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SINGLETs OF FERMIONIC GAUGE SYMMETRIES

E.A. Bergshoeff^{*} and R.E. Kallosh[†]

Theory Division, CERN

CH-1211 Geneva 23, Switzerland

M.A. Rahmanov

Lebedev Physical Institute

Moscow, 117924, USSR

ABSTRACT

We investigate under which conditions singlets of fermionic gauge symmetries which are "square roots of gravity" can exist. Their existence is non-trivial because there are no fields neutral in gravity. We tabulate several examples of singlets of global and local supersymmetry and κ -symmetry and show how in each case the gravity effect is cancelled by other gauge symmetries. We point out that the finding of κ -symmetry singlets is essential for a covariant quantization of extended objects with manifest spacetime supersymmetry. Their role in string compactification is discussed.

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* Bitnet address: BERGSHOE@CERNVM

† Bitnet address until Dec. 31 1989: KALLOSHR@CERNVM

On leave of absence from: Lebedev Physical Institute, Moscow, 117924, USSR

1. Introduction

It is well known that in supersymmetric theories all fields have superpartners. This is immediately related to the supersymmetry algebra $[Q, Q] = P$, where two supersymmetries produce a translation. Therefore the existence of singlets $Q\Phi = 0$ in general contradicts the supersymmetry algebra, since always $P\Phi \neq 0$. We thus have $Q\Phi = \epsilon\Psi$, where Ψ is the "ino" partner of Φ . However there are special cases where singlets do exist. In all cases the existence of singlets relies on the on-shell cancellation of translations or general coordinate transformations by other gauge symmetries like Lorentz, conformal and/or non-Abelian internal symmetries. A striking example is the determinant of the zweibein e and the left part of the zweibein e_z^a in the heterotic string in the Green-Schwarz formulation, which are neutral in κ -symmetry. This means that the action of two κ -symmetries on e and e_z^a gives zero. On the other hand the commutator algebra on the string coordinates X^μ is known to produce a general coordinate transformation and a κ -transformation. If this would be the complete algebra one would expect an inconsistency since both e and e_z^a transform under general coordinate transformations. The resolution of this apparent inconsistency is the existence of an additional conformal and Lorentz symmetry produced by two κ -transformations which cancel the effect of the general coordinate transformation on e and e_z^a .

We consider in general an action which is invariant under a set of gauge transformations. The commutator of two gauge transformations is equal to a linear combination of gauge transformations from the same set and/or terms which involve equations of motion, corresponding to the action. To be precise, if we have an action $S(\Phi)$ with a set of gauge symmetries $\delta\Phi^i = R_\alpha^i(\Phi)\xi^\alpha$, the commutator of two such symmetries on a generic field Φ^i has the form

$$[\delta_\beta, \delta_\alpha]\Phi^i = f_{\alpha\beta}^\gamma(\Phi)R_\gamma^i(\Phi) + \eta_{\alpha\beta}^{ij}\frac{\delta S}{\delta\Phi^j} \quad (1.1)$$

The requirement that a fermionic symmetry is the "square root" of gravity corresponds to the following property of the structure constants

$$f_{\alpha_f\beta_f}^{\gamma_g} \neq 0 \quad (1.2)$$

where α_f, β_f refer to fermionic symmetries and γ_g to general coordinate transformations. For fermionic singlets Φ^{i_s} we must have

$$\delta_{\alpha_f}\Phi^{i_s} = 0 \rightarrow [\delta_{\alpha_f}, \delta_{\beta_f}]\Phi^{i_s} = 0 \quad (1.3)$$

which implies

$$R_\gamma^{i*} f_{\alpha_f \beta_f}^\gamma + \eta_{\alpha_f \beta_f}^{i*j} S_{,j} = 0 \quad (1.4)$$

This equation looks like a reducibility condition for gauge symmetries [1], where $Z_{\alpha_i}^\alpha$ is given by $f_{\alpha_f \beta_f}^\gamma$ and η_α^{ij} is given by $\eta_{\alpha_f \beta_f}^{i*j}$. However the standard reducibility condition requires eq. (1.4) to take place for all fields Φ^i and not only for the singlets Φ^{i*} . Note that because of (1.2) we know that there is no trivial solution where all f and η are zero.

We now proceed with a presentation of several examples of singlets of fermionic symmetries. For pedagogical reasons we will first discuss singlets of global and local supersymmetry which are already given in the literature. We will then continue by discussing singlets of fermionic κ -symmetry with and without harmonic variables. The existence of such singlets is essential for a covariant quantization of extended objects with manifest spacetime supersymmetry and may be relevant for their compactification.

2. Global Supersymmetry

The existence of global supersymmetry singlets is well known and was pointed out in [2,3], where it was shown that left-handed chiral fermions in $d = 2$ have no supersymmetric partners for $(p, 0)$ supersymmetry. We explain this in terms of eq. (1.4) for the case of $(1, 0)$ supersymmetry. The Lagrangian contains matter multiplets (X^μ, Ψ^μ) and left-handed fermions Ψ^I :

$$\mathcal{L}(\text{global}) = -\frac{1}{2} \partial_z X^\mu \partial_{\bar{z}} X^\mu - \frac{1}{2} i \Psi^\mu \partial_z \Psi^\mu - \frac{1}{2} i \Psi^I \partial_{\bar{z}} \Psi^I \quad (2.1)$$

It is invariant under the supersymmetry transformations

$$\begin{aligned} \delta X^\mu &= -i\epsilon \Psi^\mu \\ \delta \Psi^\mu &= +\partial_{\bar{z}} X^\mu \epsilon \end{aligned} \quad (2.2)$$

From this we see that $\Phi^{i*} = \{\Psi^I\}$ are global supersymmetry singlets. The ϵ -commutator on X^μ produces a translation

$$[\delta_2, \delta_1] X^\mu = \xi_z \partial_{\bar{z}} X^\mu \quad (2.3)$$

with parameter $\xi_z = 2i\epsilon_2 \epsilon_1$. The commutator on the Ψ^I singlets is realized as follows

$$[\delta_2, \delta_1] \Psi^I = 0 = \delta_P(\xi_z) \Psi^I - \xi_z \partial_{\bar{z}} \Psi^I \quad (2.4)$$

In other words the translation is compensated by a term involving the field equation $\partial_{\bar{z}} \Psi^I = 0$ of the left-handed fermions.

It is maybe less well known that in $d = 2$ right-handed fermionic singlets exist as well [4,5]. In that case the Lagrangian contains besides the matter multiplets (X^μ, Ψ^μ) and the left-handed fermions Ψ^I , right-handed fermions $\chi^{\bar{I}}$ and a Lagrange multiplier field $\Sigma_z^{\bar{z}}$:

$$\mathcal{L}'(\text{global}) = \mathcal{L}(\text{global}) - \frac{1}{2}i\chi^{\bar{I}}\partial_z\chi^{\bar{I}} - \frac{1}{2}i\Sigma_z^{\bar{z}}\chi^{\bar{I}}\partial_{\bar{z}}\chi^{\bar{I}} \quad (2.5)$$

It is invariant under the supersymmetry transformations given in (2.2). Both $\chi^{\bar{I}}$ and $\Sigma_z^{\bar{z}}$ are global supersymmetry singlets. The action corresponding to (2.5) possesses an additional gauge invariance σ_z under which the singlets transform as follows:

$$\begin{aligned} \delta\chi^{\bar{I}} &= -\sigma_z\partial_{\bar{z}}\chi^{\bar{I}} - \frac{1}{2}(\partial_{\bar{z}}\sigma_z)\chi^{\bar{I}} \\ \delta\Sigma_z^{\bar{z}} &= -\sigma_z\partial_{\bar{z}}\Sigma_z^{\bar{z}} + \partial_z\sigma_z + (\partial_{\bar{z}}\sigma_z)\Sigma_z^{\bar{z}} \end{aligned} \quad (2.6)$$

Note that for constant σ_z these gauge transformations take the form of a translation. The ϵ -commutator on $\chi^{\bar{I}}$ is realized as follows:

$$[\delta_2, \delta_1]\chi^{\bar{I}} = 0 = \delta_P(\xi_z = 2i\epsilon_2\epsilon_1)\chi^{\bar{I}} - \delta(\sigma_z = 2i\epsilon_2\epsilon_1)\chi^{\bar{I}} \quad (2.7)$$

The translation is compensated by a gauge transformation. The same compensation mechanism occurs in the ϵ -commutator on $\Sigma_z^{\bar{z}}$.

We have seen that for the left-handed fermionic singlets the translation is cancelled by a field equation and for the right-handed fermionic singlets it is cancelled by a gauge transformation with constant parameter. We note that the gauge symmetry described by (2.6) is potentially anomalous. In fact the gauge system described by eq. (2.5) is only anomaly free after quantization if $\bar{I} = 1, \dots, 52$ [5].

3. Local Supersymmetry

The above example of a left-handed global supersymmetry singlet can easily be extended to the case of local supersymmetry. We just couple the matter multiplets $\{X^\mu, \Psi^\mu\}$ and $\{\Psi^I\}$ to $(1, 0)$ supergravity $\{e_z^a, e_{\bar{z}}^{\bar{a}}, \chi_z, \chi_{\bar{z}}\}$. This yields the following Lagrangian [6]

$$\mathcal{L}(\text{local}) = e\left\{-\frac{1}{2}\partial_z X^\mu\partial_{\bar{z}}X^\mu - \frac{1}{2}\Psi^\mu\partial_z\Psi^\mu - i\chi_z\Psi^\mu\partial_{\bar{z}}X^\mu - \frac{1}{2}i\Psi^I\partial_{\bar{z}}\Psi^I\right\} \quad (3.1)$$

The local supersymmetry transformations of all fields are given by

$$\begin{aligned}\delta e_a^z &= 2i\epsilon\chi_a & \delta X^\mu &= -i\epsilon\Psi^\mu \\ \delta\chi_a &= (\partial_a + \frac{1}{2}\omega_a(e, \chi))\epsilon & \delta\Psi^\mu &= (\partial_z X^\mu + i\chi_z\Psi^\mu)\epsilon\end{aligned}\tag{3.2}$$

There are two singlets : $\Phi^{i,*} = \{e_a^z, \Psi^I\}$. These singlets transform under the remaining gauge invariances of the action (3.1), i.e. general coordinate invariance ξ^a , Lorentz invariance Λ and conformal transformations ρ as follows:

$$\begin{aligned}\delta e_a^z &= \xi^b\partial_b e_a^z + (\partial_a \xi^b)e_b^z + (\Lambda + \rho)e_a^z \\ \delta\Psi^I &= \xi^a\partial_a \Psi^I + \frac{1}{2}(\Lambda - \rho)\Psi^I\end{aligned}\tag{3.3}$$

The commutator on X^μ :

$$[\delta_2, \delta_1]X^\mu = \delta(\xi^a)X^\mu + \delta(\epsilon)X^\mu\tag{3.4}$$

shows that two supersymmetry transformations produce a general coordinate transformation with parameter $\xi_z = 2i\epsilon_2\epsilon_1$ and a supersymmetry transformation with parameter $\epsilon = \xi_z\chi_z$. We next consider the commutator on the singlets:

$$\begin{aligned}[\delta_2, \delta_1]\Psi^I &= 0 = \xi_z\partial_z\Psi^I + \frac{1}{2}(\Lambda - \rho)\Psi^I - \xi_z(\partial_z\Psi^I + \frac{1}{2}e^{-1}\partial_a(ee_z^a)\Psi^I) \\ [\delta_2, \delta_1]e_a^z &= 0 = \xi^b\partial_b e_a^z + (\partial_a \xi^b)e_b^z + (\Lambda - \rho)e_a^z\end{aligned}\tag{3.5}$$

These equations tell us which field equation and which Λ, ρ rotations compensate the general coordinate transformation. We find that $\rho = 0$ and Λ is given by:

$$\Lambda = \xi_z e^{-1}\partial_a(ee_z^a)\tag{3.6}$$

So we see that in this example the gravity effect on the singlets e_a^z, Ψ^I is in one case (e_a^z) cancelled by a Lorentz rotation and in the other case (Ψ^I) by a Lorentz rotation and a field equation.

4. Fermionic κ -symmetry without harmonic variables

In this example we consider the Green-Schwarz formulation of the heterotic superstring. The Lagrangian involves the ten-dimensional superspace coordinates (X^μ, θ) , the zweibein $(e_z^a, e_{\bar{z}}^a)$ and the left-handed chiral fermions Ψ^I and is given by:

$$\mathcal{L}(\text{heterotic}) = e \{ \Pi_z^\mu \Pi_{\bar{z}}^\mu + i \partial_z X^\mu \bar{\theta} \gamma_\mu \partial_{\bar{z}} \theta - i \partial_{\bar{z}} X^\mu \bar{\theta} \gamma_\mu \partial_z \theta - \frac{i}{2} \Psi^I \partial_{\bar{z}} \Psi^I \} \quad (4.1)$$

It is invariant under the κ -transformations

$$\begin{aligned} \delta X^\mu &= i \bar{\theta} \gamma^\mu \delta \theta \\ \delta \theta &= \not{A}_{\bar{z}} \kappa_z \\ \delta e_z^a &= -4i \bar{\kappa}_z \partial_z \theta e_z^a \end{aligned} \quad (4.2)$$

The κ -symmetry singlets are e_z^a and Ψ^I . Note that also the determinant of the zweibein e is a singlet

$$\delta e = -e(e_z^a \delta e_z^a + e_{\bar{z}}^a \delta e_{\bar{z}}^a) = 0 \quad (4.3)$$

since e_z^a is a singlet and δe_z^a satisfies the relation $e_z^a \delta e_z^a = 0$. Thus the singlets are given by $\Phi^i = \{e, e_z^a, \Psi^I\}$. Under the remaining gauge invariances of the heterotic action, i.e. general coordinate transformations ξ^a , Lorentz invariance Λ and conformal invariance ρ they transform as follows:

$$\begin{aligned} \delta e_z^a &= \xi^b \partial_b e_z^a - \partial_{\bar{z}} \xi^a - (\Lambda + \rho) e_z^a \\ \delta e &= \partial_a (e \xi^a) + 2\rho e \\ \delta \Psi^I &= \xi^a \partial_a \Psi^I + \frac{1}{2} (\Lambda - \rho) \Psi^I \end{aligned} \quad (4.4)$$

The κ -commutator on θ :

$$[\delta_2, \delta_1] \theta = \delta(\xi^a) \theta + \delta(\kappa_z) \theta \quad (4.5)$$

tells us that two κ -transformations produce a general coordinate transformation and a κ -transformation with parameters given by

$$\begin{aligned} \xi^a &= 4i \bar{\kappa}_z^1 \not{A}_{\bar{z}} \kappa_z^2 e_z^a \\ \kappa_z &= 8i \kappa_z^1 \bar{\kappa}_z^2 \partial_{\bar{z}} \theta - 2i \gamma_\mu \partial_{\bar{z}} \theta \bar{\kappa}_z^1 \gamma_\mu \kappa_z^2 \end{aligned} \quad (4.6)$$

We next consider the κ -commutator on the singlets and find:

$$\begin{aligned} [\delta_2, \delta_1]e &= 0 = \partial_a(e\xi^a) + 2\rho e \\ [\delta_2, \delta_1]\Psi^I &= 0 = \xi_z \partial_{\bar{z}} \Psi^I + \frac{1}{2}(\Lambda - \rho)\Psi^I \\ &\quad - \xi_z(\partial_{\bar{z}} \Psi^I + \frac{1}{2}e^{-1}\partial_a(ee_{\bar{z}}^a)\Psi^I) \end{aligned} \quad (4.7)$$

From this we deduce that the effect of the general coordinate transformation is cancelled by a field equation and a λ , ρ rotation with parameters given by

$$\begin{aligned} \rho &= -\frac{1}{2}\partial_a(e\xi^a) \\ \Lambda &= -\frac{1}{2}\partial_a(e\xi^a) + \xi_z e^{-1}\partial_a(ee_{\bar{z}}^a) \end{aligned} \quad (4.8)$$

As a check we consider the κ -commutator on $e_{\bar{z}}^a$:

$$[\delta_2, \delta_1]e_{\bar{z}}^a = 0 = \xi^b \partial_b e_{\bar{z}}^a - \partial_{\bar{z}} \xi^a - (\Lambda + \rho)e_{\bar{z}}^a \quad (4.9)$$

Substituting the expressions for ρ and Λ we find that the right-hand side of (4.9) is given by

$$[\delta_2, \delta_1]e_{\bar{z}}^a = e^{-1}\partial_b(e\xi^{[a}e_{\bar{z}}^{b]}) = 0 \quad (4.10)$$

since $\xi^{[a}e_{\bar{z}}^{b]} = 0$ if we use the specific form of ξ^a given in eq. (4.6). Note that the commutator of two κ -transformations gives not only a general coordinate transformation but also a conformal transformation^{*}. This is a relevant information for the investigation of anomalies in the Green-Schwarz superstring [7].

We have found in this example that in the heterotic superstring two κ -transformations produce a general coordinate, a Lorentz and a conformal transformation. Most surprisingly we will find below that in the presence of harmonic variables the commutator of two κ -transformations will contain besides these gauge transformations also non-Abelian gauge symmetries which act on the harmonic variables only.

* We have established a theorem which states that even after making the most general redefinition of a κ -transformation with field dependent general coordinate transformations, Lorentz and conformal transformations, the commutator of two κ -transformations always produces a conformal transformation [7].

We have established that in contrast with the heterotic string the superparticle, the type *IIA, B* superstring and general *p*-branes in the standard formulation do not have κ -symmetry singlets[†].

5. Fermionic κ -symmetry and compactification

In this example we discuss the inclusion of right-handed fermions in the Green-Schwarz heterotic superstring. The addition of these right-handed fermions could be used to describe the compactification of the $d = 10$ Green-Schwarz superstring to $d = 4$ [8]. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}(\text{heterotic}) + e V_z^a \chi^{\bar{i}} \partial_a \chi^{\bar{i}} \quad (5.1)$$

where $V_z^a \equiv e_z^a + \Sigma_z^{\bar{z}} e_z^a$. The right-handed fermion $\chi^{\bar{i}}$ is a κ -symmetry singlet and the Lagrange multiplier field $\Sigma_z^{\bar{z}}$ transforms according to

$$\delta \Sigma_z^{\bar{z}} = 4i \bar{\kappa}_z \partial_z \theta \quad (5.2)$$

Besides general coordinate transformations ξ^a , Lorentz rotations Λ and conformal transformations ρ , the action has an additional gauge invariance σ_z . Under all these gauge transformations $\chi^{\bar{i}}$ and $\Sigma_z^{\bar{z}}$ transform as

$$\begin{aligned} \delta \chi^{\bar{i}} &= \xi^a \partial_a \chi^{\bar{i}} - \frac{1}{2} (\Lambda + \rho) \chi^{\bar{i}} - \sigma_z \nabla_{\bar{z}} \chi^{\bar{i}} - \frac{1}{2} (\nabla_{\bar{z}} \sigma_z) \chi^{\bar{i}} \\ \delta \Sigma_z^{\bar{z}} &= \xi^a \partial_a \Sigma_z^{\bar{z}} + 2\Lambda \Sigma_z^{\bar{z}} - \sigma_z \nabla_{\bar{z}} \Sigma_z^{\bar{z}} + \nabla_z \sigma_z + (\nabla_{\bar{z}} \sigma_z) \Sigma_z^{\bar{z}} \end{aligned} \quad (5.3)$$

The κ -commutator on the $\chi^{\bar{i}}$ singlets is realized as follows:

$$[\delta_2, \delta_1] \chi^{\bar{i}} = 0 = \delta_{g.c.t.}(\xi_z) \chi^{\bar{i}} - \frac{1}{2} (\Lambda + \rho) \chi^{\bar{i}} + \delta(\sigma_z = \xi_z) \chi^{\bar{i}} \quad (5.4)$$

and the general coordinate transformation is cancelled by a Λ, ρ rotation with parameters given by eq. (4.8) and a gauge transformation σ_z with parameter given by $\sigma_z = \xi_z$. This shows that the presence of the σ_z gauge symmetry is essential to provide the existence of the $\chi^{\bar{i}}$ κ -symmetry singlets.

[†] In the case of the type *IIA, B* superstring the determinant e can be made a singlet by using the conformal invariance of the theory but not e_z^a and $e_{\bar{z}}^a$.

We note that the σ_z -symmetry is potentially anomalous. The calculation of both the conformal as well as the σ_z -anomaly has been performed in [5]. It turns out that both the conformal and the σ_z -anomaly only cancel if $\tilde{I} = 1, \dots, 52$. Thus it seems that the right-handed fermions which are κ -symmetry singlets cannot help in cancelling the right-moving anomaly which arises in the process of compactification.

We thus conclude that the internal right-moving $d=2$ fermions cannot be inert under spacetime supersymmetry. This is in agreement with the situation in the N.S.R.-formulation of the heterotic string which is much better explored. In that formulation the spacetime supersymmetry charges depend on internal right-moving degrees of freedom which transform under spacetime supersymmetry [9]. This is not the case for the system described by eq. (5.1).

Concerning the compactification of the type *IIA, B* superstrings we note that we have not been able to construct any κ -symmetry singlets which could serve as internal degrees of freedom in the compactification. We therefore do not expect that the internal degrees of freedom will be inert under spacetime supersymmetry. This is again in agreement with the N.S.R. formulation of the type *IIA, B* superstring [9].

6. Fermionic κ -symmetry with harmonic variables

The covariant quantization of all extended objects with manifest space-time supersymmetry is impossible in the standard formulation. The reason for this is an infinite reducibility of the fermionic κ -symmetry. The origin of this infinite reducibility lies in the fact that the number of space-time fermionic degrees of freedom is less than the minimal dimension of the corresponding spinorial representation of the spacetime Lorentz group. However by introducing new harmonic variables it is possible for some of the κ -symmetric extended objects to perform a covariant quantization. We now discuss some examples of fermionic singlets where harmonic variables are involved.

a. Heterotic superstring with harmonic variables

The consistent quantization of the heterotic string was performed in [10] and is based on the following Lagrangian

$$\mathcal{L} = \mathcal{L}(\text{heterotic}) + \mathcal{L}(\text{harmonic}) \quad (6.1)$$

where $\mathcal{L}(\text{heterotic})$ is given in eq. (4.1) and $\mathcal{L}(\text{harmonic})$ is given by^{*}

$$\mathcal{L}(\text{harmonic}) = e\{p_{zi}(\partial_{\bar{z}}q^i - \lambda_z^A R_A^i(q)) - \mu_{z\bar{z}}^{\dot{A}} h_{\dot{A}}(q)\} \quad (6.2)$$

The world-sheet symmetries of this Lagrangian are the following: general coordinate invariance (ξ), conformal invariance (ρ), κ -symmetry (κ), Lorentz invariance (Λ) and new non-Abelian invariances ($\zeta, \tilde{\zeta}$) which act only on the new harmonic variables p, q, λ, μ . In fact the presence of these new symmetries assures that all harmonic degrees of freedom are gauge degrees of freedom, so that the physical content of the theory is the same as in the original theory (see eq. (4.1)).

Following the same presentation as in the examples above we first give the fermionic transformations of all fields. The κ -symmetry transformations of the old fields are the same as the ones given in eq. (4.2), while all new fields are singlets under the κ -symmetry. Therefore the singlets of the theory are $\Phi^i = (e, e_z^a, \Psi^I, q, p, \lambda, \mu)$. We now also give the transformation rules of all singlets under the remaining gauge transformations. Those of the old singlets (e, e_z^a, Ψ^I) are given in eq. (4.4). The transformation rules of the new singlets are as follows. Under general coordinate transformations, q^i transforms as a scalar, p_{ai} is a self-dual vector, i.e. $p_{\bar{z}i} \equiv e_z^a p_{ai} = 0$, λ_a^A is an anti-self-dual vector, i.e. $e_z^a \lambda_a^A = 0$ and $\mu_{ab}^{\dot{A}}$ is self-dual with respect to the first index, i.e. $e_z^a \mu_{a\bar{z}}^{\dot{A}} = 0$ and anti-self-dual with respect to the second index, i.e. $e_z^a \mu_{za}^{\dot{A}} = 0$. Under the other gauge transformations, i.e. Lorentz rotations, conformal transformations and the new non-Abelian gauge transformations they transform as follows:

$$\begin{aligned} \delta q^i &= R_A^i(q) \zeta^A \\ \delta p_{zi} &= -p_{zj} R_{A,i}^j \zeta^A - h_{\dot{A},i} \zeta_z^{\dot{A}} - (\rho - \Lambda) p_{zi} \\ \delta \lambda_z^A &= \partial_{\bar{z}} \zeta^A - \lambda_z^B \zeta^C f_{BC}^A - (\rho - \Lambda) \lambda_z^A \\ \delta \mu_{z\bar{z}}^{\dot{A}} &= \partial_{\bar{z}} \zeta_z^{\dot{A}} - \lambda_z^B \zeta^C p_{zi} \eta_{BC}^{\dot{A}} - 2\rho \mu_{z\bar{z}}^{\dot{A}} \end{aligned} \quad (6.3)$$

The subset of non-Abelian gauge symmetries given in eq. (6.3) forms a closed subalgebra of the full set of symmetries:

$$\begin{aligned} R_{A,j}^i R_B^j - R_{B,j}^i R_A^j &= f_{AB}^C R_C^i + \eta_{AB}^{\dot{A}} h_{\dot{A}}^i \\ R_A^i h_{\dot{A},i} &= 0 \end{aligned} \quad (6.4)$$

It turns out that in the investigation of the κ -symmetry singlets the particular form of

* The specific form of the harmonic variables occurring in eq. (6.2) was first introduced in [11].

the structure constants[†] in eq.(6.4) is irrelevant. This is most surprising in view of our experience with the earlier examples, since in this example the general coordinate transformations on all new singlets must be cancelled by the non-Abelian gauge symmetries as well. One could wonder how it is possible that the original κ -symmetry, which was formulated without any knowledge about the particular form of the gauge symmetry of the harmonic variables can produce this large gauge symmetry group. The particular mechanism which allows this to happen is due to the fact that two κ -transformations produce not only a general coordinate transformation, a κ -transformation, a Lorentz rotation and a conformal transformation whose parameters ξ^a , κ_z , Λ and ρ are given in eqs. (4.6) and (4.8) respectively but also new non-Abelian symmetries with the following field-dependent parameters:

$$\begin{aligned}\zeta^A &= -\xi^{\bar{z}}\lambda_z^A = 4i\bar{\kappa}_z^1\eta_z\kappa_z^2\lambda_z^A \\ \zeta_z^{\dot{A}} &= \mu_{zz}^{\dot{A}}\xi^{\bar{z}} = 4i\mu_{zz}^{\dot{A}}\kappa_z^1\eta_z\kappa_z^2\end{aligned}\quad (6.5)$$

If we now consider the non-Abelian symmetries (6.3) for these particular values of the parameters we observe that only the Abelian part of the transformation survives in the transformations of λ and μ , which are the gauge fields of these non-Abelian symmetries:

$$\begin{aligned}\delta_{(\zeta=\xi\lambda)}\lambda_z^A &= \partial_{\bar{z}}\zeta^A \\ \delta_{(\zeta=\mu\xi)}\mu_{zz}^{\dot{A}} &= \nabla_{\bar{z}}\zeta_z^{\dot{A}}\end{aligned}\quad (6.6)$$

Let us now show explicitly how the mechanism of compensating gravity works on the λ and μ fields. For the κ -commutators on λ and μ we find

$$\begin{aligned}[\delta_2, \delta_1]\lambda_z^A &= 0 = \xi^{\bar{z}}\partial_{\bar{z}}\lambda_z^A - (\rho + \Lambda)\lambda_z^A + \partial_{\bar{z}}\zeta^A \\ [\delta_2, \delta_1]\mu_{zz}^{\dot{A}} &= 0 = \xi^{\bar{z}}\partial_{\bar{z}}\mu_{zz}^{\dot{A}} - 2\rho\mu_{zz}^{\dot{A}} + \nabla_{\bar{z}}\zeta_z^{\dot{A}}\end{aligned}\quad (6.7)$$

One can easily check that indeed the right-hand side of these equations is zero upon using the explicit form of the parameters ξ , Λ , ρ and ζ .

On the other hand the κ -commutator on p and q does contain information about the non-Abelian symmetries since

$$\begin{aligned}\delta_{(\zeta=\xi\lambda)}q^i &= R_A^i\lambda_z^A\xi^{\bar{z}} \\ \delta_{(\zeta=\xi\lambda, \zeta^{\dot{A}}=\mu\xi)}p_{zi} &= -p_{zj}R_{A,i}^j\lambda_z^A\xi^{\bar{z}} - h_{A,i}\mu_{zz}^{\dot{A}}\xi^{\bar{z}}\end{aligned}\quad (6.8)$$

In this case the commutator of two κ -transformations also leads to a non-closure term η of the type given in eq.(1.4). These non-closure terms contain information about the

[†] Of course the particular form of the structure constants is important for the consistency of the covariant quantization [10] and for the absence of anomalies in the non-Abelian gauge symmetries [12].

nature of the non-Abelian transformations. It is not difficult to check how in these cases the gravity effect is cancelled in the κ -commutators:

$$\begin{aligned} [\delta_2, \delta_1]q^i &= 0 = \xi^{\bar{z}} \partial_{\bar{z}} q^i + R_A^i \zeta^A + \eta_z^i{}_j \frac{\delta S}{e \delta p_{zj}} \\ [\delta_2, \delta_1]p_{zi} &= 0 = \xi^{\bar{z}} \partial_{\bar{z}} p_{zi} - (\rho - \Lambda) p_{zi} - p_{zj} R_{A,i}^j \zeta^A - \zeta_z^{\bar{A}} h_{\bar{A},i} + \eta_z^i{}_j \frac{\delta S}{e \delta q^j} \end{aligned} \quad (6.9)$$

where $\eta_z^i{}_j = -\xi_z^{\bar{i}} \delta_j^{\bar{i}}$. Using the explicit values of the parameters of all gauge transformations generated by two κ -transformations and also the equations of motion one can again check that the right-hand side of eq. (6.9) is indeed zero identically.

As a result we see that the singlet property of p, q, λ and μ is reflected in the κ -commutators. A remarkable feature of this example is that the effect of gravity is compensated not only by Lorentz and conformal transformations and equation of motion terms but also by a whole set of non-Abelian gauge symmetries.

b. Type II A, B superstrings and super p-branes with harmonic variables

We consider the following Lagrangian which is a generalization of eq. (6.2) for arbitrary p-branes^{*}.

$$\mathcal{L}(\text{harmonic}) = \{p_i^a (\partial_a q^i - \lambda_a^A R_A^i) + \mu^{\bar{A}} h_{\bar{A}}\} \quad (6.10)$$

where $a = 1, \dots, p+1$. The case $p = 0$ corresponds to the superparticle and the heterotic string, $p = 1$ to the type IIA, B string and the membrane etc. . The action (6.10) is manifestly general coordinate covariant. To see this it is convenient to perform a change of variables which reintroduces the metric into the action:

$$p_i^a = e \check{p}_i^a \quad \mu^{\bar{A}} = e \check{\mu}^{\bar{A}} \quad (6.11)$$

Here $\check{p}_i^a$ transforms as a vector and $\check{\mu}^{\bar{A}}$ as a scalar. Note that all harmonic variables are inert under Lorentz rotations and conformal transformations. They are also κ -symmetry singlets: $\Phi^i = \{p_i^a, q^i, \lambda_a^A, \mu^{\bar{A}}\}$. Under the harmonic gauge transformations these singlets

^{*} We thank Paul Townsend for pointing out this possibility to us.

transform as

$$\begin{aligned}
\delta q^i &= R_A^i \zeta^A \\
\delta p_i^a &= -p_j^a R_{A,i}^j \zeta^A - \zeta^{a\dot{A}} h_{\dot{A},i} \\
\delta \lambda_a^A &= \partial_a \zeta^A + \zeta^B \lambda_a^C f_{BC}^A \\
\delta \mu^{\dot{A}} &= -\zeta^A \lambda_a^B p_j^a \eta_{AB}^{j\dot{A}} - \partial_a \zeta^{a\dot{A}}
\end{aligned} \tag{6.12}$$

We now consider the κ -commutators on the singlets Φ^{i*} . The following new feature that arises for $p \geq 1$ is the appearance of an additional gauge symmetry which is related to the reparametrization invariance but does not coincide with it

$$\begin{aligned}
\delta_{\text{add}} p_i^a &= 2\partial_b (\xi^{[a} p_i^{b]}) + 2\xi^{[a} p_j^{b]} \lambda_b^A R_{A,i}^j \\
\delta_{\text{add}} \lambda_a^A &= 2\xi^b \partial_{[a} \lambda_{b]}^A + \xi^b \lambda_b^B \lambda_a^C f_{BC}^A \\
\delta_{\text{add}} \mu^{\dot{A}} &= -\xi^a \lambda_a^A \lambda_b^B p_j^b \eta_{AB}^{j\dot{A}}
\end{aligned} \tag{6.13}$$

In the case of the superparticle and the heterotic string these symmetries are absent since $a = 1$. The additional gauge transformations are related to the other existing gauge symmetries in the theory. This relation is obtained exactly by the requirement of the existence of the singlets Φ^{i*} :

$$\begin{aligned}
[\delta_2, \delta_1] \Phi^{i*} &= 0 \rightarrow \\
\{ \delta_{g.c.t.}(\xi^a) + \delta(\zeta^A = -\xi^a \lambda_a^A) + \delta(\zeta^{a\dot{A}} = \xi^a \mu^{\dot{A}}) \\
&+ \delta_{\text{add}} + \delta(\text{on-shell trivial symmetry}) \} \Phi^{i*} = 0
\end{aligned} \tag{6.14}$$

where the on-shell trivial symmetry is defined in eq. (6.9).

It is nontrivial to covariantly quantize type II superstrings and general p-branes using the above harmonic action. This is due to the presence of the additional gauge symmetries given in eq. (6.13) and also because for $a \geq 2$ we have for every variable q^i a variable p_i^a and a variable λ_a^A . It is for instance not obvious that for $a \geq 2$ all harmonic variables can be made pure gauge by any choice of the non-Abelian harmonic gauge group. We think nevertheless that if a covariant quantization of the type II superstring and other p-branes can be performed the harmonic action presented in this paper together with the existence of harmonic fermionic singlets will play a crucial role.

7. Conclusions

In this paper the existence of singlets of fermionic gauge symmetries have been investigated because of several reasons: First of all the existence of singlets of global supersymmetry was already discussed in the literature. However there was no discussion at all of singlets of local fermionic symmetries. We think that the mere fact that there exists a whole family of such singlets as presented in this paper is a sufficient reason to study them. It is plausible that in new theories further examples of singlets will be found.

The singlets considered in this letter have the following important practical applications. First of all the simplest way to perform a covariant quantization^{*} of extended objects with spacetime supersymmetry is to use fermionic singlets as it was done for the heterotic superstring [10]. The same mechanism of covariant quantization also works in the case of the superparticle. For type *IIA, B* superstrings and general p -branes ($p \geq 1$) the existence of harmonic fermionic singlets have been established in this paper. Since at the present moment a tensor calculus for κ -symmetry which would enable one to construct generic κ -invariant actions is still lacking we are only able to use the harmonic fermionic singlets to covariantly quantize extended objects with spacetime supersymmetry. Whether or not they can be used to covariantly quantize general extended objects other than the superparticle and the heterotic string is at present under investigation.

Another practical application of the fermionic singlets considered in this paper is that they immediately reveal the structure of the gauge algebra. This is especially important because there exists in the case of κ -symmetry no underlying global Lie superalgebra. For instance using the singlets one immediately finds out that in the heterotic superstring the commutator of two κ -symmetries does not only produce a general coordinate transformation and a κ -transformation but also a conformal transformation as well as non-Abelian gauge symmetries. Furthermore using the existence of singlets we found new gauge invariances which were not observed before like the additional gauge symmetries in the harmonic action for super p -branes ($p \geq 1$) given in eq. (6.13). An interesting consequence of this structure of the commutator algebra is that it leads to relations between κ -symmetry, conformal and non-Abelian gauge symmetry anomalies. The specific form of the relationship between these anomalies is now under investigation [7].

Finally we have pointed out in Section 5 that right-handed fermions which are κ -symmetry singlets cannot contribute to the cancellation of conformal anomalies in the

^{*} By covariant quantization we mean Lorentz covariance in the target space as well as reparametrization invariance of the world-volume of the extended object.

compactification of the Green-Schwarz heterotic superstring. This is in accordance with the corresponding NSR superstring theory in $d = 4$ where the internal right-handed fermions are not neutral under supersymmetry [9].

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References

- [1] I.A. Batalin and G.A. Vilkovisky, Phys. Rev. **D28**, 2567 (1987)
- [2] M. Sakamoto, Phys. Lett. **B151** (1985) 115
- [3] C.M. Hull and E. Witten, Phys. Lett. **B106** (1985) 398
- [4] M. Grisaru, L. Mezincescu and P. Townsend, Phys. Lett. **B179** (1986) 35
- [5] C.M. Hull, Phys. Lett. **B212** (1988) 437
- [6] E. Bergshoeff, E. Sezgin and H. Nishino, Phys. Lett. **B166** (1986) 141
- [7] E. Bergshoeff, R. Kallosh and M. Rahmanov, *Anomalies in the Green Schwarz Superstring*, in preparation.
- [8] S.J. Gates Jr., P. Majumdar, R.N. Oerter and A.E. van de Ven, University of Maryland preprint UMDEPP 89-66 (December 1988)
- [9] W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. **B287** (1987) 477 ; T. Banks, L. Dixon, D. Friedan and E. Martinec, Nucl. Phys. **B299** (1988) 613 ; for a review see J.H. Schwarz, *Superconformal Symmetry and Superstring Compactification*, Caltech preprint CALT-68-1531 (1989).
- [10] R. Kallosh and M. Rahmanov, Phys. Lett. **B209** (1988) 133; Phys. Lett **B214** (1988) 549
- [11] E. Nissimov, S. Pacheva and S. Solomon, Nucl. Phys. **B296** (1988) 462
- [12] M.T. Grisaru and D. Zanon, Phys. Lett. **B218** (1988) 26