IS THE UNIVERSE EUCLIDEAN ?

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ABSTRACT

We argue that a Lorentzian approach to the quantum behaviour of gravity is healthier than a Euclidean approach. In a Lorentzian formulation we show how it is possible to obtain a convergent kernel describing the quantum evolution of the universe from a path integral. We also present two straightforward examples which illustrate the advantages of the Lorentzian formalism. Its implications for wormhole dynamics are discussed.

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1. INTRODUCTION

There is a controversy stirring at present regarding the correct formulation of quantum gravity. It is concerned with the range of validity of certain methods in Euclidean space. These methods are used extensively in some of the recent work purporting to demonstrate the vanishing of the cosmological constant [1] through the interaction of our universe with wormholes [2]. The aim of this paper is to provide a case against the Euclidean methods, and for a Lorentzian approach to the theory.

Of course, quantum gravity is not an experimental science – energies on the order of 10^{19} GeV are simply not accessible for inspection. It is still not clear if it is even a theoretical science, due to almost insurmountable technical problems with fitting general relativity into a quantum mechanical framework. It has been suggested that the two theories are at heart incompatible. Quantum mechanics is inherently non-local, and this goes against the grain of relativity.* However, string theory has made remarkable inroads to this much-sought-after goal of contemporary physics, even though the physical principles upon which it is based are not yet known. It might well be that a firm conceptual basis for string theory and the reconciliation of quantum mechanics with relativity are one in the same thing.

Our only testing ground for theories at energies close to the Planck scale is at the early stages of our universe. Inflationary models of cosmology give clear reasons for why the universe we see today is homogeneous, isotropic, and devoid of such nasties as monopoles. One of the basic mechanisms in inflation is that of quantum tunnelling through a barrier. It is important to stress that there are really two usages of the word “tunnelling”. One is the tunnelling of a scalar field through a potential barrier, on a classical background space. The other is where the space itself undergoes a transition from one geometry to another.

In the first case the dominant contribution to the tunnelling amplitude comes from a semi-classical approximation [4]. In this approximation calculations of the decay rate are most easily performed by Wick rotating the time axis to be imaginary. This can be thought of as dynamics in a Euclidean spacetime. The solution can be rotated back and re-interpreted as a physical Lorentzian process. However, it is not clear how to proceed with this Euclidean picture when higher order quantum corrections are to be included. Fortunately, there do exist other methods for analyzing barrier penetration [5] and they can be employed if it is

* For a readable account of this see the essays of Bell [3], numbers 16 and 18.

felt that the Euclidean approach is deficient. Regarding Euclidean space as somehow having a *fundamental reality* therefore has little theoretical basis – Wick rotating recklessly will give wrong results. The picture is a helpful metaphor only when used properly.

The second case, of topology change via tunnelling of the space itself, is best discussed in the context of quantum cosmology [6]. In this situation questions of which description of space (Lorentzian or Euclidean) is more fundamental are unavoidable. Our discussions will be based in quantum cosmology. In the next section we give a short primer on the subject which may be skipped by experts. In section 3 we show that a convergent Green’s function can be derived from a Lorentzian path integral without recourse to Wick rotation. Section 4 discusses Euclidean formulations of quantum cosmology. Thereafter we present instructive examples which highlight the differences between the two formalisms, and we show that the Lorentzian viewpoint is the more fundamental.

2. A QUANTUM COSMOLOGY PRIMER

It suffices for the study of cosmology to assume that the universe is homogeneous. We know that at a large enough scale one place looks much like another. However, it need not be isotropic. A suitable choice of Lorentzian metric is then,

$$ds^2 = -\mathcal{N}_L^2 dt \otimes dt + h_{ij} e^i \otimes e^j \quad (2.1)$$

where $\mathcal{N}_L$ and h_{ij} are functions of the parameter t only, and $\{e^i\}$ is the basis for a homogeneous spatial three-metric. $\mathcal{N}_L$ is called the *lapse function* – it relates t to the proper time. The h_{ij} represent any anisotropies the space may have. The action for gravity is the usual Einstein action,

$$S_L = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^4x \sqrt{-g} (R - 2\Lambda) + \frac{1}{8\pi G_N} \int_{\partial\mathcal{M}} d^3x \sqrt{h} (K - K^0) \quad (2.2)$$

where the last (surface) term is included if the manifold has a boundary [7]. $\mathcal{M}$ is a manifold with metric given by (2.1), and $\partial\mathcal{M}$ a space-like boundary to the manifold. K is just the trace of the second fundamental form on $\partial\mathcal{M}$, *i.e.* $K = h^{ij}K_{ij}$, and for our choice of metric $K_{ij} = -\dot{h}_{ij}/2\mathcal{N}_L$. K^0 is the trace of the second fundamental form of $\partial\mathcal{M}$ when imbedded in flat space. This last term is arbitrary, but if the metric is asymptotically flat it is often convenient to renormalize K so that it vanishes at infinity and the action is finite.

At any given time slice, the three-metric h_{ij} describes the spatial geometry on this chosen slice. Such slices can be thought of as initial or final boundaries of some four-geometry. Between any two slices there are many four-geometries which induce the appropriate three-metrics on these boundaries. Most of these are not extrema of the action. The basic idea of quantum cosmology is that we treat the boundaries as end points of a path. Then the h_{ij} 's are the metric configurations at these end points. The amplitude to propagate from one three-geometry to another is a path integral over all four-geometries (weighted by the action) with fixed end geometries. Classical paths which are solutions to the Einstein equations are expected to dominate this path integral. In the more general case (where we do not assume homogeneity) the functional space of all h_{ij} 's is called *superspace*. The amplitude is a functional defined on superspace.

We further restrict ourselves to *minisuperspace*, for the sole reason that we are then able to perform explicit calculations. It is hoped that any conclusions we may draw will be generally applicable to the more complete theory in superspace. In that case the h_{ij} can be expressed in terms of a few parameters only, which represent the degrees of freedom of our chosen classical cosmology. One such useful parameter is $v = (\det h)^{\frac{1}{2}}$ which scales as the volume of the three-space. The significance of this conformal factor lies in the fact that it is a positive quantity, and that the classical singularity occurs at $v = 0$. Let us denote these degrees of freedom (the co-ordinates of the minisuperspace) generically by $\{q^\alpha\}$.

The gravitational action will then depend only on the q^α , their first derivatives with respect to t , and the lapse function $\mathcal{N}_L$ i.e. $S_L = S_L(q^\alpha, \dot{q}^\alpha; \mathcal{N}_L)$. (For the moment, we shall not consider the addition of matter. The extension to this case is left until later.) If there are boundaries to the spacetime manifold the surface contribution to the action always removes $\ddot{q}^\alpha$ terms upon integration by parts – this is why it is introduced in the first place. We are also implicitly assuming that any higher curvature terms present can be treated in a similar manner. In addition, the action depends only quadratically on the velocities.

The Lorentzian conjugate momenta are $p_\alpha = \frac{\partial S_L}{\partial \dot{q}^\alpha}$, leading to a re-expression of the action

$$S_L = \int_i^f (p_\alpha \dot{q}^\alpha - \mathcal{N}_L \mathcal{H} dt) \quad (2.3)$$

The integral is from the initial configuration of the three-geometry labelled by q_i^α to the final one q_f^α . The Hamiltonian is independent of the t parameter, depending only on the co-

ordinates and their momenta. There is no intrinsic meaning to time in quantum cosmology due to the reparametrization invariance of general relativity; t simply labels a path from the initial to the final configurations of the three-geometry. We can choose another labelling by a different choice of the lapse function. This is reflected in the absence of derivatives of $\mathcal{N}_L$, which is then a Lagrange multiplier. Variation of $\mathcal{N}_L$ implies that the classical Hamiltonian is constrained to vanish,

$$\mathcal{H}(p_\alpha, q^\alpha) = 0 \quad (2.4)$$

With the above assumptions about the action, the gravitational Hamiltonian looks like

$$\mathcal{H}(p_\alpha, q^\alpha) = \frac{1}{2\mu} f^{\alpha\beta}(q) p_\alpha p_\beta + V(q) \quad (2.5)$$

where μ is proportional to M_P^2 . The $f^{\alpha\beta}$ ought really to be thought of as the scale factors of a curvilinear co-ordinate system, rather than as a metric. Its signature is typically $(-++\dots)$, the negative sign being associated with the scale factor v . Canonical quantization is achieved by the usual replacement of p_α by $-i\nabla_\alpha$. The Wheeler–DeWitt equation for the cosmological wavefunction in the purely gravitational case is the quantum mechanical version of (2.4),

$$\left[-\frac{1}{2\mu} f^{\alpha\beta}(q) \nabla_\alpha \nabla_\beta + V(q) \right] \Psi(q) = 0 \quad (2.6)$$

There is still a problem of operator ordering. However, most of our analysis which follows shall rely simply on semi-classical approximations to wavefunctions, and in this case the ambiguities do not give rise to any serious difficulties[★]. We refer the reader to the discussion of ref. [9]. We are then faced with the task of imposing suitable boundary conditions on this wavefunction. Before delving into the controversy surrounding this issue we should like to discuss the path integral approach to quantum cosmology.

★ If one *does* regard $f^{\alpha\beta}$ as a metric and demand covariance under co-ordinate transformations then ∇_α should be taken to be the *covariant* derivative in the (curved) minisuperspace. This automatically fixes the operator ordering in (2.6). In addition, demanding invariance under co-ordinate dependent redefinitions of the lapse function implies that one must add a term to the Wheeler–DeWitt operator of the form $-\frac{(d-2)}{8(d-1)} \mathcal{R}$ where $\mathcal{R}$ is the scalar curvature of the minisuperspace and d is the dimension of the minisuperspace. This is just the requirement that the operator be conformally covariant. For a discussion of this see ref. [8].

3. THE KERNEL OF QUANTUM COSMOLOGY

In conventional quantum mechanics and quantum field theory the path integral (or sum-over-histories) and operator approaches complement each other [10]. The Schrödinger equation can be derived from the path integral, and the path integral can be constructed from the Schrödinger equation. One is regarded as no more fundamental than the other. However, the path integral approach does confer certain advantages, as we have learnt from its use in gauge theories. By analogy with the usual procedure, we would expect the evolution kernel in quantum cosmology to be a path integral of the form,

$$K(q_f^\alpha; q_i^\alpha) \sim \int \mathcal{D}q^\alpha \mathcal{D}p_\alpha \mathcal{D}\mathcal{N}_L e^{iS_L}$$

i.e. we sum over all paths with the given initial and final configurations of the three-geometry. The action is in the form of (2.3) and the p_α integral is free at the end-points. Due to the reparametrization invariance of the internal path parameter t through the lapse function, we have to integrate out this unphysical degree of freedom to prevent the over-counting of equivalent paths. This requires gauge fixing and the introduction of ghosts as per usual. All said and done [8], in the gauge $\dot{\mathcal{N}}_L = 0$ the result of integrating over the ghost fields is simply a factor $[t_f - t_i]$. The functional integral over $\mathcal{N}_L(t)$ reduces to an ordinary integral over the single value of $\mathcal{N}_L$. The kernel is actually

$$K(q_f^\alpha; q_i^\alpha) = \int_0^\infty d\mathcal{N}_L [t_f - t_i] \int \mathcal{D}q^\alpha \mathcal{D}p_\alpha e^{iS_L} \quad (3.1)$$

The remaining path integral in (3.1) can be expressed in terms of the more familiar kernel of quantum mechanics. First observe that since $\mathcal{N}_L$ is fixed for the internal path integral we may rescale the parameter t in (2.3). Then S_L depends on the lapse parameter only through the rescaled initial and final times,

$$S_L = \int_{T_i}^{T_f} \left(p_\alpha \frac{dq^\alpha}{dT} - \mathcal{H} \right) dT$$

with $T_{i,f} = \mathcal{N}_L t_{i,f}$. Regarding the coordinates and momenta as functions of this new variable we still have $q^\alpha(T_{i,f}) = q_{i,f}^\alpha$ at the end points. The path integral is then formally that of

the familiar quantum mechanical kernel,

$$\int \mathcal{D}p_\alpha \mathcal{D}q^\alpha e^{iS_L} = \langle q_f^\alpha, T_f | q_i^\alpha, T_i \rangle \quad (3.2)$$

where the $|q^\alpha, T\rangle$ are state vectors in the Heisenberg picture. Since the Hamiltonian is time independent this amplitude depends on T only through the difference $(T_f - T_i)$. We are free to choose the parameter T_i to be initially zero. Thus (3.1) is formally

$$K(q_f^\alpha; q_i^\alpha) = \int_0^\infty dT \langle q_f^\alpha, T | q_i^\alpha, 0 \rangle \quad (3.3)$$

and it is this expression that we shall take to be a starting point for our discussions on the relative merits of the Lorentzian and Euclidian formulations of quantum cosmology.

It is at this stage in the development that problems associated with giving meaning to the symbols in (3.3) become unavoidable. So far we have relied on standard formal manipulations and have derived a very compact expression for our kernel. At first glance it seems quite reasonable, and in keeping with our notions of time in quantum cosmology: we propagate from one configuration to the other, the paths being parametrized by a time T , and then we sum over all the parametrizations. However, as we shall point out, the usual procedures to ensure convergence of the integrals and identify a ground state do not apply here. The reason for this is that we are no longer able to perform a Wick rotation to complex time. As the signature of the metric has dynamical significance in general relativity we cannot be certain that we can change its sign with impunity. We should therefore regard any success in implementing these procedures as a luxury, rather than as a direct consequence of the construction of the theory.

Let us first look at the kernel and see if it is a Green's function corresponding to the Hamiltonian operator. By definition,

$$\hat{\mathcal{H}}_f \langle q_f^\alpha, T | = i \frac{\partial}{\partial T} \langle q_f^\alpha, T |$$

If we then apply the Wheeler-De Witt operator to (3.3), acting on the final coordinate, the integral can be readily performed,

$$\hat{\mathcal{H}}_f K(q_f^\alpha; q_i^\alpha) = i \left[\langle q_f^\alpha, T | q_i^\alpha, 0 \rangle \right]_0^\infty \quad (3.4)$$

If we can arrange so that $\langle q_f^\alpha, T | q_i^\alpha, 0 \rangle \rightarrow 0$ as $T \rightarrow \infty$ then we would immediately discover

that

$$\hat{\mathcal{H}}_f K(q_f^\alpha; q_i^\alpha) = -i\delta(q_f^\alpha; q_i^\alpha)$$

implying that K is a Feynman Green's function.^{*} So we focus our attention on the amplitude as $T \rightarrow \infty$.

Insert a complete set of eigenstates of the Hamiltonian, $\mathcal{H}|n\rangle = E_n|n\rangle$, into the amplitude to write

$$\langle q_f^\alpha, T | q_i^\alpha, 0 \rangle = \sum_n \phi_n(q_f^\alpha, T) \phi_n^*(q_i^\alpha, 0)$$

where $\phi_n(q^\alpha, T) = \langle q^\alpha, T | n \rangle$. As the eigenvectors' T development is simply $\phi_n(q^\alpha, T) = e^{-iE_n T} \phi_n(q^\alpha, 0)$ we can express the amplitude as a sum over energy states,

$$\langle q_f^\alpha, T | q_i^\alpha, 0 \rangle = \sum_n e^{-iE_n T} \phi_n(q_f^\alpha, 0) \phi_n^*(q_i^\alpha, 0) \quad (3.5)$$

It is customary at this stage to Wick rotate $T \rightarrow -i\tau$ and take the limit $\tau \rightarrow \infty$ so that only the state with lowest energy survives. Indeed it is in this manner that the vacuum-to-vacuum amplitude arises in quantum field theory. In our case we would prefer that the amplitude vanish, which would ordinarily occur if all energies were positive. The reason that it is difficult to arrange this here is that the energy levels can be *negative* and (3.5) blows up. In conventional quantum mechanics it is easy to reset the zero of energy, if there is a lower bound on any such negative energy levels. Needless to say, in gravity energy is absolute and cannot be reset. What is even more alarming is that the energy levels are *unbounded from below* in gravitational systems.

This stems from the negative signature to the metric in (2.5). The term in the Hamiltonian for the momentum conjugate to v is negative. An ingenious way to circumvent this disaster was proposed by the authors of ref. [11]. They proposed that the (conformal) scale factor should be rotated to lie along the imaginary axis. This would enable us to put a

* The delta function is that appropriate to the minisuperspace co-ordinate system and can therefore contain some geometrical factors. It is normalized in the sense that $\int \mathbf{d}q \delta(q; q') = 1$, where $\mathbf{d}q$ is the natural volume element of the minisuperspace. If we demand general covariance then $\mathbf{d}q$ would be the invariant volume element *i.e.*

$$\mathbf{d}q = (-\det f_{\alpha\beta})^{\frac{1}{2}} \prod_\alpha dq^\alpha$$

lower bound on the energy (the potential term could be negative over some finite region). However, (3.5) would still blow up in general. Indeed, there are arguments of a different sort which suggest that this proposal only works in a restricted class of models [12].

In our opinion the most straightforward way to accommodate the vanishing of the $T \rightarrow \infty$ amplitude is to introduce a Feynman $-i\epsilon$ term into the action. Thus in (3.1) the action should be replaced by

$$S_L = \int_{T_i}^{T_f} \left(p_\alpha \frac{dq^\alpha}{dT} - [\mathcal{H} - i\epsilon] \right) dT \quad (3.6)$$

The states $|q^\alpha, T\rangle$ then satisfy the differential equation

$$(\dot{\mathcal{H}}_f - i\epsilon) \langle q_f^\alpha, T | = i \frac{\partial}{\partial T} \langle q_f^\alpha, T |$$

which in turn implies that in (3.5) we replace E_n by $E_n - i\epsilon$. Clearly the amplitude vanishes as $T \rightarrow \infty$ no matter what the oscillatory behaviour of each term might be. Thus the kernel is taken to satisfy the Lippmann-Schwinger equation [13],

$$(\dot{\mathcal{H}}_f - i\epsilon) K(q_f^\alpha; q_i^\alpha) = -i\delta(q_f^\alpha; q_i^\alpha) \quad (3.7)$$

We will now summarize what we have learnt in this section. We started off with a path integral between fixed initial and final three geometries, which in standard quantum theory would result in the evolution kernel. Such a path integral can be gauge-fixed to remove the over-counting of paths deformable into each other by a different choice of lapse function. There are many ways to label the same path between the end configurations and we have had to remove this degree of freedom. “Time” has no absolute meaning in quantum cosmology; it has been reduced to the role of an internal parameter of a path.

Contrast this with conventional quantum mechanics, where time is fixed. There time is an classical variable external to the system. Like it or not, we observers are constantly moving into the future (or away from the past) and our whole concept of measurement is deeply rooted in notions of time. In field theory the range of this rigid time is taken from past to future infinity. This limit selects the state of lowest energy and places it in

a privileged position. Also in basic quantum mechanics the retarded kernel allows us to evaluate a wavefunction at a time t given its value at an earlier time t' ,

$$\psi(\mathbf{x}, t) = \int G_R(\mathbf{x}, t; \mathbf{x}', t') \psi(\mathbf{x}', t') d^3 \mathbf{x}'$$

Both times are external classical labels to the kernel *i.e.* we have to specify the initial and final times. In our case, the absence of an external time leads to a departure from some familiar notions of kernels. The first is that any reference to a state of lowest energy is absent. This is fortunate as there exists none. The second is that the kernel is not transitive unlike the case above where

$$G_R(\mathbf{x}_3, t_3; \mathbf{x}_1, t_1) = \int G_R(\mathbf{x}_3, t_3; \mathbf{x}_2, t_2) G_R(\mathbf{x}_2, t_2; \mathbf{x}_1, t_1) d^3 \mathbf{x}_2$$

and $t_3 \geq t_2 \geq t_1$. The reason why our kernel is not transitive can be traced to the integration over the lapse function. The lapse function does not appear as a label for the end configurations.

In (3.1) we integrate over positive values of $\mathcal{N}_L$. We do not allow it to flip sign, in keeping with its behaviour in the classical theory (the metric depends on $\mathcal{N}_L^2$ and when we take the square root of the determinant of the metric we choose the positive root). This naturally leads to the Lippmann-Schwinger equation of (3.7) as we have demonstrated. The kernel so defined finds its use in theories where we “third quantize” gravity. Given a source, $\rho(q^\alpha)$, the wavefunction which is a solution to

$$(\hat{\mathcal{H}} - i\epsilon)\Psi(q^\alpha) = \rho(q^\alpha)$$

is simply

$$\Psi(q^\alpha) = i \int d q' K(q^\alpha; q^{\alpha'}) \rho(q^{\alpha'})$$

If it is possible to develop a perturbative expansion, or something like it, in third-quantized gravity the kernel would play the same role as a Feynman propagator does in second quantized field theory [10]. We hope to return to such uses of the kernel in a future publication.

4. EUCLIDEAN FORMULATIONS OF QUANTUM COSMOLOGY AND MOMENTUM REPRESENTATIONS

Difficulties with giving meaning to path integrals with an oscillating integrand are usually resolved by Wick rotating to Euclidean space. The rotation turns an imaginary exponent, iS_L , into a negative one, $-S_E$, and more often than not S_E is positive definite. In gravity it is not. Moreover, in quantum gravity we are integrating over geometries (or metrics representing these geometries). Wick rotation *changes the class of geometries* from Lorentzian to Euclidean, and there is a topological distinction between these two classes. It is unclear if, once we have actually performed the path integral for the Euclidean theory, we can analytically continue our results back to Lorentzian space correctly.

Hartle and Hawking, in their seminal paper [6], went one step further. They suggested that the theory should be formulated in Euclidean space at the outset, and that we should sum over closed compact Euclidean geometries without boundary (the so-called “no boundary” proposal). The advantage of this is that the resulting path integral yields a wavefunction rather than a kernel,

$$\Psi_H(q^\alpha) = \int_{CC} \mathcal{D}q^\alpha e^{-S_E} \quad (4.1)$$

and it is a solution to the Wheeler-DeWitt equation. There is only one end configuration since the geometry is closed. We get boundary conditions on the wavefunction for free.

There still arises the question of convergence of this path integral. The Euclidean action is equivalent to the Lorentzian Hamiltonian (at least in the cases we consider here) and is unbounded from below in cosmological circumstances. The positive action theorem [14] cannot be invoked to remedy the situation [12]. However, if we are prepared to accept semi-classical approximations to the path integral as having some validity it is possible to develop this approach further. Indeed, it might only be in this approximation that the Euclidean space approach makes any sense at all.

We perform a steepest descent analysis of the integral and choose our contour in such a way as to pass through selected stationary points of the action. In general there are many saddle points and many possible contours [15]. With different choices of contour we can extract the Hartle-Hawking wavefunction (whose characteristic property is that it is real)

or the Vilenkin [16] wavefunction (whose characteristic feature is that it behaves as $e^{-i\tilde{S}}$ in oscillating regions, where $\tilde{S}$ is a positive phase). Each contour imposes its own boundary conditions on the wavefunction.

Recall the trouble we encountered in the previous section when attempting a Wick rotation in the Lorentzian kernel. We found that such a procedure was accompanied by severe complications when trying to obtain the convergence of our expression. Our only way out was to resort to the introduction of a Feynman $-i\epsilon$ term. Ordinarily this term has the same effect as the Wick rotation in enforcing convergence of the path integral. We found that only one, the $-i\epsilon$, had any chance of success in quantum cosmology. It should not be too great a surprise to us to learn that if the theory is formulated in Euclidean space *a priori* we have to resort to an approximation which avoids any reference to the complete quantum behaviour of the path integral. If we have to rely on the semi-classical approximation, then why not simply employ it in the Lorentzian formulation and choose integration contours accordingly. It would make progress and understanding much easier.

A useful tool for calculations, especially if one of the co-ordinates is cyclic, is transformation to the momentum representation. If a co-ordinate is cyclic then its conjugate momentum is conserved and may be used to label the states of the system. As an example, suppose we want to transform one of our variables, q^β say, in the Lorentzian kernel of (3.1). Denote all the other co-ordinates by $\tilde{q}^\alpha$ i.e. $\{q^\alpha\} = \{q^\beta, \tilde{q}^\alpha\}$. Then

$$\tilde{K}(p_{\beta f}, \tilde{q}_f^\alpha; p_{\beta i}, \tilde{q}_i^\alpha) = \int dq_f^\beta dq_i^\beta e^{-ip_{\beta f} q_f^\beta} K(q_f^\alpha; q_i^\alpha) e^{+ip_{\beta i} q_i^\beta}$$

with no sum on β of course. It is straightforward to show that $\tilde{K}$ is a path integral

$$\tilde{K}(p_{\beta f}, \tilde{q}_f^\alpha; p_{\beta i}, \tilde{q}_i^\alpha) = \int_0^\infty d\mathcal{N}_L[t_f - t_i] \int \mathcal{D}q^\alpha \mathcal{D}p_\alpha e^{iS_L^p}$$

where $S_L^p = S_L - p_{\beta f} q_f^\beta + p_{\beta i} q_i^\beta$ is the action appropriate to keeping $\tilde{q}^\alpha$ and p_β fixed at the end points, and that the integration is over all paths with $\tilde{q}^\alpha$, p_β fixed at the ends and $\tilde{p}_\alpha$, q^β unrestricted. With the $-i\epsilon$ prescription this still yields a Lippmann-Schwinger equation. We can transform any number of co-ordinates, and even at only one of the ends if so desired. The inverse transformation is also a Fourier transform, although we have to be careful if some of the co-ordinates (such as the scale parameter) do not have an infinite range.

The Euclidean case is quite different. The action is obtained by the replacement of the Lorentzian lapse function by that appropriate for a Euclidean metric, $\mathcal{N}_L \rightarrow -i\mathcal{N}_E$. This is tantamount to a Wick rotation and $S_E(q^\alpha, \dot{q}^\alpha; \mathcal{N}_E) = -iS_L(q^\alpha, \dot{q}^\alpha; -i\mathcal{N}_E)$ in the notation of section 2. The Euclidean momenta, which we denote by capital letters to distinguish them from the Lorentzian momenta, are $P_\alpha = \frac{\partial S_E}{\partial \dot{q}^\alpha}$ and are $-i$ times the p_α . In this case the momentum representation is achieved through a Laplace transform. In the case of a Euclidean wavefunction, for example,

$$\tilde{\Psi}(P_\beta, \tilde{q}^\alpha) = \int dq^\beta e^{+P_\beta q^\beta} \Psi(q^\alpha)$$

so that if $\Psi \sim e^{-S_E}$ then $\tilde{\Psi} \sim e^{-S_E^P}$. The inverse transform is a Bromwich integral which requires a contour of integration for definition [15].

The most apparent difference between the two formulations occurs when some of the co-ordinates are cyclic. The conjugate momenta are conserved. For example, in the case of a Euclidean wavefunction we would use P_β to label the states $\tilde{\Psi}(P_\beta, \tilde{q}^\alpha)$, and in the Lorentzian case the kernel would be of the form

$$\tilde{K}(p_{\beta f}, \tilde{q}_f^\alpha; p_{\beta i}, \tilde{q}_i^\alpha) = \delta(p_{\beta f}; p_{\beta i}) \tilde{K}_2(\tilde{q}_f^\alpha; \tilde{q}_i^\alpha)$$

However, the constant Lorentzian and Euclidean momenta differ by a factor of $-i$. Which one is real? Surely this depends on which formulation we consider to be most fundamental, encapsulating as it should the true physical nature of quantum cosmology. In the next two sections we present a couple of simple examples which, to our minds, highlight some of the advantages of the Lorentzian formulation.

5. BIANCHI I COSMOLOGY

In a previous paper [17] we discussed the construction of an anisotropic Bianchi I quantum cosmology. There are three degrees of freedom in this model: the scale factor v , and two which parametrize the anisotropy of the space, $\beta_{\pm}$. The closed three-space has the geometry of a three-torus whose volume is V_3 say. The action can be written as

$$S_L = \frac{\mu}{2} \int \left[\frac{v}{\mathcal{N}_L} \left(-\frac{1}{9} \left(\frac{\dot{v}}{v} \right)^2 + \dot{\beta}_+^2 + \dot{\beta}_-^2 \right) - \frac{v \mathcal{N}_L \Lambda}{3} \right] dt \quad (5.1)$$

where $\mu = 3V_3/4\pi G_N$, and the corresponding Hamiltonian is

$$\mathcal{H} = -\frac{9v}{2\mu} p_v^2 + \frac{1}{2\mu v} (p_+^2 + p_-^2) + \frac{\mu v \Lambda}{6} \quad (5.2)$$

As the co-ordinates $\beta_{\pm}$ are cyclic, their corresponding momenta $p_{\pm}$ are constants of the motion. We could solve the Wheeler-DeWitt equation from (5.2) to get the wavefunctions labelled by $p_{\pm}$ *i.e.* solve to obtain $\Psi(v, p_{\pm})$. In doing so we keep the $p_{\pm}$ real constants. The solution is typically a Bessel function.

Now let us consider the construction of the wavefunction from the perspective of Hartle and Hawking[★] in keeping with our earlier study. First, we examine the Euclidean action. This is similar to (5.1) but with the final, cosmological constant, term reversed in sign. We can choose the gauge $\mathcal{N}_E = 1$. We find that the Euclidean momenta $P_{\pm}$ are conserved and that the classical equations of motion are

$$\begin{aligned} v(t) &= \frac{3\sqrt{P^2}}{\mu H} \sin Ht \\ \beta_{\pm}(t) &= \frac{P_{\pm}}{3\sqrt{P^2}} \ln \left[\tan \frac{Ht}{2} \right] \end{aligned} \quad (5.3)$$

where $H = \sqrt{3\Lambda}$ and $P^2 = P_+^2 + P_-^2$. Inserting these solutions into the Euclidean action would result in a real expression. Clearly if we evaluate $\Psi(v, P_{\pm})$ for $v > \hat{v} = 3\sqrt{P^2}/\mu H$ we would expect oscillating behaviour *i.e.* the stationary points of the Euclidean action occur for imaginary action as there are then no real classical solutions like (5.3). This is similar to the Robertson-Walker model discussed in the original Hartle-Hawking paper [6] (in our model $\hat{v}$ replaces their cube of the inverse Hubble length).

★ In doing so we have to extend the class of geometries in the path integral of (4.1) to include closed, finite volume.

Indeed, transforming to the momentum representation, the semiclassical approximation to the momentum space wavefunction is

$$\hat{\Psi}(P_v, P_{\pm}) \sim N_P e^{-S_E^P}$$

where the dominant Euclidean action is

$$S_E^P = \frac{\sqrt{P^2}}{3} \left(1 - f \left(\frac{9P_v}{\mu H} \right) \right)$$

with $f(x) = \ln(x + (1 + x^2)^{\frac{1}{2}})$. Transforming back to get $\Psi(v, P_{\pm})$ requires a contour of integration in the complex P_v space. If we are considering P^2 real then we can choose the cuts and contours of Hartle and Hawking to find two semiclassical wavefunctions,

$$\begin{aligned} v < \hat{v} \quad \Psi_{<}(v, P_{\pm}) &= \mathcal{A}_{<} \exp \left(\frac{\sqrt{P^2}}{3} (x - \tanh^{-1} x) \right) \\ v > \hat{v} \quad \Psi_{>}(v, P_{\pm}) &= \mathcal{A}_{>} \cos \left(\frac{\sqrt{P^2}}{3} (y - \tan^{-1} y) - \frac{\pi}{4} \right) \end{aligned} \quad (5.4)$$

where $x = \sqrt{1 - (v^2/\hat{v}^2)}$ and $y = \sqrt{(v^2/\hat{v}^2) - 1}$.

However, on physical grounds we would expect the wavefunction to have a strong oscillatory behaviour for small v , since in the classical (Lorentzian) theory the universe would be in a period of Kasner-type evolution. The above Euclidean wavefunctions in (5.4) do not behave in this way. This is because we have assumed throughout their construction that the Euclidean momenta $P_{\pm}$ are real. If we rotate these back to Lorentzian space we have to impose a different set of cuts in the P_v plane [17]. It is still possible in that case to define a Hartle-Hawking contour and we find that there is only one wavefunction,

$$\Psi_{\star}(v, p_{\pm}) = \mathcal{A}_{\star} \cos \left(\frac{\sqrt{p^2}}{3} \left\{ \sqrt{1 + \left(\frac{v}{v^{\star}} \right)^2} - \coth^{-1} \sqrt{1 + \left(\frac{v}{v^{\star}} \right)^2} \right\} + \frac{\pi}{4} \right) \quad (5.5)$$

where now $v^{\star} = 3\sqrt{p^2}/\mu H$ with $p^2 = p_+^2 + p_-^2$. The wavefunction oscillates over the complete range of v .

This is understandable from inspection of (5.2). If $p_{\pm}$ are real then the effective potential[†] is always positive. Taking account of the sign of the momentum term, this would correspond to free motion. Therefore we would expect oscillating quantum solutions. If we had taken $p_{\pm}$ imaginary the effective potential has a zero at $v = \hat{v}$. Free motion results for $v > \hat{v}$ and bound motion for $v < \hat{v}$. Moreover, it is clear that $p_{\pm}$ should be real for stationary states. It is the Lorentzian momenta that satisfy the canonical quantization rules *i.e.* $p_{\pm} \rightarrow -i \frac{\partial}{\partial \beta_{\pm}}$. For a wavefunction that depends on $\beta_{\pm}$ only through a phase the eigenvalues of these momenta must be real.

We could have obtained this wavefunction more directly from the Lorentzian formalism. Extremizing the action appropriate to keeping v , $p_{\pm}$ fixed at the end points we find that the classical path is $p_{\pm} = \text{constant}$, and $v(t) = v^* \sinh Ht$. Inserting this solution back into the action, the semiclassical approximation to the wavefunction is

$$\Psi(v, p_{\pm}) = \mathcal{A}_+ e^{-i\tilde{S}} + \mathcal{A}_- e^{+i\tilde{S}}$$

where $\tilde{S}$ is the argument of (5.5) up to constants (which may be absorbed into the $\mathcal{A}_{\pm}$). The first term corresponds to a universe expanding forward in time *i.e.* v is the value of the scale factor at the *final* end of the classical path. Similarly, the second term represents a universe that is expanding backwards in time, since v is specified at the initial end. This is equivalent to a universe contracting forward in time. The contours of integration in the Euclidean analysis determine the relative phase of $\mathcal{A}_{\pm}$. However, it is also possible to fix the phases by hand in the Lorentzian case. For the Vilenkin wavefunction we would want only forward expanding spaces so we could set $\mathcal{A}_- = 0$. For the Hartle-Hawking wavefunction we want an equal admixture of both spaces and a real solution to the Wheeler-DeWitt equation. This fixes $\mathcal{A}_{\pm}$ to be such that (5.5) results.

† We use “effective potential” in the classical sense, such as occurs when a particle has conserved angular momentum.

6. THE GIDDINGS AND STROMINGER WORMHOLE

Up till now we have considered only purely gravitational systems. The introduction of matter is quite simple. We just add the action for the matter fields to the gravitational action. Variation of the total action with respect to the metric yields the Einstein equations as usual. The Hamiltonian now includes that of the matter, and the constraint from variation of the lapse function is $\mathcal{H}_{\text{Grav}} + \mathcal{H}_{\text{Matter}} = 0$. Wavefunctions are functionals of both the end points' three-geometries and matter configurations at these end points.

In the case of the Giddings-Strominger wormhole [18] the matter is a three-index antisymmetric field strength $H_{\mu\nu\lambda}$, which is a remnant of ten-dimensional supergravity after compactification. The (Lorentzian) action is

$$S_L^H = -\frac{f^2}{2} \int \mathbf{H} \wedge \star \mathbf{H}$$

where $\mathbf{H}$ is the corresponding three-form and f has the dimensions of mass (it becomes the axion scale). The three-form is dual to an axion field, $\mathbf{H} = \star d\phi/f$, so equivalently

$$S_L^H = S_L^\phi = \frac{1}{2} \int d\phi \wedge \star d\phi$$

We choose the three-space geometry to be that of a three-sphere, S_3 , of radius $a(t)$. As our space is homogeneous we have $\phi = \phi(t)$ and also $\mathbf{H} = \eta(t) \mathbf{e}^1 \wedge \mathbf{e}^2 \wedge \mathbf{e}^3$, where the $\mathbf{e}^i$ are the dimensionless basis forms for S_3 . Then we can write

$$\begin{aligned} S_L^H &= -\frac{1}{2} \int u_L(t) \eta^2(t) dt \\ S_L^\phi &= +\frac{1}{2} \int w_L(t) \dot{\phi}^2(t) dt \end{aligned} \tag{6.1}$$

where the geometric factors are $u_L = 2\pi^2 f^2 \mathcal{N}_L / a^3$ and $w_L = 2\pi^2 a^3 / \mathcal{N}_L$. If we continue these to Euclidean space, both matter actions are positive,

$$S_E^H = \frac{1}{2} \int u_E \eta^2 dt, \quad S_E^\phi = \frac{1}{2} \int w_E \dot{\phi}^2 dt$$

where u_E, w_E have the same form as u_L, w_L but with $\mathcal{N}_L$ replaced by $\mathcal{N}_E$. However, the

(Euclidean) Hamiltonians at first glance appear to be of the opposite sign,

$$\mathcal{H}_E^H = -\frac{\pi^2 f^2 \eta^2}{a^3}, \quad \mathcal{H}_E^\phi = +\frac{P_\phi^2}{4\pi^2 a^3}$$

where $P_\phi = w_E \dot{\phi}$ is the Euclidean momentum conjugate to ϕ . Since ϕ is cyclic then P_ϕ is conserved.

As the stress tensor is diagonal and $T_{00} = \mathcal{H}_E/(2\pi^2 a^3)$ then some confusion exists regarding the eigenvalues of the Euclidean space Ricci tensor. We need a negative eigenvalue of the Ricci tensor for a gravitational instanton. Certainly the form of $\mathcal{H}_E^H$ allows for this, but what of $\mathcal{H}_E^\phi$? The authors of [18] have stated that taking the dual and continuation to Euclidean space do not commute. The resolution of this apparent paradox is that it is the Lorentzian momentum p_ϕ , not the Euclidean momentum P_ϕ , that is a real conserved quantity. Then P_ϕ^2 is negative and both Hamiltonians are of the same sign.

This can be seen as follows from a Lorentzian point of view. As $\mathbf{H}$ is the curl of a two-form potential then $d\mathbf{H} = 0$. Thus integrating $\mathbf{H}$ over a three-sphere implies that

$$\int_{S_3} \mathbf{H} = \frac{n}{f^2} \tag{6.2}$$

and so $\eta = n/(2\pi^2 f^2)$, a constant. In string theory n is an integer [19], but in general it is just a conserved charge. Equivalently, since ϕ is cyclic we have the conservation of p_ϕ . The constraint on $\mathbf{H}$ is translated into $p_\phi = n/f$. Therefore in string theory p_ϕ is discretely quantized. It is convenient to transform to the momentum representation [20] and we find that

$$S_L^H = S_L^{p_\phi} = -\frac{n^2}{4\pi^2 f^2} \int \frac{\mathcal{N}_L}{a^3} dt \tag{6.3}$$

and it is these actions, rather than (6.1), which we should continue to Euclidean space. Both result in a negative Euclidean Hamiltonian.

It is the Lorentzian momentum which labels the states of the system, reflecting as it does the topological constraint (6.2) on the three-form. The Euclidean space equation of motion for the radius a is obtained from the total (gravitational plus matter) Hamiltonian

and is

$$\dot{a}^2 - 1 = -\frac{r^4}{a^4}$$

where r is the radius of the wormhole (in our notation $r^4 = n^2/(3\pi^3 f^2 M_P^2)$). The solution has $a \sim |t|$ as $t \rightarrow \pm\infty$, indicating an asymptotically flat Euclidean space which can be patched onto another manifold with curvature much less than r^{-2} . Also at $a = r$ we have $\dot{a} = 0$ which says that r is the minimum radius for a three-sphere and is a throat of the four-dimensional geometry. The interpretation of this instanton in Lorentzian space is a tunnelling process between states of given initial and final Lorentzian momenta for the axion. We will say more on this in the next section.

7. DISCUSSION

Euclidean space methods are extremely useful when calculating expressions whose physical meaning is understood at the outset. Such is the case of a scalar tunnelling through a potential barrier. We expect to find a rate for tunnelling between given classical turning points. We know the form of the solutions at the turning points and it is then only the amplitude for traversal that we need to calculate. For this isolated segment of parameter space inside the barrier we can Wick rotate and calculate the number. However, we should not consider the Euclidean space as having any real existence.

This is more acute in quantum cosmology, or any situation where there could be topology change. For example, consider a Robertson-Walker universe with an axion as described in the last section. The (Lorentzian) Hamiltonian is

$$\mathcal{H} = -\frac{1}{2\mu a} p_a^2 + V(a)$$

where a is the radius of the three-space and the potential has the form,

$$V(a) = \frac{\alpha}{a^3} - \beta a + \gamma a^3$$

with α, β, γ positive. The first term is due to the axion and $\alpha \sim n^2/f^2$ from (6.3). The second is from the curvature of the three-space and $\beta \sim M_P^2$. The last is a cosmological constant term and $\gamma \sim \Lambda M_P^2$.

In general there are three regions: $V < 0$ for $a_1 < a < a_2$ and also $V \geq 0$ for $a \leq a_1$, $a \geq a_2$. Since $\mathcal{H} = 0$ then a_1 and a_2 are the classical turning points of the motion. If we were to solve the Wheeler-DeWitt equation we would find oscillating solutions for the regions $a < a_1$ (corresponding to Lorentzian universes which expand to or contract from a maximum radius a_1) and $a > a_2$ (corresponding to universes which contract to or expand from a minimum radius a_2). For $a_1 < a < a_2$ the solutions do not oscillate. Should we regard these solutions as corresponding to universes with a Euclidean signature? The answer is no. They are instantons, tunnelling configurations which match up to Lorentzian solutions at the turning points.

Another property of solutions in the tunnelling region is that they are incoherent. It can be shown [21] that for a wavefunction $\Psi(q^\alpha)$ (with more than one parameter) there are strong correlations amongst the parameters in oscillating regions, at least in the semi-classical approximation. The correlations are most strong along paths in parameter space which correspond to classical solutions with a Lorentzian signature. However, there are no such correlations amongst parameters in tunnelling regions — paths corresponding to classical Euclidean solutions are not enhanced. The distance along each strongly correlated Lorentzian path can be labelled by a parameter which we may call “time” [22]. In this way we extract classical Lorentzian solutions as regions of parameter space for which the wavefunction is strongly peaked. This is not the case for Euclidean paths. The quantum mechanics places those solutions with an innate sense of Lorentzian time in a privileged position.

We now come to the implications for wormhole dynamics. The original papers on the interaction of wormholes with universes [2] assumed that the universe was Euclidean, somehow being an analytic continuation of a Lorentzian universe. This was because wormholes were thought of as Euclidean manifolds with asymptotically flat regions which could then be patched onto large enough Euclidean universes. The cosmological constant of these Euclidean universes is driven to zero [1] by the dynamics of the wormholes. It has recently been strongly argued by Unruh [23] and Veneziano [24] that the Euclidean approach is in trouble — the presence of wormholes makes the effective action unbounded from below in a manner much more severe than previously anticipated. However, by a judicious choice of integration contour the singular vacua of [1] may be avoided, resulting in a suppression of the cosmological constant [24] rather than its vanishing. The range of validity of the Euclidean formulation is more restricted than first believed.

On the other hand, there do exist Lorentzian-based discussions of the cosmological constant and wormholes. The original paper of Banks [25] considers the bifurcation of Lorentzian universes, and there the cosmological constant is suppressed rather than zero.[★] This seems to be characteristic of Lorentzian analyses --- there are less compelling reasons for selecting the singular vacuum of [1] and instead the path integral is dominated by saddle points for which Λ is small. In addition, Rubakov has taken a Lorentzian stance [27] when attempting to third quantize the theory, and we believe that progress should be made by pursuing these lines of thought.

Even though Euclidean methods are helpful for solving certain problems they are by no means generally applicable. In quantum gravity, however, the choice of a Lorentzian or Euclidean space is not a matter of methodology but rather a reflection of the basic principles of the theory. In this paper we have discussed some of the pros and cons of a Euclidean formulation. We have presented a couple of examples which, we hope, illustrate some of its shortcomings. Moreover, we have shown that there does exist a prescription for ensuring the convergence of a Lorentzian path integral, in sharp contrast to the Euclidean case --- the Euclidean action is irredeemably unbounded from below. However, it is not clear at this early stage in its development if the Lorentzian approach can be made to work either.

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[★] See also the recent work of Strominger [26].

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