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EXPERIMENTAL STUDY OF RESONANCES

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1. INTRODUCTION

The fields used to guide and focus beams in circular accelerators and storage rings are generally designed with great care. However, some unwanted nonlinearities are inevitable due to small production errors in magnets or saturation effects. In addition, lumped nonlinear elements are frequently added to the linear lattice for specific purposes. For example, large synchrotrons and storage rings require chromaticity correction, where lumped sextupole lenses are introduced into the lattice whereas high-intensity machines may require octupoles to provide Landau damping in order to maintain transverse stability. Colliding beam storage rings suffer from strong nonlinearities introduced by the global electromagnetic fields themselves as the beams cross one another.

Nonlinearities in circular machines cause two main effects. First of all, the transverse variation in focusing strength causes a variation of tune with particle amplitude, the nonlinear detuning. Secondly, for certain combinations of radial, vertical and synchrotron tunes, nonlinear resonances can be excited. These resonances can have a profound influence on the stability of the particle dynamics and generally steps must be taken either to avoid them or to compensate for them. Occasionally they can be put to good use, for example to provide a slow controlled spill of particles out of a proton synchrotron.

2. NONLINEAR RESONANCES

One of the first things a newcomer encounters in the field of particle accelerators is the "tune diagram", like the one shown in Fig. 1, crossed with nonlinear resonance lines. For the moment, neglecting synchrotron motion, nonlinear resonances occur for combinations of radial and vertical tune satisfying relationships of the form

$$n_x Q_x + n_z Q_z = p \quad (1)$$

Fig. 1 Tune diagram showing sum resonance lines up to and including the 8th order.

where $n_x + n_z = N$, the "order" of the resonance and p is the "azimuthal harmonic".

The theory of nonlinear resonances is now well refined^{1),2)} and the reader is referred to the literature for details. Here, a few results are stated without proof.

The "excitation strength" κ of a resonance of order N is given by

$$\kappa = \frac{1}{2\pi} \int_0^{2\pi R} d\theta B_x^{|n_x/2|} B_z^{|n_z/2|} \exp i[n_x \mu_x + n_z \mu_z] * \begin{matrix} K_z^{N-1} & n_z \text{ even} \\ K_x^{N-1} & n_z \text{ odd} \end{matrix} \quad (2)$$

$$\text{where,} \quad K_z^{N-1} = \frac{R^2}{B_\rho} \frac{\partial^{N-1} B_z}{\partial x^{N-1}} \quad \text{normal } (n_z \text{ even})$$

$$K_x^{N-1} = \frac{R^2}{B_\rho} \frac{\partial^{N-1} B_x}{\partial x^{N-1}} \quad \text{skew } (n_z \text{ odd})$$

where μ and B are the usual lattice functions and θ is the azimuthal variable. A resonance of the form $n_x Q_x + n_z Q_z = p$ is excited by the p^{th} azimuthal harmonic of the distribution of $2N$ -pole elements, normal if n_z is even and skew (tilted by $\pi/2N$) if n_z is odd. The important point is that for a given distribution of nonlinear elements around the ring, only the p^{th} Fourier component of the distribution drives the resonance.

One of the most powerful ways of investigating nonlinear resonances experimentally is to correct them.

Due to the fact that a given resonance is driven only by a single Fourier component of the azimuthal distribution of nonlinear elements this can be achieved by introducing a set of lumped multipoles with the following constraints:

- 1) Install $2N$ -pole elements, normal if n_z is even, skew if n_z is odd
- 2) Choose element positions such that the product $\beta_x^{n_x/2} \beta_z^{n_z/2}$ is a maximum in order to minimize the required corrector strengths
- 3) Distribute the correctors to produce a pure as possible p^{th} azimuthal harmonic.

The amplitude and phase of the required correctors gives the resonance strength.

Especially in low-energy high-intensity synchrotrons, it is important to correct the strong low-order resonances because the self space-charge and image forces experienced by the particles induce a large tune spread, the "incoherent Laslett detuning" which makes it impossible to stay clear of them. A good example³⁾ is the CERN PS Booster, where the high-intensity beam at its injection energy of 50 MeV has almost half an integer of vertical tune spread (Fig. 2).

Fig. 2

The large shaded area indicates the area of the tune space occupied by particles at high intensity in the PS Booster

A comprehensive campaign of resonance correction in the Booster has been carried out by Schindl. Figure 3 shows the result of correcting the third order (sextupole) resonances. The technique is to use a low intensity beam to cross a given resonance at a certain time and to observe the beam loss. The amplitude and phase of the appropriate Fourier harmonic is then varied by empirically adjusting the strengths of a pair of correcting sextupoles. Of course, with only two correcting elements, many Fourier harmonics are excited as well as the desired one and this can cause problems to minimize the loss. For the case of the Booster the first step was to correct the resonance $Q_H + 2Q_V = 15$ using a pair of normal sextupoles, trace b). Next, the resonance $3Q_V = 16$ was corrected using a pair of skew sextupoles. However, the result of this correction was to excite the unwanted resonance $2Q_H + Q_V = 14$ which is also driven by skew sextupole fields, c). Finally, a second pair of skew sextupoles was used to simultaneously correct both skew resonances. The end result is almost perfect transmission, d). From the required correctors one can then work out the resonance strengths.

Fig. 3 Resonance correction in the PS Booster³⁾.

- a) beam transmission without correction.
- b) $Q_H + 2Q_V = 15$ corrected
- c) two-lens compensation of $3Q_V = 16$ with the consequence that $2Q_H + Q_V = 14$ is excited
- d) Simultaneous compensation of $3Q_V = 16$ and $2Q_H + Q_V = 14$ together with point b).

Resonance correction is not only necessary in small low energy machines. In the early days of the 500 GeV main ring at Fermilab the machine performance at its injection energy of 8 GeV was severely limited by the third-order resonances (Fig. 4a)). Transmission was greatly improved by correcting all four third-order stopbands $3Q_H = 61$, $2Q_H + Q_V = 61$, $Q_H + 2Q_V = 61$, $3Q_V = 61$ (Fig. 4 b))⁴⁾. This was accomplished using a number of independently powered sextupoles, both skew and normal, to generate the appropriate 61st Fourier harmonics.

Fig. 4 Scan of the tune diagram in the region of the third-order resonances in the FNAL main ring⁴⁾
a) before and b) after correction of the four third-order resonances

One of the easiest resonances to observe is the "main diagonal", the linear coupling resonance $Q_H - Q_V = p$. This is excited when the fractional parts of the radial and vertical tunes are close together. It is driven by skew gradients, mainly from tilt errors in normal quadrupoles, vertical orbit errors in sextupoles or experimental solenoids, resulting in a coupling of the transverse motion between the two phase planes. It can easily be observed by kicking the beam in one plane and observing the transverse difference signal on a pickup in the other plane. Figure 5 shows an example of such a measurement from the ISR²⁾. The beam was kicked in the radial plane and the transverse motion observed on both radial and vertical pickups. The beating between the two planes can easily be observed. From the amplitude and frequency of the modulation the strength of the coupling can be computed.

When the working point is placed close to a resonance, the circular trajectories in the normalized phase plane become distorted. For example, near a third-order resonance the phase space can be divided into stable and unstable areas by a triangular "separatrix". Inside the separatrix particle trajectories are closed but strongly distorted into a triangular shape whereas outside, the trajectories are

Fig. 5 Observation of the linear coupling resonance
The beam is kicked in the horizontal plane (lower trace)

unbounded. It is possible to observe the projection of this separatrix by measuring the transverse beam profile with a fast wire scanner. Figure 6 shows an example⁵⁾ from the SPS where the machine tune was placed close to a resonance and the beam kicked transversely to fill the stable area.

Fig. 6 Transverse profile of a beam near a third-order resonance. The phase-space plot was obtained by particle tracking.

The whole phase space can be plotted by kicking a beam and then measuring the transverse displacement turn-by-turn simultaneously on two pickups separated in phase by 90° . Plotting the position on one pickup against the position on the other is essentially the same as plotting position against angle. Figure 7 shows the

Fig. 7 Phase-space trajectories measured at SPEAR plotted by kicking the beam and measuring the position turn-by-turn on the orthogonal pickups. For small kick amplitudes the trajectories are circular whereas for large amplitude the phase space shows the characteristic distortion of a third-order resonance.

result of such a measurement⁶⁾ at SPEAR. For small kick amplitude the trajectories are almost circular whereas at larger amplitudes they become more and more distorted.

Observation with Schottky noise

The betatron frequencies on a difference pickup are given by

$$f_{\beta} = (n \pm Q) f_0 \quad (3)$$

where f_0 is the revolution frequency.

The width of the betatron lines for a debunched beam is given by

$$\Delta f_{\beta} = [(n \pm Q)\eta \pm Q\xi] f_0 \Delta p/p \quad (4)$$

where, $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_T^2}$

and γ_T is the "transition" energy of the machine.

The betatron lines with the negative sign correspond to the "slow" waves and those with the positive sign to the "fast" waves. Above transition ($\eta < 0$). The slow waves are narrower than the fast waves.

The corresponding longitudinal frequencies f_L observed on a sum pickup or off-centred difference pickup, are

$$f_L = n f_0 \quad (5)$$

with width

$$\Delta f_L = n \eta f_0 \Delta p/p. \quad (6)$$

Observation of these lines, either through their natural Schottky noise or through beam excitation, can reveal a great deal about beam behaviour.

Figure 8 shows a set of Schottky scans made on a coasting beam in the SPS with the tune such that particles straddle fourth-order resonances. The top trace shows the 247th longitudinal harmonic. From the width of the line, the momentum spread can be computed.

$$\frac{\Delta p}{p} = \frac{\Delta f}{n \eta f_0}$$

with this case $f_0 = 43.375$ kHz, $\eta = 1.83 \times 10^{-3}$, giving $\Delta p/p = \pm 1.8 \times 10^{-3}$. The next two traces show the radial and vertical Schottky bands. All three of these

Fig. 8

Longitudinal and transverse Schottky signals from a coasting beam in the SPS. The top trace shows the 247th revolution harmonic. The next two traces show the radial and vertical Schottky bands.

The tunes corresponding to the three depleted regions can be measured very precisely. They correspond to the three octupole resonances $4QH = 107$, $3QH + QV = 107$, $2QH + 2QV = 107$.

signals show three depleted regions. From the transverse signals the tunes corresponding to the depleted regions can be measured very precisely whereas from the longitudinal signal the corresponding relative momenta can be measured.

The depleted regions correspond to the three octupole resonances $4Q_H = 107, 3Q_H + Q_V = 107, 2Q_H + 2Q_V = 107$. The strongest of these is the skew octupole resonance $3Q_H + Q_V$. These "markers" give a very accurate measurement of the chromaticity since the tunes and relative momenta are known precisely.

Even higher-order resonances have been identified using Schottky scan techniques at the ISR. Figure 9 shows some examples⁷⁾ where resonances of order 5 to 12 manifest themselves as peaks in the transverse distribution rather than depleted regions. This is because the nonlinear detuning (variation of tune with amplitude) is strong enough compared to the resonance excitation strengths to trap particles in stable islands at tune values corresponding to the resonant tunes, resulting in an enhancement of the Schottky signals.

Transverse Schottky scan, showing 5th (and 8th) order resonances.

A family of 8th and 11th order resonances.

$$Q_H = 8 + \frac{11}{12} \frac{10}{11} \frac{9}{10} \frac{8}{9}$$

Identification of resonances from orders 8 to 12

Fig. 9

Transverse Schottky signals from coasting beams at the ISR. Resonances of order 5 to 12 can be identified as peaks in the Schottky signals rather than the depleted regions shown in the previous example. The nonlinear detuning is strong enough to stabilise particle trajectories.

3. BEAM-BEAM RESONANCES

Up to now, we have only considered resonances excited by azimuthal variations in field nonlinearities due to imperfections in the guide fields. In a colliding beam storage ring, much stronger nonlinearities can be introduced by the electromagnetic fields of the beams themselves as they cross one another. In fact, this beam-beam interaction has proved to be one of the most fundamental limitations in performance of both lepton (bunched beam) hadron storage rings⁸⁾.

One of the first and cleanest direct observations⁹⁾ of beam-beam resonances was made at the ISR using a variant of the Schottky technique where the beam was excited transversely in order to enhance the signal. By measuring the amplitude and phase of the beam response to the applied excitation (beam transfer function) a great deal of information can be obtained. Figure 10 shows the amplitude of the vertical transfer function of one beam as the current in the other beam is gradually increased. Resonances of order 7 to 9 slowly appear as features on the BTF signal as the beam-beam interaction becomes stronger.

Fig 10

Amplitude of the vertical beam transfer function in one ring as the current in the other ring is increased. Beam-beam resonances of order 7 to 9 become visible⁹⁾.

In the ISR the beam-beam interaction was weak and the associated nonlinear detuning produced a strong stabilisation as particle amplitudes increased thereby moving the tune off resonance. However, in a bunched-beam storage ring like the SPS proton-antiproton collider, particles are continuously changing their tune due to their synchrotron motion so the nonlinear detuning is considerably less effective in stabilising the beams. Beam-beam resonances of much higher order can prove to be catastrophic for beam survival, especially when the emittances of the two beams are widely unbalanced. For example, Fig. 11 shows a scan of the working diagram in the vicinity of a 16th-order resonance in the SPS proton-antiproton collider under particularly unfavourable conditions¹⁰⁾ with the emittance of one of the beams more than a factor of two higher than that of the other. The enhanced loss rate (measured from the experimental background) is clearly correlated to the tune.

Fig. 11 Experimental background as a function of radial tune at the SPS collider. The proton beam emittance was a factor of 2 larger than that of the antiproton beam ¹⁰⁾

With bunched beams, some tune modulation due, for example, to a non-zero chromaticity or a modulation in the strength of the beam-beam force depending on the longitudinal position of a particle in a bunch is unavoidable. Such a periodic modulation, say at the synchrotron frequency Q_s causes the main resonance at $n_x Q_x + n_z Q_z = p$ to split into F.M. sidebands at values of the tunes corresponding to

$$n_x Q_x + n_z Q_z + k Q_s = p$$

These are called synchro-betatron resonances.

In hadron storage rings the synchrotron tune Q_s is very low ($< 3 \times 10^{-3}$) so these individual sideband resonances cannot be clearly identified. However, in lepton storage rings, Q_s is generally at least on order of magnitude larger so it may be possible to probe synchrotron sideband resonances individually. One striking example ¹¹⁾ of this came from the storage ring DORIS I. In this machine, strong synchro-betatron resonances were excited because the bunches crossed at an angle (Fig. 12) so that the beam-beam kick depended strongly on the longitudinal position of a particle in the bunch. Varying the vertical tune then revealed many resonances where sharp losses were observed at tune values corresponding to the resonance condition (Fig. 12). In several measurements, the following 25 vertical resonances were observed:

Fig. 12 Beam losses as a function of vertical tune in Doris¹¹⁾.

$$\begin{aligned}
 &6 + 3Q_s, 6 + 4Q_s \\
 &49/8, (49 + Q_s)/8, (49 - 2Q_s)/8 \\
 &43/7, (43 \pm Q_s)/7, (43 - 2Q_s)/7 \\
 &37/6, (37 - Q_s)/6, (37 - 2Q_s)/6 \\
 &31/5, (31 \pm Q_s)/5, (31 - 2Q_s)/5, (31 \pm 3Q_s)/5, (31 \pm 4Q_s)/5 \\
 &25/4, (25 - Q_s)/4, (25 - 2Q_s)/4 \\
 &28 - 3Q_x, Q_x - 1 - Q_s
 \end{aligned}$$

4. ION INDUCED RESONANCES

Resonances excited by trapped ions have been observed in the Antiproton Accumulator (AA) at CERN. The negative potential well of the antiproton beam traps ions which can cause both tune shifts and resonance excitation. The ions are removed by clearing electrodes but longitudinal variations in potential caused by changes in both transverse beam profile and vacuum chamber shape make it difficult to remove isolated ion pockets. These resonances are quite similar to beam-beam resonances. In particular, very-high-order resonances can be excited by the strongly nonlinear field of the trapped ion cloud. For example, Fig. 13 shows a measurement of anomalous heating of the beam in the vicinity of 15th-order resonances¹²⁾. A stored proton beam does not show this behaviour because electrons are much easier to remove from the positive potential well of the beam.

Fig. 13 Transverse heating rate as a function of tune in the Antiproton Accumulator¹²⁾. The enhanced heating due to 15th order resonances is clearly visible.

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