EE

CERN - LEP - BI 89-09

c1

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/LEP-BI/89-09

POLARIZATION IN LARGE e^+/e^- STORAGE RINGS

M. Placidi

ABSTRACT After a general reminder of the nature of polarization phenomena in high energy storage rings, the course reviews the laws governing the spin motion in magnetic fields and the origin of depolarizing mechanisms; a description of simple spin rotators is given as an application of the spin precession laws. The principles of operation of laser polarimeters are discussed and optimization criteria are introduced. Finally, the application of polarimetry to the calibration of accelerator parameters and particle resonances completes the review.

CM-P00065101

Lecture given at the Joint US-CERN School
on Beam Observation, Diagnostics and Correction.
October 1988, Capri, Italy

GENEVA
January 1989

POLARIZATION IN LARGE e^+/e^- STORAGE RINGS

M. Placidi

CERN, Geneva, Switzerland

ABSTRACT After a general reminder of the nature of polarization phenomena in high energy storage rings, the course reviews the laws governing the spin motion in magnetic fields and the origin of depolarizing mechanisms; a description of simple spin rotators is given as an application of the spin precession laws. The principles of operation of laser polarimeters are discussed and optimization criteria are introduced. Finally, the application of polarimetry to the calibration of accelerator parameters and particle resonances completes the review.

1. INTRODUCTORY REMARKS

While of extreme interest for precision measurements around the Z^0 resonance, the possibility of obtaining a useful degree of transverse and longitudinal polarization in high energy storage rings is the result of a challenge with several problems; the most striking ones are listed below :

- Depolarizing effects increase with the spin tune $\nu_s = a\gamma$ hence with the beam energy [1]
- The absolute rms energy spread $\sigma_E \sim E^2$ increases quadratically with the beam energy and may become comparable to the integer spin-resonance spacing $\Delta E_s = 440.65$ MeV
- Reduction of radiated energy losses down to acceptable limits requires the use of low dipolar fields in the magnetic structure of the accelerator; as a consequence the natural transverse radiative polarization time ($\sim B^{-3}$) is very long, comparable with the beam life time
- Asymmetric wigglers proposed to speed up the natural polarization rate introduce a significant enhancement of the natural energy spread
- Reversing the beam polarization from the transverse to longitudinal direction, as required by some experimental programmes, needs the implementation of spin rotators in the interaction regions and their integration in the main structure of the accelerator without altering the original properties of the beams.

2. INTEREST IN POLARIZATION

Polarized e^+/e^- beams are very important for accelerator as well as for experimental physics.

Transverse (vertical) polarization would provide an absolute energy calibration of the accelerator to an accuracy about one order of magnitude better than that allowed using other methods (magnetic measurements). This is of great interest not only from the machine point of view but for the experimental physics too, since it also provides a precise determination of particle resonances (section 8).

As stressed in the Summary of Ref. #[2] / Vol.I, helicity effects would play an important role at e^+/e^- energies around 50 GeV, where weak interactions characterize the physics potential, differently from the "QED-dominated" physics available with lower energy storage rings.

Longitudinally polarized e^+/e^- beams can be used as a precision probe of the Standard Electroweak Model through the right-left asymmetry A_{LR} of the Z^0 production.

The experimental programme of the e^+/e^- storage ring LEP I is based on collisions of longitudinally polarized beams around the Z_0 energy. About 10^6 Z events are expected in a 100 days experimental programme with an integrated luminosity of $4 \cdot 10^4 \text{ nb}^{-1}$ and a $\approx 50\%$ polarization level. The statistical precision on the asymmetry A_{LR} would then be of the order of $2 \cdot 10^{-3}$.

3. TRANSVERSE POLARIZATION

The degree of transverse polarization in e^+/e^- storage rings is the result of an equilibrium between two conflicting mechanisms :

- The Sokolov-Ternov (ST) radiative spin-flip effect [3]
- Depolarization from spin-orbit coupling.

The natural radiative ST mechanism originates from an asymmetry in the spin-flip probability connected with the quantum emission of synchrotron radiation photons in the guiding dipole magnetic fields. The e^+/e^- beams become vertically polarized with spins parallel (e^+) or antiparallel (e^-) to the magnetic field. Their projection onto the field vector B defines the degree of beam polarization.

Due to the very low spin-flip probability ($\sim 10^{-11}$ the photon emission rate) the polarization time is very long for large storage rings with low field dipoles. The ideal asymptotic level from spin-flip asymmetry and the natural polarization time are given by :

$$\begin{aligned}
P_{\infty} &= 8/5\sqrt{3} = 92.4\% \\
\tau_p &= 1.63 (R/\rho) \rho^3/E^5 \sim B^{-3} \quad [\text{min}]
\end{aligned}
\tag{1}$$

where R is the average accelerator radius and ρ the dipole bending radius.

In a real machine the spins precess under the effect of electric and magnetic fields other than the main vertical guide field (section 4); as a consequence depolarizing mechanisms originate which modify the natural polarization time and the maximum attainable level of transverse polarization.

At the equilibrium the buildup of the polarization can be described by an exponential behaviour :

$$P_e(E, t) = \frac{\pm P_{\infty} (1 - e^{-t/\tau})}{1 + \tau_p/\tau_d}
\tag{2}$$

where

$1/\tau = 1/\tau_p + 1/\tau_d$ is the effective polarization time

$\tau_p(E)$ is the natural radiative polarization rise time, and

τ_d describes the effect of the depolarizing mechanisms.

The beam polarization (2) can be expressed in terms of the measurable time constant τ_x as :

$$P_e(E, t) = \pm (\tau_x / \tau_p) P_{\infty} (1 - e^{-t/\tau_x})
\tag{2}$$

3.1 Polarization rate enhancement

For large e^+/e^- storage rings such as LEP I, the natural polarization time would be quite long :

$$\begin{aligned}
\tau_p &= 6.64 / (E/100)^5 \quad \rightarrow 5.5 \text{ h at } 46.5 \text{ GeV} \\
&\quad \rightarrow 1.4 \text{ h at } 60 \text{ GeV}
\end{aligned}
\tag{1}$$

Asymmetric wigglers [4] can be used to speed up the polarization rate. Their configuration is a sequence of vertical field dipoles with opposite directions subject to the condition $\int B \, dl = 0$ (fig.1). The central element provides a strong field B^+ in the same direction as the guide field, to enhance the polarization rate; the depolarization introduced by the reverse field B^- in the side elements, responsible for a reduction of the overall polarization level, can be kept small if they can be made sufficiently long so that $B^- \ll B^+$. This mechanism is actually based on the dependence of the polarization rate on $\int B^3 \, dl$ and not on $\int B \, dl$ as can be gathered from equ. (1). A large B^+ / B^- ratio is then required to reduce the polarization time without penalizing the asymptotic value.

As an example, the natural polarization time in LEP is compared in fig. 2 to that attainable with the use of wigglers. It can be seen that at $E = 46.5$ GeV the time of about 90 min. predicted with the use of the "design" dipole wigglers [5], foreseen to control the LEP beam emittance and damping time, can be reduced to some 35 min by adopting recently proposed [6] dedicated wigglers.

Improving the polarization rate is of great importance in correcting depolarizing effects in a time short compared to the beam life time, hence to improve the polarization level available for experiments.

Fig. 1. Symmetric / asymmetric wigglers

Fig. 2. Polarization time in LEP

4. POLARIZATION PRECESSION

4.1 The Thomas-BMT equation

Spinning particles for high energy physics beams, protons, muons and electrons, have spin $1/2$.

A magnetic moment $\vec{\mu}$ is associated to the spin $\vec{s}$ of a charged particle of mass m and charge e :

$$\vec{\mu} = g \frac{e}{2mc^2} \vec{s} \quad (3)$$

which causes the particle to experience a torque $\vec{N}$ when passing through a magnetic field $\vec{B}$:

$$\vec{N} = \vec{\mu} \times \vec{B} \quad (4)$$

The value of the factor g is 2 for point-like fermions in Dirac's theory. Corrections from quantum electrodynamics (QED) to the classic g -value are expressed through a gyromagnetic anomaly a such that $g = 2(1 + a)$.

Values of a are given in the table below.

Particle	Anomaly $a = (g - 2)/2$
positron/electron	$+ 1.159652 \cdot 10^{-3}$
muon	$+ 1.165924 \cdot 10^{-3}$
proton	$+ 1.792846$
deuteron	$- 0.1429878$

Specifying the torque as the time rate of change of the angular momentum colinear to the magnetic moment, the relation (4) can be written, with $\vec{\Omega} = -(ge/2mc^2) \vec{B}$:

$$\frac{d\vec{s}}{dt} = g \frac{e}{2mc^2} \vec{s} \times \vec{B} = \vec{\Omega} \times \vec{s} \quad (5)$$

This is the Thomas-BMT [7,8] spin precession equation for a particle at rest in a magnetic field $\vec{B}$.

Detailed treatment of equ. (5) in the most general electromagnetic environment can be found in [1]. We restrict ourselves in the following considerations to the influence on spin motion from magnetic fields.

To describe the spin evolution of relativistic particles subject to forces from magnetic fields when moving in a storage ring, equ. (5) has to be transformed into the laboratory frame. The rate of precession of the polarization vector relative to a reference frame rotating in such a way that one of its axes is always aligned with the particle velocity is then described by the BMT equation (5) with

$$\vec{\Omega} = -(e/\gamma mc^2) [(1 + a) \vec{B}_{\parallel} + a \gamma \vec{B}_{\perp}] \quad (6)$$

where γ is the Lorentz factor and $\vec{B}_{\parallel}$, $\vec{B}_{\perp}$ the longitudinal and transverse $\vec{B}$ components on the particle trajectory: while e^+/e^- are deflected by a magnetic field, their spin precesses around the field direction.

The first term of equ. (6) represents the Larmor precession which takes place when the field is parallel to the direction of motion. The "anomalous" part describes the Thomas precession in fields orthogonal to the momentum; because of the factor γ it dominates the Larmor effect at relativistic energies, where the factors in γ cancel and the polarization precession rate becomes energy-independent.

4.2 Effect of Magnets on Polarization

Combining the Thomas-BMT equation with the Lorentz equation for charged particles moving in a magnetic field :

$$\frac{d\vec{p}}{dt} = q (\vec{v} \times \vec{B})$$

we can relate the precession rotation angle ϑ_p to the reference trajectory bending angle ϑ_0 produced by a magnetic field transverse to the particle momentum p

$$\vartheta_0 = c (e/E) \int B_{\perp} dl = (0.2997925 / p) \int B_{\perp} dl \quad [\text{Tm/GeV/c}] \quad (7)$$

to derive the well known relation :

$$\vartheta_p \approx (a\gamma) \vartheta_0 = (c a/mc^2) \int B_{\perp} dl \quad (8)$$

The rotation of the precession vector for relativistic particles is independent of energy if produced by a transverse magnetic field. Some interesting consequences of equ. (8) can be readily derived :

1) A 90° rotation of the precession axis requires a transverse field strength :

$$\left. \begin{aligned} \int B_{\perp} dl &= (mc^2/a c) \vartheta_p = 1.470 \vartheta_p = 2.309 \text{ Tm} \quad [e^+/e^-] \\ 1.746 \vartheta_p &= 2.742 \text{ Tm} \quad [p] \end{aligned} \right\} \quad (9)$$

2) Longitudinal fields are a factor $a\gamma/(1+a) \approx (a\gamma)$ less effective in rotating the spin vector [equ. (6)] hence the field strengths (9) should be a factor $(a\gamma)$ larger to produce the same rotations :

$$\int B_{\parallel} dl = (a\gamma) \int B_{\perp} dl \quad (10)$$

The spin rotations are in this case energy dependent and equ. (10) sets a limit to practical application of longitudinal fields for spin manipulation purposes (section 5).

3) After one machine revolution ($\vartheta_0 = 2\pi$) the total precession angle is $\oint \vartheta_p = 2\pi (a\gamma)$; this defines the spin tune as the number of spin precessions per revolution, in analogy with the synchrotron and the betatron tunes :

$$\nu_s = (1/2\pi) \oint \vartheta_p dl = a \gamma \quad (11)$$

In contrast to the betatron tunes, the spin tune is energy dependent.

5. DEPOLARIZING MECHANISMS

The periodic solution of the Lorentz equation for particles circulating in a storage ring defines a closed orbit which depends on the particle energy and which is referred to as a reference orbit for transverse oscillations.

In the same way the succession of magnetic fields influences the spin motion of the particles according to the Thomas-BMT precession equation : torques replace forces and the periodic solution of equ. (5) defines a reference set of orthogonal axes (l, m, n) for particles on the closed orbit, [9] to which arbitrary spin rotations are referred. The n-component is parallel/antiparallel to the direction of the main guide dipolar field respectively for e^+/e^- . For perfect planar machines it represents the periodic solution of (5) and defines a local precession axis, periodic on one revolution for particles on the reference closed orbit :

$$\vec{n}_0(\vartheta) = \vec{n}_0(\vartheta + 2\pi). \quad (12)$$

5.1 Perturbed spin motion with synchrotron radiation.

The significant modification in the state of e^+/e^- during the quantum emission is the energy loss. After photon emission the energy dependent closed orbit is changed and the particle begins to execute betatron and synchrotron oscillations. The sequence of magnetic fields has now a different history from that on the reference closed orbit and causes perturbations to spin motion. In a real machine with orbit errors the vector $\vec{n}$ is no more perfectly vertical and depends on the azimuth ϑ and on particle energy γ : $\vec{n}_0(\vartheta) \rightarrow \vec{n}(\vartheta, \gamma)$. Depolarizing effects are then described by the time evolution of $\vec{n}(\vartheta, \gamma)$ and its energy dependence through the spin-orbit coupling function $\vec{\Gamma} = \gamma \partial \vec{n} / \partial \gamma$ which is a measure of the depolarizing influence of synchrotron radiation emission. The fast photon emission and the slow energy recovery from the RF accelerating system induce the so called spin diffusion. After the quantum emission the particle spin starts precessing around a perturbed axis

$$\vec{n}(\gamma + \delta\gamma) = \vec{n}_0(\gamma) + \vec{\Gamma} \delta\gamma / \gamma \quad (13)$$

and it keeps doing so even if, after some synchrotron damping times, the energy recovery process makes $\vec{n}$ return

to $\vec{n}_0$. However, since the spin evolution only depends on the magnetic history, there is no damping for the spin motion and the spin projection on $\vec{n}_0$ is reduced.

Depolarization from spin diffusion is balanced by the ST spin-flip effect; both originate from synchrotron radiation emission.

5.1.1 Resonant depolarization

The cumulative effect of the perturbations from localized machine errors, harmonic with the revolution frequency, and from betatron and synchrotron oscillations, aperiodic to preserve oscillation stability, give rise to **resonant depolarization** when the spin precession frequency ν_s is commensurate with the general combination of oscillation frequencies :

$$\nu_s = a\gamma = \ell_0 + \ell_x Q_x + \ell_y Q_y + \ell_s Q_s \quad (14)$$

where $a\gamma$, Q_x , Q_y , Q_s are the spin, betatron and synchrotron tunes, and ℓ 's integers including zero.

The integer resonance $a\gamma = \ell_0$, produced by machine errors, is harmonic with the revolution period and defines the integer spin resonance spacing

$$\begin{aligned} \Delta E_s = mc^2 / a = & \quad 440.65 \text{ MeV} \quad [e^+/e^-] \\ & 523.35 \text{ MeV} \quad [p] \end{aligned} \quad (15)$$

Their structure is rather complicated, as shown in fig. 3, where the SPEAR polarization data, recently interpreted by S. Mane [10], give a picture of the real spin behavior in a storage ring.

Fig. 3. Interpretation of the resonant depolarizing spectrum in SPEAR [10]

5.2 Depolarization with colliding beams

Particles interacting off-axis with an incoming bunch of equal or opposite charge are subject to strong toroidal magnetic fields orthogonal to their trajectories as sketched in fig. 4. Two major depolarizing mechanisms are likely to be produced during the beam-beam interaction : the classical BMT spin precession from the transverse collective fields, and the ST spin-flip effect from beamstrahlung. Both can affect transverse (fig. 4a) or longitudinal (fig. 4b) polarization states.

Experimental results from PETRA seem to indicate that transverse e^+/e^- polarization can survive at bunch densities close to the beam-beam limits. An investigation of the effects on longitudinally polarized beams has been performed recently [11] in the case of linear colliders.

Fig. 4. Depolarization from beam-beam interaction for (a) vertical and (b) longitudinal case

5.3 Interaction with the experimental solenoids

Solenoids affect transverse and longitudinal polarization [12] through the main field and its components on the particle momentum associated with the beam divergences x' , y' (fig. 5). Their effect on spin motion has to be compensated in view of experimentation with polarized beams, adopting schemes based on closed orbit bumps.

Fig. 5. Depolarization from solenoids

5.4 Compensation of depolarizing effects

Single-beam depolarization is mainly resonant and essentially originates from an imperfect vertical closed orbit. Strategies adopted to optimize vertical polarization use simulation programs to cancel the closed orbit harmonics which have been shown [13] to be responsible of the deviations of the $\bar{n}$ axis :

$$|\delta n| \sim \sum_{\ell} \frac{(a_k^2 + b_k^2)}{(\ell - \nu_s)^2} \quad (16)$$

where a_k, b_k are the Fourier components of the vertical closed orbit in the spin precession frame.

Four orbit bumps each over three LEP I 60° cells, where the spin preceeds by $\approx 2\pi + \pi/4$ at the Z^0 spin tune $a_\gamma(Z^0) = 104.8$, are suggested [14] to correct the two closest harmonics of the closed orbit (harmonic spin matching), complemented by further control of the spurious vertical dispersion harmonics. Predicted vertical polarization levels for LEP I are shown in fig. 6.

Fig. 6. Polarization after (a) orbit correction (b) harmonic spin-matching and (c) partial compensation of synchrotron satellites

6. SPIN MANIPULATION : ROTATORS

The natural polarization direction in e^+/e^- storage rings is parallel/antiparallel to the dipolar magnetic guide field perpendicular to the beam direction. To match the interest for spin physics in longitudinal polarization special magnetic devices are required to rotate the precession axis into the longitudinal direction in the experimental interaction regions and back to the vertical direction in the arcs.

The simplest family of these **Spin Rotators** produces 90° rotations of the precession axis. The energy dependence of the spin tune $\nu_s = \gamma a$ makes these devices "chromatic" : the 90° rotation is exact only at a given beam energy and the rotator can be used in a rather narrow energy range around it.

This problem can in principle be solved with the use of more complicated configurations of horizontal and vertical bending magnets called **Siberian Snakes**, from the Soviet authors Derbenev, Kondratenko and Skrinsky [15]. A Siberian Snake rotates the spin vector by 180° about an arbitrary axis lying in a plane containing the horizontal and the longitudinal axes of the accelerator, thus always inverting the vertical component of the spin vector.

Extensive accounts of the properties of **Snakes** and **Serpents** can be found in [1] and [16]. In the following, two examples of spin rotators are given to illustrate the concepts expressed above.

6.1 Solenoid spin rotator

The basic configuration makes use of a solenoid followed by a horizontally bending dipole. A vertical spin vector is rotated into the horizontal direction by the solenoid and into the longitudinal direction by the dipole. The combination of two configurations in an **antisymmetric scheme** (opposite field polarities at each side of the symmetry point) clearly restores the initial spin direction at the exit of the system even if this is not excited at the design beam energy. The helicity at the symmetry point can be reversed by simply inverting the solenoid polarity, i.e. without changing the beam trajectory, as is the case for transverse field rotators. Another advantage is that no vertical displacement is introduced in the machine layout, a positive point in view of the limited dimensions of accelerator tunnels.

From eqs. (8) and (10) the Larmor precession in a solenoid is inversely proportional to beam energy so that the required field strength for high energy particles is about $(a\gamma)$ times the strength of a transverse field rotator. Unless very high field (superconducting) solenoids are used, the application of these rotators is limited to low energy rings.

The evolution of the precession vector in an antisymmetric spin rotator is shown in fig. 7.

An example of their application is in the SLAC/SLC complex where the longitudinal spin of the electrons from the Linac has to be put into the vertical direction in the LTR beam line before entering the North Damping Ring and turned again into longitudinal in the RTL beam line, prior to reinjection in the Linac.

Fig. 7. Spin vector evolution in an antisymmetric solenoid spin rotator

6.2 Transverse field spin rotators

Practical spin rotators at high energies make use of transverse magnetic fields. The Richter-Schwitters (RS) spin rotator [17] is an antisymmetric configuration of four vertically deflecting dipoles around the Interaction Point (IP) where the precession vector is meant to be rotated into the longitudinal direction.

Recalling that the spin is rotated by an angle $(a\gamma)$ times the particle deflection [equ. (8)], the $\Psi = 90^\circ$ spin rotation requires a net vertical angle for the beam trajectory at the IP :

$$\varphi = \Psi/a\gamma \Rightarrow 14.885 \text{ mrad for LEP at } 46.5 \text{ GeV} .$$

This aspect may constitute a drawback for large experimental detectors installed in lower energy rings.

Since the relationship between amplitude of spin rotation and orbit geometry is energy dependent, the 90° spin rotation is obtained exactly only at a given energy, unless the geometry is made to vary with energy. Moreover, the helicity at the IP cannot be switched from positive to negative without inverting the signs of the deflections (variable layout) and hence the slope of the orbit in the experiments. Despite these "limitations" the RS scheme constitutes the simplest rotator for high energy rings, with evident advantages for acceptable costs and possibility of spin-matched optical solutions, providing a correct spin orientation together with the compensation of radiation-induced spin diffusion.

The geometry of an RS spin rotator, "spin-matched" for the LEP interaction regions [18,19], is sketched in fig. 8.

Fig. 8. Richter-Schwitters spin rotator scheme.

7. THE PRINCIPLES OF COMPTON POLARIMETRY

Originally suggested by Baier and Khoze [20], the laser polarimeter has become a part of the standard equipment in e^+/e^- storage rings above 1 GeV. The first example worked at SPEAR [21] and since then almost all the accelerator laboratories operating with e^+/e^- storage rings have adopted the laser polarimeter as a powerful diagnostics tool.

The laser polarimeter is based on spin-dependent Compton scattering of circularly polarized photons from high energy positron/electron beams. In the scheme of fig. 9 circularly polarized laser light is directed against the beam and the vertical distribution of the backscattered photons is observed.

If the e^+/e^- beam is vertically polarized an up-down asymmetry in the γ distribution occurs depending on the helicity of the laser photons, while for longitudinally polarized e^+/e^- a difference in the scattering cross section is observed. An Asymmetry function is defined in both cases as

$$A = \frac{n_R - n_L}{n_R + n_L} \quad (17)$$

where n_R and n_L are the recoil γ -rates for the two helicities of the incoming photons.

Fig. 9. Schematic of the laser polarimeter

Fig. 10. Notations for Compton kinematics

7.1 The spin-dependent Compton cross section

Calculated by Lipps and Tolhoek [22] the spin-dependent differential Compton scattering cross section $\sigma_C(P_e, P_\varphi)$ reads, in the e^+/e^- rest frame with the notations of fig. 10 :

$$d\sigma_C / d\Omega = 1/2 (r_0 k' / k_0')^2 (\Phi_0 + \Phi_2) \quad (18)$$

with

$$\Phi_0 = (1 + \cos^2 \vartheta') + (k_0' - k')(1 - \cos \vartheta')$$

$$\Phi_2 = -\xi_3 (1 - \cos \vartheta') [\vec{\xi} \cdot (\vec{k}_0' \cos \vartheta' + \vec{k})]$$

$$\vec{\xi} \equiv (\xi_1, \xi_2, \xi_3) = e^+/e^- \text{ polarization vector}$$

$$\vec{\xi} \equiv (0, 0, \pm P_\varphi) = \text{circularly polarized light vector}$$

$$r_0 = e^2 / m_e c^2 \text{ (classical electron radius)}$$

The photon momenta k', k_0', k_0 in units of electron rest mass and the scattering angles ϑ, ϑ' are related by the kinematic relations :

$$\left. \begin{aligned} 1/k' - 1/k_0' &= 1 - \cos \vartheta' \\ k_0' / \gamma k_0 &= 1 + \cos \vartheta \approx 2 \\ k &\approx \gamma k' (\cos \vartheta' - 1) \approx 2 \gamma^2 k_0 (\cos \vartheta' - 1) / [1 + k_0' (1 - \cos \vartheta')] \\ \gamma \vartheta &\approx \sin \vartheta' / (1 - \cos \vartheta') = \cot(\vartheta'/2) \end{aligned} \right\} \quad (19)$$

The cross section (18) depends on the beam polarization through the spin-dependent term Φ_2 .

For vertical e^+/e^- polarization $[\vec{\xi} \equiv (0, 0, \pm P_e)]$:

$$\Phi_2 = \pm P_\varphi P_e \cos \varphi' (1 - \cos \vartheta') k' \sin \vartheta' \quad (20)$$

shows a φ' asymmetry proportional to the beam polarization level P_e and maximum at $\vartheta' = \pi/2$.

For longitudinal e^+/e^- polarization $[\vec{\xi} \equiv (0, \pm P_e, 0)]$:

$$\Phi_2 = \pm P_\varphi P_e (1 - \cos \vartheta') (k' + k_0') \cos \vartheta' \quad (21)$$

shows no φ' asymmetry. A difference in the scattering cross section exists however from left- and right-handed photons which is maximum for $\vartheta' = \pi$.

Introducing the cross section (18), shown in fig. 11 as a function of the CM scattering angle ϑ' for P_e and $P_\varphi = 1$, the asymmetry (17) can be written as

$$A(P_e, P_\varphi) = \frac{(d\sigma/d\Omega)_R - (d\sigma/d\Omega)_L}{(d\sigma/d\Omega)_R + (d\sigma/d\Omega)_L} = \frac{\Phi_2}{\Phi_0} = P_e P_\varphi \Pi(\vartheta', k_0') \quad (22)$$

The function $\Pi(\vartheta', k_0')$, i.e. the asymmetry for $P_e, P_\varphi = 1$, is also referred to as the **Analyzing power**.

For the transverse case the analyzing power Π_T has a maximum around $\vartheta' = \pi/2$ and thus the strongest information on P_e comes from photons backscattered at angles $\vartheta \approx \pm 1/\gamma$ in the laboratory frame [equ. (19)₄].

The longitudinal analyzing power Π_L is maximum at $\vartheta' = \pi$. The dependence of $\Pi_{T,L}(\vartheta', k_0')$ on the CM scattering angle is shown in fig. 12.

Fig. 11. Spin-dependent Compton cross section vs CM scattering angle ϑ'

Fig. 12. Dependence of the analyzing power Π_T and Π_L on the CM scattering angle ϑ'

The analyzing power of a laser polarimeter depends on the incoming photon energy k_0' and then on the e^+/e^- and the photon energies E and E_φ in the laboratory system :

$$k_0' = (1 + \cos\vartheta) \gamma k_0 = (1 + \cos\vartheta) E E_\varphi / (mc^2)^2 \quad (23)$$

For a given energy E it can be optimized by properly choosing the photon energy $E_\varphi = hc/\lambda_\varphi$.

The energy E_γ of the recoil γ 's [equ. (19),] in the laboratory frame

$$E_\gamma = \frac{2 \gamma^2 E_\varphi (\cos\vartheta' - 1)}{1 + k_0' (1 - \cos\vartheta')} \quad (24)$$

depends quadratically on the beam energy, reaching a maximum of $\approx 2/3 E$ for γ 's emitted backwards. The information on the beam transverse polarization is carried on by the γ 's backscattered at $\pm \pi/2$ with energy :

$$E_\gamma(\vartheta' \approx \pi/2) = \frac{E k_0'}{1 + k_0'} \quad (24')$$

i.e. about $E/2$ for $k_0' \approx 1$. The spatial distribution of the recoil γ 's at the detector is shown in fig.13 for longitudinal and vertical beam polarization [23]. In the first case the asymmetry is on the backscattered intensities. In the second case the distribution of the high energy γ 's is shown separated from the spin-dependent one, the shape of which is modified by the laser light helicity.

Fig. 13. Spatial distributions of Compton scattered γ 's at the detector

7.2 Optimization

From the preceding considerations it follows that the measurement of the transverse polarization is more critical than that of the longitudinal. It seems then useful to analyze the criteria which influence the choice of the parameters of a transverse laser polarimeter.

7.2.1 The Laser Interaction Region (LIR). For a given e^+/e^- polarization level the measurable asymmetry depends on the optical beam parameters at the illumination point and on the distance D of the detector. For a given spatial resolution of the detector the angular resolution and the analyzing power are optimized by choosing the largest possible value for D with the condition [21] :

$$D_{\text{opt}} = (\beta / \alpha)_{\text{LIR}} \quad (25)$$

where α and β are the beam Twiss parameters at the LIR. In particular $\alpha = -\beta'/2$ has to be positive so that the angular resolution is optimized by illuminating the beam in a point where the β -function is near a maximum.

7.2.2 The asymmetry. The maximum of the analyzing power $\Pi_T(\vartheta', k_0')$ for $\vartheta' \approx \pi/2$ reads :

$$\Pi_T^*(k_0') \equiv \Pi_T(\vartheta' = 90^\circ) = \frac{k_0'}{1 + k_0' + k_0'^2} \quad (26)$$

The asymmetry is then at an optimum for

$$k_0' \approx 2 E E_\varphi / (m_0 c^2)^2 = 9.5 E / \lambda_\varphi \approx 1 \quad (23')$$

where $E_\varphi \lambda_\varphi = 1.24 \cdot 10^3 \text{ eV nm}$.

This optimum asymmetry condition allows one to choose the laser wavelength according to the beam energy. It prescribes the use of a laser in the visible range ($\lambda_\varphi = 532 \text{ nm}$) for energies around 50 GeV (LEP I case) and in the UV ($\lambda_\varphi = 308 \text{ nm}$) around 30 GeV (HERA case). Figure 14 shows the energy dependenc of Π^* in the transverse and the longitudinal case for the two laser wavelengths. As already anticipated, the choice of the parameters optimizing the transverse analyzing power always provides a larger value for Π_L^* .

Fig. 14. Energy dependence of $\Pi_T^* (k_0)$ and $\Pi_L^* (k_0)$

7.2.3 The luminosity. For collisions between bunched beams with gaussian distributions in three dimensions the luminosity is given by :

$$L = f \frac{N_e N_\phi}{\Sigma} = f L^* \quad (27)$$

For the laser polarimeter N_e is the electron bunch charge, N_ϕ the laser pulse intensity, Σ the interaction area and the interaction frequency f is the smallest between the laser repetition rate and the bunch revolution frequency. L^* is the luminosity per interaction.

If the β -functions at the LIR are not too small the beam widths can be assumed constant and the interaction area Σ for beams colliding at an angle $2\delta_0$ is given by

$$\Sigma = \frac{2\pi \sqrt{(\sigma_x^2 + \sigma_{\phi x}^2) [(\sigma_y^2 + \sigma_{\phi y}^2) \cos^2 \delta_0 + (\sigma_z^2 + \sigma_{\phi z}^2) \sin^2 \delta_0]}}{\cos \delta_0} \quad (28)$$

where $\sigma_{x,y,z}$ and $\sigma_{\phi x, \phi y, \phi z}$ are the rms dimensions of the electron bunch and of the laser pulse.

The contribution to Σ from the longitudinal rms beam dimensions is reduced by choosing a small value of the interaction angle δ_0 , which would also produce a long interaction region. This requirement has to be balanced with the importance of reducing the sensitivity to the vertical orbit misalignments.

The number N_φ of photons per laser pulse is given by

$$N_\varphi = E_L / E_\varphi = 5 \cdot 10^{15} E_L \lambda_\varphi \quad (29)$$

where E_L is the laser pulse energy in Joules. Once λ_φ is chosen to optimize the asymmetry, the pulse energy E_L defines the luminosity of the interaction.

7.2.4 The Laser. According to the background context defined by the position of the LIR and the detector in the machine, two different methods can be adopted to illuminate the particle beam.

In the **single-photon method** a low energy, high repetition frequency laser (pulsed or cw) is used. The backscattered γ -rate per interaction is of the order of ≈ 1 Hz or less and the maximum rate is then given by the e^+/e^- bunch revolution frequency. A calorimeter is used to detect positions and energies of single recoil γ 's and appropriate energy cuts are introduced to select the medium- E_γ energy range where the P_e information is present [fig. 13b)]. The enhanced performance can however be spoiled by the background from the accelerator environment (synchrotron radiation and gas bremsstrahlung) if not carefully controlled.

In the **multi-photon method** several 10^4 γ 's per interaction are produced by a high peak power laser and the maximum γ -rate is limited by its repetition rate [24]. The method provides a more efficient background rejection and a fast information on changes in the asymmetry, comparable with the polarization rate. With a bunch of γ 's reaching the detector no energy selection is possible and the spatial distributions of fig. 13b) are superimposed (fig. 15).

The measurable asymmetry is reduced in comparison with the previous method since reversing the laser helicity affects the shoulders of the γ -distribution, the central part being unaffected.

Measuring the centroid shift of the distributions for the two laser helicities provides information on the beam polarization independent from the vertical e^+/e^- beam dimensions at the LIR [25].

Fig. 15. Recoil γ distribution in the multiphoton method

8. CALIBRATION OF ACCELERATOR PARAMETERS AND PHYSICS EXPERIMENTS

By introducing an artificially generated spin resonance and recording the associated changes in the γ -rate asymmetry, the mean energy of the circulating beams and other accelerator parameters can be determined to a great accuracy [26]. If the accelerator energy is calibrated in conjunction with physics experiments, hadronic resonances can be determined to the accuracy of the electron rest mass since the $e^\pm$ gyromagnetic anomaly

$$a(e^\pm) \equiv (g-2)/2 = 1159652188 \cdot 10^{-12}$$

is known to a precision of some 10^{-9} [27].

A depolarizing device generating an RF radial magnetic field induces a controlled spin resonance on the circulating beam without perturbing the machine operation. The resonance occurs at spin tunes

$$\nu_s = n + (f_d / f_{\text{rev}}) \quad (30)$$

where n is any integer.

The asymmetry is monitored as a function of the driving frequency f_d and the resonance condition

$$f_{\text{dr}} = |a\gamma - n| f_{\text{rev}} \quad (31)$$

defines the fractional part of the spin tune $\delta\nu_s = |a\gamma - n|$ which, from equ.(11) and (15), prescribes the correction

$$\delta E_{\text{cal}} = (mc^2/a) \delta\nu_s \quad (32)$$

to the nominal machine energy. As an example of precise resonance measurement, fig. 16 shows the T' experiment results from the ARGUS and Crystal Ball collaborations at DORIS [28].

ACKNOWLEDGEMENTS

I would like to thank Phil Bryant and Mel Month for the invitation to give this course at the US-CERN School, and Stuart Turner for the careful and patient effort in reading the manuscript. I am particularly grateful to the Local Organizing Committee for the friendly welcome in Capri.

Fig. 16. Energy calibration and precise resonance measurements at DORIS

REFERENCES

- [1] B.W. Montague, *Physics Reports* Vol.113 No.1 (Nov. 1984).
- [2] G. Alexander et al., (Ed.), *POLARIZATION AT LEP*, CERN 88-06 (Sept. 1988).
- [3] A.A. Sokolov and I.M. Ternov, *Sov. Phys. Dokl.* 8 (1964) 1203.
- [4] J.M. Paterson, J.R. Rees and H. Wiedemann, *SPEAR-186 and PEP 125* (1975).
- [5] J.M. Jowett, and T.M. Taylor, *IEEE Trans. NS-30* (1983).
- [6] A. Blondel, J.M. Jowett, *LEP Note 606* (May 1988).
- [7] L.H. Thomas, *Phil. Mag.* 3 (1927).
- [8] V. Bargmann, L. Michel, V.L. Telegdi, *Phys. Rev. Lett.* 2 (1959).
- [9] A.W. Chao, *Nucl. Instr. Meth.* 180 (1981) 29.
- [10] S. Mane, *Proc. Conf. on H.E. Spin Physics Minneapolis* (1988).
- [11] K. Yokoya, P. Chen, *SLAC-PUB-4692* (Sept. 1988).
- [12] D.P. Barber et al., *DESY 82-076* (1982).
- [13] R. Rossmanith, R. Schmidt, *DESY 84-105* (1984).
- [14] J.P. Koutchouk, T. Limberg, in Ref.[2], p.204.
- [15] Ya. Derbenev, A.M. Kondratenko and A.N. Skrinsky, *Sov. Phys. Doklady* 15 (1970).
- [16] T. Fieguth, *SLAC/AP-52* (April 1986).
- [17] R. Schwitters, B. Richter, *SLAC Technical Note PEP-87 and SPEAR Note 175* (1974).
- [18] A. Blondel, E. Keil in Ref.[4], Vol.2, p.250.
- [19] J. Buon, *LAL-RT 88-02* (Feb. 1988).
- [20] V.N. Baier, V.A. Khoze, *Sov. J. Nucl. Phys.* 9 (1969) 238.
- [21] D.B. Gustavson et al., *Nucl. Instr. Meth.* 165 (1979) 177.
- [22] F.W. Lipps, H.A. Tolhoek, *Physica XX* (1954) p.85 and p. 395.
- [23] G. Barbagli, INFN Florence, Italy, private communication.
- [24] M. Placidi, R. Rossmanith, *Nucl. Instr. Meth.* A274 (1989) 79-94.
- [25] E. Locci, *LEP Note 617* (Dec. 1988).
- [26] Y. Derbenev et al., *Particle Accelerators* 18 (1980).
- [27] R.S. Van Dyck Jr. et al., *Phys. Rev. Letters* 59 (1987) 26.
- [28] D.B. Barber et al., *Physics Letters* 135B (1984) 498.