OIDE EFFECT AND RADIATION IN BENDING MAGNETS

Oscar Blanco $^{1,2}$, Rogelio Tomas$^1$, Philip Bambade$^2$
Organisation Européenne pour la Recherche Nucléaire (CERN), Geneva – Switzerland$^1$
Laboratoire de L’accélérateur Lineaire (LAL), Université Paris Sud, CNRS/IN2P3, Orsay – France$^2$

Abstract

Including radiation effects during lattice design optimization is crucial in high energy accelerators. Oide effect and radiation in bending magnets are reviewed aiming to include them in the optical design process to minimize the IP beam size. The Oide double integral is expressed in simpler terms in order to speed up calculations, concluding in how longer quadrupoles with lower gradients may help reducing the Oide effect. Radiation in bending magnets is reviewed for linear lattices, generalizing to the case when the final dispersion is different from zero and making comparisons with theoretical results and particle tracking. An agreement between the theory, the implemented approximation included in MAPCLASS2 and the six-dimensional tracking in PLACET has been found.

Geneva, Switzerland
November 2014
Oide effect and Radiation in bending magnets

O. BLANCO\textsuperscript{1,2}, R. TOMAS\textsuperscript{1}, P. BAMBADE\textsuperscript{2}

Organisation Européenne pour la Recherche Nucléaire (CERN), Geneva – Switzerland\textsuperscript{1}
Laboratoire de L’accélérateur Lineaire (LAL), Université Paris Sud, CNRS/IN2P3, Orsay – France\textsuperscript{2}

Abstract
Including radiation effects during lattice design optimization is crucial in high energy accelerators. Oide effect and radiation in bending magnets are reviewed aiming to include them in the optical design process to minimize the IP beam size. The Oide double integral is expressed in simpler terms in order to speed up calculations, concluding in how longer quadrupoles with lower gradients may help reducing the Oide effect. Radiation in bending magnets is reviewed for linear lattices, generalizing to the case when the final dispersion is different from zero and making comparisons with theoretical results and particle tracking. An agreement between the theory, the implemented approximation included in MAPCLASS2 and the six-dimensional tracking in PLACET has been found.

1 Introduction
In order to achieve design luminosities, linear colliders feature nanometer IP beam spot sizes. Radiation effects are crucial during the design stage of the lattices, where effects can be evaluated by tracking particles through the lattice or by analytical approximations. However, during the design process this effect is measured at the end, when the optic parameters characterizing the lattice are set. In order to include both, radiation and optic parameters, during the design optimization process, radiation phenomena is reviewed. This document addresses two particular radiation phenomena: the Oide effect \cite{1} and the radiation caused by bending magnets \cite{2}.

The beam size is calculated as the sum of the contributions from linear transport, non-linear aberrations and radiation effects. The calculated second moment, $\sigma^2$, can be expressed as the sum of the second moment of each effect. Then, the total beam size is calculated as:

$$
\sigma^2 = \sigma_0^2 + \sigma_i^2 + \sigma_{\text{rad}}^2
$$

where, $\sigma_0$ is the linear beam size ($\sqrt{\epsilon \beta}$), $\sigma_i$ is contribution from aberrations and $\sigma_{\text{rad}}$ arises from the radiation within magnets: $\sigma_{\text{oide}}$ for quadrupoles and $\sigma_{\text{bend}}$ for dipole magnets.

The most important contribution to the vertical beam size due to radiation comes from the Oide effect in the final quadrupole QD0 which is addressed in Section $2$. In horizontal plane, the largest effect due to radiation originates in dipole magnets. This is addressed in Section $3$.

2 Oide effect
The Oide effect is the contribution to beam size due to radiation while particles traverse quadrupole magnets with gradually reduced momentum \cite{1}. For the vertical plane,

$$
\sigma_{\text{oide}}^2 = \frac{110}{3\sqrt{6\pi}} \frac{\lambda_e}{2\pi} \frac{\gamma^5 F(\sqrt{kL}, \sqrt{kl^*})}{2\pi} \left( \frac{\epsilon}{\beta^*} \right)^{5/2}
$$

where

$$
F(\sqrt{kL}, \sqrt{kl^*}) = \int_0^{\sqrt{kL}} \left| \sin \phi + \sqrt{kl^*} \cos \phi \right| \int_0^\phi \left( \sin \phi' + \sqrt{kl^*} \cos \phi' \right)^2 d\phi' d\phi
$$

(3)
and $\lambda_e$ is the Compton wavelength of the electron, $r_e$ is the classical electron radius, $\gamma$ is the relativistic factor, $\epsilon$ is the geometrical beam emittance, $\beta^*$ is the Twiss parameter at the observation point (in this case, the IP), and $k$, $L$ and $l^*$ are the quadrupole gradient, the quadrupole length and the distance to the observation point measured from the closest magnet face.

### 2.1 Solving the Integrals

The inner integral over $\phi'$ can be solved because it has a known primitive.

$$
\int_{0}^{\phi} (\sin \phi' + \sqrt{kL^*} \cos \phi')^2 d\phi' = \frac{\phi}{2} \left( (\sqrt{kL^*})^2 + 1 \right) + \frac{\sin(2\phi)}{4} \left( (\sqrt{kL^*})^2 - 1 \right) + \sqrt{kL^*} \sin^2 \phi
$$

(4)

The Eq. (4) can now be expressed as one integral.

$$
F(\sqrt{kL}, \sqrt{kL^*}) = 
\int_{0}^{\sqrt{kL}} |\sin \phi + \sqrt{kL^*} \cos \phi|^3 \left( \frac{\phi}{2} \left( (\sqrt{kL^*})^2 + 1 \right) + \frac{\sin(2\phi)}{4} \left( (\sqrt{kL^*})^2 - 1 \right) + \sqrt{kL^*} \sin^2 \phi \right)^2 d\phi
$$

The squared factor in brackets is always positive because all inner terms are real. The term inside the absolute value is also always positive, therefore, the integrand is always positive.

Considering the function:

$$
|\sin \phi + \sqrt{kL^*} \cos \phi| = \begin{cases} 
\sin \phi + \sqrt{kL^*} \cos \phi, & \text{if } \sin \phi + \sqrt{kL^*} \cos \phi \geq 0 \\
-(\sin \phi + \sqrt{kL^*} \cos \phi), & \text{if } \sin \phi + \sqrt{kL^*} \cos \phi < 0 
\end{cases}
$$

(6)

sign changes at every point $\phi_n = \arctan(-\sqrt{kL^*}) \pm n\pi, \ n \geq 1$.

It is possible to split the integration interval $i$ times, being $i$ the number of $\phi_n$ solutions where $0 < \phi_n < \sqrt{kL}$. On each of those intervals, the absolute value definition can be removed and replaced by the corresponding expression in Eq. (6), having only a difference in sign. By defining the primitive $F$ in an interval where the factor inside the absolute value is positive it is possible to evaluate $F$ as it is shown in Eq. (7).

$$
F(\sqrt{kL}, \sqrt{kL^*}) = F_{10} + F_{\phi_1} + F_{\phi_2} + F_{\phi_3} + \cdots \pm F_{\phi_i}
$$

(7)

The change of signs in each interval is only given by the absolute value definition, then, it is simpler to add the absolute value of each contribution.

$$
F(\sqrt{kL}, \sqrt{kL^*}) = |F_{10}| + |F_{\phi_1}| + |F_{\phi_2}| + |F_{\phi_3}| + \cdots + |F_{\phi_i}|
$$

(8)

If we know the primitive $F$ and we are able to calculate the $\phi_n$s in the integration interval, then, it is possible to calculate the factor $F$ without using an approximate integrator. The double integration has been simplified to a primitive evaluation.

### 2.2 The primitive $F$

The primitive $F$ exists and it has been calculated using Maxima [3] and Wolfram Alpha Mathematica [4] software.

$$
F = \frac{1}{1209600.0} \{ 1323 \cos(5\sqrt{kL}) - 675 \cos(7\sqrt{kL}) \\
+ \sqrt{kL}(378000 \sin(\sqrt{kL}) + 21000 \sin(3\sqrt{kL}) - 7560 \sin(5\sqrt{kL})) \\
+ \sqrt{kL^*}(23625 \sin(\sqrt{kL}) + 4725 \sin(3\sqrt{kL}) - 14175 \sin(5\sqrt{kL}) + 4725 \sin(7\sqrt{kL})
$$

2
+ √kL(−37800 cos(5√kL))
+ (√kL)^2(−75600 sin(3√kL) + 226800 sin(√kL))]
+ (√kl^*)^2[−49707 cos(5√kL) + 14175 cos(7√kL)
+ √kL(1587600 sin(3√kL) − 172200 sin(3√kL) + 68040 sin(5√kL))]
+ (√kl^*)^3[−80325 sin(√kL) − 144725 sin(3√kL) + 82215 sin(5√kL) − 23625 sin(7√kL)
+ √kL(37800 cos(5√kL))
+ (√kL)^2(680400 sin(√kL) − 126000 sin(3√kL))]
+ (√kl^*)^4[68985 cos(5√kL) − 23625 cos(√kL)
+ √kL(2041200 sin(√kL) − 205800 sin(3√kL) + 37800 sin(5√kL))]
+ (√kl^*)^5[−458325 sin(√kL) − 43225 sin(3√kL) − 25893 sin(5√kL) + 14175 sin(7√kL)
+ √kL(68040 cos(5√kL))
+ (√kL)^2(680400 sin(√kL) − 25200 sin(3√kL))]
+ (√kl^*)^6[−945 cos(5√kL) + 4725 cos(7√kL)
+ √kL(831600 sin(√kL) − 12600 sin(3√kL) − 37800 sin(5√kL))]
+ (√kl^*)^7[−354375 sin(√kL) + 5425 sin(3√kL) − 1323 sin(5√kL) − 675 sin(7√kL)
+ √kL(−7560 cos(5√kL))
+ (√kL)^2(226800 sin(√kL) + 25200 sin(3√kL))]
+ cos(√kL)((√kl^*)^2 + 1)(4725)((√kl^*)^5(√kL)(80)
+ (√kl^*)^4(155 − 48(√kL)^2)
+ (√kl^*)^3(√kL)(64)
+ (√kl^*)^2(182 − 96(√kL)^2)
+ √kl^*(−16√kL)
+ (√kL)^2(−48) + 75]
+ cos(3√kL)(−175)((√kl^*)^7(√kL)(120)
+ (√kl^*)^6(3)(144(√kL)^2 + 71)
+ (√kl^*)^5(√kL)(744)
+ (√kl^*)^4(720(√kL)^2 + 347)
+ (√kl^*)^3(√kL)(−24)
+ (√kl^*)^2(144(√kL)^2 − 473)
+ √kl^*(−648)√kL
+ (√kL)^2(−144) − 31]

This equation was coded in Python and included in MapClass2 [5,8].

In order to confirm that the code implementation gave the same result than the original double integral, both expressions were numerically evaluated a hundred times with random values assigned to √kl^* and √kL. Results show always agreement with relative difference lower than 10^{-3}.
\[ \epsilon_N \in \mathbb{N} \ \gamma \ \sigma_0 \ k \ L \ l^* \ F \ \sigma_{oide} \]

<table>
<thead>
<tr>
<th>Lattice</th>
<th>( \epsilon_N ) (nm)</th>
<th>( \gamma ) ((10^5))</th>
<th>( \sigma_0 ) (nm)</th>
<th>( k ) ((m^{-2}))</th>
<th>( L ) (m)</th>
<th>( l^* ) (m)</th>
<th>( F )</th>
<th>( \sigma_{oide} ) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIC 3 TeV</td>
<td>20</td>
<td>2935.0</td>
<td>0.7</td>
<td>0.116</td>
<td>2.73</td>
<td>3.5</td>
<td>4.086</td>
<td>0.85</td>
</tr>
<tr>
<td>CLIC 500 GeV</td>
<td>25</td>
<td>489.2</td>
<td>2.3</td>
<td>0.077</td>
<td>3.35</td>
<td>4.3</td>
<td>4.115</td>
<td>0.08</td>
</tr>
<tr>
<td>ILC 500 GeV</td>
<td>40</td>
<td>489.2</td>
<td>5.7</td>
<td>0.170</td>
<td>2.20</td>
<td>4.3</td>
<td>9.567</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 1: Beam size and radiation beam size contribution for three lattices. \( \epsilon_N \) is the normalized emittance.

2.3 Oide Beam size contribution

The Oide beam size contribution depends on combination of beam and optics parameters. If none of the beam parameters is to be changed then \( F \) can be used as a figure of merit of the optics as it is calculated only from \( k, L \) and \( l^* \). The target is to reduce it as much as possible and two energy cases are analyzed: 3 TeV \((l^* = 3.5 \text{ m})\) and 500 GeV \((l^* = 4.3 \text{ m})\).

Columns \( \sigma_0, \sigma_{oide} \) and \( F \) in Table 1 show that CLIC 3 TeV and 500 GeV [9][10] have larger contributions to beam size even having a lower \( F \) value than ILC 500 GeV [11].

In order to evaluate the minimum possible \( F \) for \( L \) and \( l^* \) given, the minimum \( k \) required to get the particles focused is when the Twiss function \( \alpha \) is zero just at the quadrupole opposite face to the IP.

Fig. [1a] shows the ratio squared between the beam size contribution due to Oide effect and the linear beam size for three cases: when \( k \) is the minimum required to get particles focused (to get \( \alpha_y = 0 \) at QD0 opposite side to the IP), when \( k \) is calculated as thin lens \((k = \frac{1}{L l^*})\), and the current QD0 status. Fig. [1b] shows the \( k \) values for the previous mentioned cases.

The Oide contribution to beam size is of the same order of the linear beam size. It might be possible to reduce it by doubling the current quad length. Quad lengths larger than 10 m do not lead to further improvements with the current parameters.

Fig. [2] is the corresponding to Fig. [1] for the 500 GeV case. The current design contributes less than 4% of the total beam size, concluding that the current QD0 length with the CLIC 500 GeV parameters does not need adjustment.
Fig. 1: Oide effect beam size contribution for CLIC 3 TeV design parameters. (a) $\sigma_{oide}^2$ normalized to designed linear beam size as a function of quad length for the minimum focusing $k$ (when $\alpha_y = 0$ at the quadrupole opposite side to the IP), for $k$ calculated as thin lens ($k = \frac{1}{L_{ll}}$) and the current QD0. (b) $k$ in the three previous cases for comparison.
Fig. 2: Oide effect beam size contribution for CLIC 500 GeV design parameters. (a) $\sigma_{oide}^2$ normalized to designed linear beam size as a function of quad length for the minimum focusing $k$ (when $\alpha_y = 0$ at the quadrupole opposite side to the IP), for $k$ calculated as thin lens ($k = \frac{1}{Ll}$) and the current QD0. (b) $k$ in the three previous cases for comparison.
3 Radiation in a bending magnet

The theory developed in [2] will be first rewritten in order to clarify all terms used, making the proceeding generalization straightforward.

3.1 Theoretical approximation

Assuming a lattice that can be described by transport matrices in the form written in the Eq. (9), radiation effects can be calculated by the model in [2].

\[
\begin{pmatrix}
    x_2 \\
    x'_2
\end{pmatrix} =
\begin{pmatrix}
    C(s_1, s_2) & S(s_1, s_2) & R_{16}(s_1, s_2) \\
    C'(s_1, s_2) & S'(s_1, s_2) & R_{26}(s_1, s_2)
\end{pmatrix}
\begin{pmatrix}
    x_1 \\
    x'_1
\end{pmatrix}
\]  

(9)

Being \( \Delta x_i = R_{16}(s_i, s_p)(-u)/E \), the deviation at the observation point \( s_p \) due to the \( i \)th photon of energy \( u \) radiated at some point \( s_i \), and \( E \) the beam energy.

The first rhs term in Eq. (10) is the sum over the \( N \) photons radiated during the time \( T \) for the particle to cross the magnet. \( N(T) \) describes the probability distribution of photon emission. As we are interested only in the second order moment, the mean \( x_0 = (\sum_{i=1}^{N(T)} \Delta x_i) \) is subtracted from the sum, obtaining \( \langle x \rangle = 0 \), \( \sigma^2_{\text{bend}} = \langle x^2 \rangle \), being \( x \) the horizontal transverse displacement from the reference orbit of a particle at the observation point.

\[
\sum_{i=1}^{N(T)} \Delta x_i - x_0 = x
\]  

(10)

The photon emission follows a Poisson distribution as a consequence of the normalized radiation spectrum and photon number spectrum of synchrotron radiation used in [12] Section 5. For any Poisson-like distribution \( N^2 = \langle N \rangle \). The beam size contribution due to radiation has two components of variability: the spread of \( \Delta x_i \) due to the energy emission \( u \) and the number of times the emission process occurs \( N^1 \). Therefore, it is calculated as in Eq. (11)

\[
\sigma^2_{\text{bend}} = \langle x^2 \rangle - \langle x \rangle^2 = \langle x^2 \rangle
\]  

(11)

\[
= \langle N \rangle \sigma^2_{\Delta x} + \langle \Delta x \rangle^2 \sigma^2_N
\]  

(12)

\[
= \langle N \rangle (\langle \Delta x \rangle^2) - \langle N \rangle (\langle \Delta x \rangle)^2 + \langle \Delta x \rangle^2 \langle N \rangle
\]  

(13)

\[
= \langle N \rangle (\langle \Delta x \rangle^2)
\]  

(14)

Where the \( i \) sub-index has been removed intentionally because the photon number emission is extracted from a continuous function of \( u \) the photon energy and either \( T \) or \( s/c \), where \( c \) is the speed of light.

The rate of emission of photons is calculated as in Eq. (15) where \( K_{5/3} \) is the modified Bessel function, \( u_c = \frac{3}{2} \frac{hc}{\rho} \) called the critical energy which depends on the relativistic factor \( \gamma \), the reduced Planck constant \( \hbar \) and the particle trajectory curvature \( \rho \), and \( P_\gamma = \frac{2\gamma_P mc^2}{\hbar \rho} \) is the instantaneous radiated power where \( r_e \) is the classical electron radius and \( m \) is the electron mass.

\[
n(u, s) = \frac{P_\gamma}{u_c^2} \left[ \frac{9\sqrt{3}}{8\pi} \int_{u/u_c}^{\infty} K_{5/3}(\xi) d\xi \right]
\]  

(15)

Using \( \Delta x(s) = (-u/E)R_{16}(s, s_p) \) the second moment is calculated by integration over the entire space and energies.

\[
\sigma^2_{\text{bend}} = \int_{0}^{T} \int_{0}^{\infty} [\Delta x(u, s)]^2 n(u, s) dudT
\]  

(16)

\footnote{Using statistics notation, \( \sigma = V(x) \) and \( \langle x \rangle = E(x) \), then, a process with two components of variability has a variance expressed as \( V(x) = E(V(x|N)) + E(E(x|N)) \). The term \( (x|N) \) denotes the evaluation of the \( x \) variable for a given \( N \)}
\[ \Delta x(s)_{total} = \frac{-u}{E} \eta(s_p) = \frac{-u}{E} \left[ C(s, s_p) \eta(s) + S(s, s_p) \eta'(s) + R_{16}(s, s_p) \right] \]

\[ \eta(s_p) = \sqrt{\frac{\beta s_p}{\beta_s}} \left[ \eta_s \cos \Delta \phi_{s,s_p} + (\alpha_s \eta_s + \beta_s \eta'_s) \sin \Delta \phi_{s,s_p} \right] + R_{16}(s, s_p) \]

where \( \alpha, \beta \) and \( \phi \) are the optics parameters and the subscripts indicate the evaluation point. The equations derived in [2] assume \( \alpha_{s_p} = 0, \eta_{s_p} = 0 \) and \( \eta'_{s_p} = 0 \), which are not valid during the lattice optimization process. From Eq. (19) and (21) is clear that the contribution to beam size due to radiation now can be calculated as:

\[ \sigma_{bend}^2 = C_2 \int_0^{s_p} \frac{E^5}{\rho^2} \left\{ \sqrt{\frac{\beta s_p}{\beta_s}} \left[ \eta_s \cos \Delta \phi_{s,s_p} + (\alpha_s \eta_s + \beta_s \eta'_s) \sin \Delta \phi_{s,s_p} \right] - \eta_{s_p} \right\}^2 ds \tag{22} \]

Eq. (22) was included in MapClass2 in order to be used during lattice design and optimization.

### 3.2 Generalization when final dispersion is not zero

It is common to have direct access to Twiss functions over the lattice, therefore, it is also convenient to use Eq. (9) to calculate \( R_{16} \) from the off-momentum function \( \eta \) and lattice parameters. Measuring from the reference orbit, the kick propagation from \( s \) to \( s_p \) can be written in terms of the general transport matrix, giving

\[ = \frac{1}{c} \int_0^{s_p} \int_0^\infty \left[ -\frac{u}{E} R_{16}(s, s_p) \right]^2 n(u, s) duds \tag{17} \]

\[ \sigma_{bend}^2 = C_2 \int_0^{s_p} \frac{E^5}{\rho^2} R_{16}(s, s_p)^2 ds \tag{19} \]

where \( C_2 = \frac{55}{24 \sqrt{3} (m c^2)^6} \approx 4.13 \times 10^{-11} \) m²GeV⁻⁵ is a constant coming from the emission rate integration already derived by Sands.

### 3.3 One dipole

A theoretical calculation has been derived for the case of one sector magnet (\( \rho, L, \theta \)) and a sector magnet plus a drift (\( L_{drift} \)). Beam energy loss is negligible compared with beam energy \( E \).

For a sector magnet, \( R_{16} = \rho(1 - \cos \theta) \), the radiation effect is calculated as follows

\[ \sigma_{bend}^2 = C_2 E^5 \int_0^\theta \frac{1}{\rho^3} \left[ \rho(1 - \cos(\theta - \chi))^2 \right] d\chi \tag{23} \]

\[ = C_2 E^5 \left[ \frac{1}{4} (6 \theta - 8 \sin \theta + \sin(2\theta)) \right] \tag{24} \]

\[ = C_2 E^5 \left[ \frac{\theta^5}{20} - \frac{\theta^7}{168} + \frac{\theta^9}{2880} - \frac{17\theta^{11}}{1330560} + O(\theta^{13}) \right] \tag{25} \]

In the case of a drift after the bending magnet and defining \( j = \frac{L_{drift}}{\rho} = \frac{L_{drift} \theta}{\rho} \)

\[ \sigma_{bend}^2 = C_2 \int_0^\theta E^5 \left[ 1 - \cos(\theta - \xi) + \frac{L_{drift}}{\rho} \sin(\theta - \xi) \right]^2 d\xi \tag{26} \]

\[ = \frac{C_2}{4} E^5 \left[ (1 - j^2) \sin(2\theta) - 8 \sin \theta + 4j(1 - \cos \theta)^2 + (6 - 2j^2) \theta \right] \tag{27} \]
\[ C_2 E^5 \left[ \frac{j^2 \theta^3}{3} + \frac{j \theta^4}{4} - \frac{(4j^2 - 3) \theta^5}{60} - \frac{j \theta^6}{24} + \frac{(16j^2 - 15) \theta^7}{2520} + \frac{j \theta^8}{320} \\
- \frac{(64j^2 - 63) \theta^9}{181440} - \frac{17j \theta^{10}}{120960} + \frac{(256j^2 - 255) \theta^{11}}{19958400} + \frac{31j \theta^{12}}{7257600} \\
- \frac{(1024j^2 - 1023) \theta^{13}}{3113510400} + O(\theta^{14}) \right] \]  

Eqs. (24) and (27) will be used to normalize the results from MAPCLASS2 and PLACET [13] where some care should be taken due to numerical precision.

### 3.4 MAPCLASS2 and PLACET results

In PLACET 0.99.01 two different implementations of radiation exist: the first one will be called ‘default’ and the second ‘six_dim’. ‘Default’ calculates radiation by segmenting the dipole in shorter pieces, what is called thin dipole approximation, while ‘six_dim’ does not make any sectioning of the dipole. Results from these implementations and from MAPCLASS2 can be seen in Fig. 3, normalized to the theoretical values in Eqs. (24) and (27). The energy is 1500 GeV in all cases.

Calculation of radiation effect in PLACET was done by subtracting the squared beam size from two trackings with same input parameters except for radiation ON/OFF. As PLACET 0.99.01 does not give an uncertainty value, fitting was made once on the raw particle data with total agreement with the PLACET reported value, therefore, the error bars come from statistics assuming error of \( \frac{1}{\sqrt{M}} \) with \( M \) the number of tracked particles, 100000 particles in all cases.

All required Twiss functions were calculated using ‘ptc_twiss’ 5 dim in MAD-X [14]. However, they have discrepancies for very low angles as it is shown in Fig. 3(a) and (b) showing abrupt changes close to \( \theta = 7.5 \times 10^{-6} \) rad. The validity of the model for such low angles is still under discussion.
Fig. 3: Beam size increase due to radiation normalized to theoretical value assuming negligible energy loss. (Left) ‘Default’ radiation option, (Right) ‘Six_dim’ option in PLACET 0.99.01. Plots (a) and (b) correspond to $L = 10$ m for a dipole only. Plots (c) and (d) correspond to $\theta = 10^{-4}$ rad for a dipole only. Plots (e) and (f) correspond to $L = 10$ m and $\theta = 10^{-4}$ rad while varying the drift length. Beam energy is 1500 GeV in all cases.
3.5 Validity for the FFS

The radiation model presented is valid when the average number of photons radiated per particle $\langle N \rangle$ is enough to characterize the overall effect in position by its second moment, where $\langle N \rangle = C_1 E \theta$ with $C_1 = 20.61 \text{ GeV}^{-1}$.

The CLIC FFS design is composed by magnets with bending angles shown in Table 2 for 3 TeV and Table 3 for 500 GeV. Although the average value of photons emitted per magnet is low, these are grouped in long sections with common bending angle. The third column indicates the quantity of magnets used in each of those sections.

<table>
<thead>
<tr>
<th>$\theta$ ($\mu$rad)</th>
<th>$\langle N \rangle$</th>
<th>Qty.</th>
<th>$\theta$ ($\mu$rad)</th>
<th>$\langle N \rangle$</th>
<th>Qty.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.07</td>
<td>70</td>
<td>8.3</td>
<td>0.08</td>
<td>70</td>
</tr>
<tr>
<td>3.9</td>
<td>0.24</td>
<td>20</td>
<td>27.5</td>
<td>0.28</td>
<td>20</td>
</tr>
<tr>
<td>17.2</td>
<td>1.06</td>
<td>10</td>
<td>135.0</td>
<td>1.39</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2: Bending angles in CLIC 3 TeV. Table 3: Bending angles in CLIC 500 GeV.

3.6 Cases where the model fails

Particle tracking gave dissimilar results when compared with theory for magnetic fields lower than $5 \times 10^{-3}$ T as is shown in Fig. 3(b). When calculating the average number of photons emitted by particle $\langle N \rangle$, it seems that there is a connexion between the one photon threshold and the dissimilarity, see Fig. 4 where the magnetic field was fixed at $5 \times 10^{-3}$ T. Changes in the model might need to be revisited on this magnetic field range.
Fig. 4: Result from tracking, theoretical calculation with the mean number of photons emitted by particle superimposed. Magnetic field is fixed at $5 \times 10^{-3}$ T and $E = 1500$ TeV.
4 Conclusion

The Oide effect was revisited, obtaining a faster implementation using analytical result. It is possible to reduce the radiation effect in QD0 by doubling its length in the current CLIC 3 TeV design. CLIC 500 GeV features a negligible contribution from the Oide effect.

Radiation in sector bending magnets has been analyzed, resulting in an explicit equation to evaluate the case when dispersion function $\eta$ is different from zero at the observation point. Simple cases with one sector bend and a sector bend plus a drift were calculated analytically showing that MAPCLASS2 results are in agreement with theory. PLACET showed different results depending on the radiation method used. There was better agreement between ‘six_dim’ tracking and theoretical radiation effects and as result, ‘six_dim’ has become the default option since PLACET 0.99.02.

This work has been developed as part of the program to perform computational optimization of lattices. It is foreseen to be used for the FFS optimization of future colliders and could lead to luminosity improvement since this effect has not been included in the design optimization before. The case with low number of photons emitted per particle showed a small disagreement which is under investigation as the current design uses bend angles around $10^{-5}$ rad.

Acknowledgements

My acknowledgement to Hector García for his help and time for discussions, to Angela Luna for clarifications on statistics, to Andrea Latina and Daniel Schulte for letting me know how the PLACET code works and for explanations.

References