{NC}^D = \begin{pmatrix} 1.0000 & -0.0001i & 0.0004i & 0.0037i \\ 0.0001i & 0.9997 & 0.0018 & 0.0160 \\ -0.0004i & 0.0018 & 0.9877 & -0.1104 \\ -0.0037i & 0.0160 & -0.1104 & 0.0126 \end{pmatrix} \quad (15b)$$

Note that the $\bar{c}tZ$ and $\bar{t}cZ$ couplings are at the level of 0.03 and suggest that the FCNC Z decay, $Z \rightarrow \bar{c}t + \bar{t}c$, may be observable in the future at LEP and SLC. The large value for the $\bar{X}tZ$ and $\bar{t}XZ$ couplings suggests that the $X \rightarrow tZ$ decay will compete favorably with the $X \rightarrow YW$ decay, as seen from (15b) and (12b).

By starting from the Fritzsch form of the mass matrices extended to four standard quark families or to three families plus a pair of isosinglet quarks, we have studied the viability of these two models in explaining both the “observed” KM mixing matrix and large $B_d - \bar{B}_d$ mixing. For this purpose, we have imposed restrictions only on the accurately known V_{ud} , V_{us} , V_{cd} and V_{cb} elements of the KM mixing matrix. We have also set the heavy quark masses at $m_X = 250$ GeV and $m_Y = 240$

GeV, comparable to the infrared fixed points, so as to minimize radiative corrections to the ρ parameter and renormalization effects on the mass matrices which will be reported elsewhere. Our results indicate that, for either standard model extension, good agreement can be obtained with a top quark mass of $m_t \simeq 70$ GeV, if two Higgs doublets are used for the four doublet version and just minimal Higgs structure for the isosinglet version. The flavor-changing neutral current matrices appearing in the isosinglet case are of considerable interest in rare Z decay studies at the near-operational e^+e^- colliders. The 70 GeV top mass is to be contrasted with our previous result of 94 or 91 GeV (88 GeV with renormalization effects included) in the presence of just three standard doublet quark families. If we drop the heavy quark masses to 150 GeV and 140 GeV, respectively, we find the top quark mass can be lowered to approximately 60 GeV. This suggests rather strongly that the appearance of a top quark mass significantly lower than 90 GeV can be a strong hint of new physics beyond the standard model, as we now know it.

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Fig.	$m_t(1\text{GeV})$ (GeV)	m_t^{phys} (GeV)	$\phi_{A'}$	$\phi_{B'}$	ϕ_D	$J \times 10^4$	δ	$m_X^2 V_{Xd} ^2 R_X$	$ V_{Xs} ^2/ V_{Xd} ^2$	x_s	$ V_{ub} / V_{cb} $	B_K	ϵ'/ϵ $\times 10^4$
Four-Family Doublets with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$													
1a	110	71	85°	8°	*	0.27	98°	1.92	19.0	13.5	0.058	0.04	
Three-Family Doublets, Two Isosinglets with Minimal Higgs Structure													
1b	110	71	85°	0°	*	0.29	99°	1.71	18.9	12.0	0.053	0.05	
Three-Family Doublets, Two Isosinglets with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$													
1b	70	46	85°	18°	*	0.30	92°	2.04	18.9	14.2	0.065	0.05	
Fig.	$m_t(1\text{GeV})$ (GeV)	m_t^{phys} (GeV)	$\phi_{A'}$	$\phi_{B'}$	ϕ_D	$J \times 10^4$	δ	$m_t^2 V_{td} ^2 R$	$ V_{ts} ^2/ V_{td} ^2$	x_s	$ V_{ub} / V_{cb} $	B_K	ϵ'/ϵ $\times 10^4$
Three-Family Fritzsch Model with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$													
1a	150	94	85°	0°		0.30	99°	1.86	17.2	11.9	0.053	0.51	8
New Three-Family Model of Ref. 4 with Minimal Higgs Structure													
1d	145	91	80°	2°	10°	0.30	157°	1.79	6.9	4.6	0.149	0.72	13
New Three-Family Model of Ref. 4 with Two-Doublet Higgs Structure and $m_\eta = 50$ GeV, $v_2/v_1 = 1.0$													
1d	110	71	80°	14°	45°	0.33	153°	2.06	6.8	5.2	0.156	0.60	13
1d	125	80	80°	1°	0°	0.30	154°	2.41	5.9	5.3	0.179	0.61	15
Experimental Results and Theoretical Predictions													
Ref.		?				0.30	?	2.0 ± 0.5	?	?	0 - 0.17	0.66 ± 0.10	33 ± 11
						24		15			25	21	26

Table I. Values of parameters associated with selected points in the plots of Figs. 1, with a comparison of the three-family models of ref. 4. The results are insensitive to the values of ϕ_D and $\phi_{D'}$ in the four-family cases considered here.

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Figure Caption

Figure 1: Phase angle $\phi_{B'}$ vs. $m_t(1\text{GeV})$ and m_t^{phys} plots showing the physically-allowed KM matrix annular rings and $B_d^0 - \bar{B}_d^0$ mixing bands, the latter single-hatched for the standard Higgs model and double-hatched for the two-doublet Higgs model. Here (a) refers to the four doublet model and (b) to the three doublet plus two isosinglet model. The masses for the fourth-family heavy quarks have been set at 250 GeV for the charge $2/3$ X quark and 240 GeV for the charge $-1/3$ Y quark. Equal VEV's have been used for the double Higgs model with 50 GeV for the charged Higgs mass.