EXPERIMENTAL TESTS OF PERTURBATIVE QCD

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1. - INTRODUCTION

Quantum Chromodynamics (QCD) (1), the $SU(3)$ gauge theory of coloured quarks and gluons is the present theory of strong interactions. Today it stands as a main building block of the standard model of the known interactions of fundamental particles (except gravity) based on the gauge group $SU(3) \otimes SU(2) \otimes U(1)$. The most striking physical properties of QCD are asymptotic freedom and confinement. The first property (2) is rigorously established and consists in the fact that the effective coupling decreases logarithmically at short distances. This is the basis for perturbative QCD which is relevant for processes involving large momentum transfers. Colour confinement is the property that the potential energy between coloured charges increases approximately linearly at large distances, so that only colour singlet states can be produced and observed. Originally a mere conjecture (3), with time the basis for colour confinement has become more solid partly from experiments on quarkonium spectroscopy (4) (which are consistent with a potential that increases at long distances) and especially from lattice simulations (5) [for example, the calculation (6) of the potential between static colour charges at large distances and the study (7) of the deconfinement phase transition]. Although not yet proved it is by now at least quite plausible that confinement is really implied by the theory.

Precise experimental tests of QCD are clearly as important as tests of the electroweak sector of the standard theory. Yet testing QCD is even more difficult than testing the electroweak theory. In fact the latter interactions are so weak that perturbation theory is always reliable at present energies. Moreover, leptons as well as photons, W 's and Z 's are at the same time the fields in the Lagrangian and the particles observed in our detectors. On the contrary, in QCD the perturbative approach,

which is essentially the only viable method for extracting from the theory testable quantitative predictions, is only valid for processes with large momentum transfers. Even in the most favourable cases the expansion is only slowly converging because of the relatively large values of the QCD coupling $\alpha_s(Q) \gg \alpha_{QED}$. Moreover QCD is a theory of confined quarks and gluons (the "partons") while only hadrons are actually observed. Except for the rather limited set of completely inclusive experiments, in general non-perturbative effects connected with soft parton cascades and hadronization tend to obscure the underlying simplicity of the parton dynamics. On one hand, in spite of its limitations perturbative QCD provides a rich testing ground for the theory as will be concisely reviewed in this article. On the other hand, among non-perturbative methods, QCD on the lattice (5) is extremely promising for understanding crucial properties such as confinement, broken chiral symmetry, hadronic masses and matrix elements. However, computer limitations impose very drastic restrictions on the lattice size, on the accuracy of the extrapolation to the continuum limit and on the possibility of implementing the effects of quark loops on the various observables. As a consequence up to now lattice calculations have been more useful for adding important elements to our understanding of the theoretical foundations of QCD rather than for offering direct possibilities of comparison with experiment.

The difficulty of testing QCD is reflected in the fact that no single process or experiment by itself provides a clear-cut and precisely quantitative experimental proof of the theory, at least when practical limitations of feasible experiments are taken into account. There are no analogues in QCD of the $g-2$ experiment for the muon or the electron, of the Lamb shift and so on in QED. In view of the uncertainties connected with any given experiment, our confidence in QCD rests on the overall set of convergent positive indications which arise from an ever increasing

number of different experiments with all possible beams and targets in a wide range of large energies and momentum transfers. As a result the integrated experimental evidence in favour of QCD is quite impressive by now.

In particular, QCD provides a solid field theoretical basis for the parton approach. The beautiful "naive" parton model of Bjorken, Feynman and others (8) has evolved into the "QCD improved" parton model (9). This powerful approach has become such a familiar and widespread tool for everyday practice in high energy physics that one is led to take all its new successes as granted and, in a way, obvious. Actually the very definite and characteristic pattern of successful predictions given by the parton model already provides quite a solid ground of experimental evidence for QCD. In addition to inheriting the beautiful experimental score of the parton approach, QCD is also tested by an increasing list of phenomena which go beyond the naive parton model and therefore provide an additional and qualitatively different experimental basis for the specific dynamical structure of the theory.

The present article is devoted to a review of the comparison of QCD with experimental data. The theory of QCD has been summarized in recent textbooks (10) and review articles (11,12). Here only that minimum of formalism is reported which is convenient for a clear and self-contained exposition of the experimental tests of QCD.

The QCD theory is specified by the Yang-Mills (13) Lagrangian corresponding to the unbroken colour gauge group SU(3) with quark matter fields in the fundamental three-dimensional representation of SU(3):

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} \sum_{A=1}^8 F_{\mu\nu}^A F^{\mu\nu A} + \sum_{j=1}^f \bar{q}_j (i \not{D} - m_j) q_j \quad (1.1)$$

with

$$\begin{aligned} F_{\mu\nu}^A &= \partial_\mu g_\nu^A - \partial_\nu g_\mu^A - g_s c_{ABC} g_\mu^B g_\nu^C \\ D_\mu &= \partial_\mu + i g_s \sum_{A=1}^8 t^A g_\mu^A \end{aligned} \quad (1.2)$$

where g_μ^A ($A=1, \dots, 8$) are the gluon fields, t^A are the generators in the quark representation, normalized according to $\text{Tr}(t^A t^B) = \frac{1}{2} \delta^{AB}$. In turn the normalization of t^A fixes the coupling g_s and the structure constants c_{ABC} given by $[t^A, t^B] = i c_{ABC} t^C$. Gauge fixing and ghost terms are to be added for a correct quantization of the theory (14), but are omitted here. Also omitted is the P, T and CP violating θ term (15)

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \sum_{A=1}^8 F^{\mu\nu A} F^{\rho\sigma A} \quad (1.3)$$

of instantonic origin (16) whose empirical smallness ($\theta < 10^{-8}$) (17) is not understood and represents an important conceptual problem for the standard model (18).

The selection of SU(3) as colour gauge group is unique in view of (a) the fact that the group must admit complex representations because it must be able to distinguish a quark from an antiquark (there are meson states made up of $q\bar{q}$ but not similar qq bound states). (b) There must be colour singlet (because we see no colour replicas of known hadrons), completely antisymmetric baryonic states made up of qqq in order to solve the statistics puzzle for the lowest-lying baryons of spin 1/2 and 3/2 in the 56 of (flavour) SU(6) (19). (c) As we shall see, the number of colours for each kind of quarks must be in agreement with the data on the total hadronic e^+e^- cross-section and on the $\pi^0 \rightarrow 2\gamma$ rate and also, to some extent, on other processes like lepton pair production and the semi-leptonic branching ratio of the τ lepton. Within simple groups (a)

restricts the choice to $SU(N)$ with $N > 3$, $SO(4N+2)$ with $N > 1$ [SO(6) has the same algebra as $SU(4)$], and E_6 , and then (b) and (c) directly lead unambiguously to $SU(3)$ with each flavour of quarks in a fundamental representation of the group. Too many coloured quarks not only would violate (c) but could also spoil asymptotic freedom (2). In particular the $\pi^0 \rightarrow 2\gamma$ rate is fixed by the Adler-Bell-Jackiw anomaly (20) to the value (21):

$$\begin{aligned} \Gamma(\pi^0 \rightarrow 2\gamma)_{TH} &= N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_{\pi^0}^3}{32 \pi^3 f_\pi^2} = \\ &= (7.61 \pm 0.02) \left(\frac{N_c}{3}\right)^2 eV \end{aligned} \quad (1.4)$$

obtained for $f_\pi = 131.69 \pm 0.15$ MeV (22). N_c is the number of quark colour replicas and $Q_{u,d}$ are the u,d quark charges. The theoretical error is expected to be of order m_π^2/M^2 (due to the soft-pion extrapolation from $q^2 = m_\pi^2$ down to $q^2=0$) where M is the scale of variation of the relevant form factor ($M \sim 1$ GeV). The experimental value is given by (22):

$$\Gamma(\pi^0 \rightarrow 2\gamma)_{exp} = (7.3 \pm 0.2) eV \quad (1.5)$$

This beautiful result is a great achievement for the theory and a remarkable proof that $N_c = 3$.

$\alpha_s = (g_s^2/4\pi)$, the analogue in QCD of the QED fine structure constant, is widely used. The precise definition of the renormalized coupling must be specified. Technically the most practical definition is in the context of dimensional regularization (23,24) through the procedures of minimal subtraction (MS) (25) or of modified minimal subtraction ($\overline{MS}$) (26). Unless explicitly stated, we shall always refer to the $\overline{MS}$ definition of α_s in the following. Note that in the perturbative region the light quark masses can be neglected in most cases. Thus the definition of α_s refers to the massless theory. The renormalization group

formalism (27) leads to the concept of running coupling. The running coupling $\alpha_s(Q)$ is a function of the energy scale Q . Loosely speaking $\alpha_s(Q)$ acts as an effective coupling for QCD interactions at distances of order $1/Q$. The running coupling is defined by the relation

$$t = \int_{\alpha_s(\mu)}^{\alpha_s(Q)} \frac{d\alpha}{\beta(\alpha)} \quad (1.6)$$

where $t = \ln Q^2/\mu^2$ and the initial value $\alpha_s \equiv \alpha_s(\mu)$ has been specified.

The β function, obeying the relation

$$\frac{d}{dt} \alpha_s(Q) = \beta[\alpha_s(Q)] \quad (1.7)$$

can be computed in perturbation theory (2,28,29)

$$\begin{aligned} \beta(\alpha) &= -b_f \alpha^2 (1 + b'_f \alpha + b''_f \alpha^2 + \dots) = \\ &= -\left(11 - \frac{2}{3}f\right) \frac{\alpha^2}{4\pi} \left[1 + \frac{153 - 19f}{2(33 - 2f)} \frac{\alpha}{\pi} + \right. \\ &\quad \left. + \frac{3}{32(33 - 2f)} \left(2857 - \frac{5033}{9}f + \frac{325}{27}f^2\right) \left(\frac{\alpha}{\pi}\right)^2 + \dots\right] \end{aligned} \quad (1.8)$$

The values of b_f (2) and b'_f (28) do not depend on the renormalization prescription, while b''_f (29) and the following terms change with the definition of α_s [however, b''_f as given in Eq. (1.8) is the same (30) in the $\overline{MS}$ and $\overline{MS}$ prescriptions]. Numerically, one obtains for $f = 5$:

$$\beta(\alpha) = -0.610 \alpha^2 \left[1 + 1.261 \frac{\alpha}{\pi} + 1.475 \left(\frac{\alpha}{\pi}\right)^2 + \dots\right] \quad (1.9)$$

When the first two terms in the expansion of $\beta(\alpha)$ are included in Eq. (1.6) defining $\alpha_s(Q)$, the integral can be performed and one obtains:

$$\frac{1}{\alpha_s(Q)} = \frac{1}{\alpha_s} + b_f t - b'_f \ln \frac{\alpha_s(Q)}{\alpha_s} \quad (1.10)$$

To the required accuracy this equation can be solved in the form:

$$\alpha_s(Q) = \alpha_{0s}(Q) [1 - b'_f \alpha_{0s}(Q) \ln Q^2/\Lambda^2 + o(\alpha_{0s}^2(Q))] \quad (1.11)$$

where

$$\alpha_{0s}(Q) = (b_f \ln Q^2/\Lambda^2)^{-1} \quad (1.12)$$

and the parameter μ appearing in $\alpha_s \equiv \alpha_s(\mu)$ has been traded for the parameter Λ :

$$\ln \frac{\mu^2}{\Lambda^2} = \frac{1}{b_f \alpha_s} + \frac{b'_f}{b_f} \ln b_f \alpha_s \quad (1.13)$$

As b_f and b'_f do not depend on the renormalization prescription the functional form of the running coupling is universal up to terms of order $\alpha_s^2(Q)$ included. Different definitions of the coupling imply different values of $\alpha_s(\mu)$, hence of Λ . Thus one talks of $\Lambda_{\overline{MS}}$, $\Lambda_{\overline{MS}}$, etc., and for us $\Lambda \equiv \Lambda_{\overline{MS}}$. Clearly the difference only appears at next-to-leading order, because the variation of α_s appears at order α_s^2 . Thus only sufficiently precise measurements analyzed in terms of theoretical formulae developed up to non-leading accuracy can really lead to a determination of $\Lambda_{\overline{MS}}$.

Note that b_f , b'_f , ... depend on the number of excited quark flavours. In fact when working at large but fixed Q , with Q much larger than the masses of some light quarks and much smaller than the masses of some heavy quarks, then according to an intuitive decoupling theorem (31), the relevant number of flavours f is that of light quarks. In fact in QCD the theory obtained by dropping out heavy quarks is still renormalizable and the couplings do not grow with masses (unlike in spontaneously broken gauge theories where the couplings of Higgses and of longitudinal gauge boson modes increase with masses). Then the behaviour of amplitudes with light external particles is dictated in this range of Q by the set of diagrams where all internal lines are also light. For example, for $Q <$

$a m_b$ with $a \sim 1-2$ the large Q behaviour of $\alpha_s(Q)$ is determined by b_4 and b'_4 , while b_5 and b'_5 are relevant for $\alpha_s(Q)$ at $Q \gg a m_b$. Of course, the physical coupling is continuous so that some matching prescription must be specified. Experiments on e^+e^- annihilation at PETRA or TRISTAN work at $Q \sim 30-50$ GeV well above the b threshold. Their results are naturally compared with the asymptotic formula for $f = 5$. One can extrapolate their determination of $\alpha_s(Q)$ below the b threshold by setting, for example, (we only consider Q values above c threshold for simplicity)

$$\alpha_s(Q) = \alpha_{s,5}(Q) \cdot \theta(Q - a m_b) + \alpha_{s,4}(Q) \cdot \theta(a m_b - Q) \quad (1.14)$$

with

$$\alpha_{s,5}(Q) \approx \alpha_s(Q, 5) \quad (1.15)$$

$$\frac{1}{\alpha_{s,4}(Q)} = \frac{1}{\alpha_s(Q, 4)} + \frac{1}{\alpha_s(a m_b, 5)} - \frac{1}{\alpha_s(a m_b, 4)} \quad (1.16)$$

where

$$\alpha_s(Q, f) = \frac{1}{b_f \ln Q^2/\Lambda_f^2} \left[1 - \frac{b'_f \ln Q^2/\Lambda_f^2}{b_f \ln Q^2/\Lambda_f^2} \right] \quad (1.17)$$

Note that the constant addition in Eq. (1.16) is subleading with respect to the logarithmically growing terms in $1/\alpha_s(Q, 4)$ so that the correct asymptotic behaviour is not spoiled. This constant is fixed so that

$$\alpha'_{s,4}(a m_b) = \alpha_{s,5}(a m_b) \quad (1.18)$$

On the other hand, the results of experiments on deep inelastic scattering or on T decays are naturally expressed in terms of the asymptotic form for $\alpha_{s,4}(Q)$:

$$\alpha'_{s_4}(Q) = \frac{1}{b_4 \ln Q^2/\Lambda_4^2} \left[1 - \frac{b'_4 \ln Q/\Lambda_4^2}{b_4 \ln Q^2/\Lambda_4^2} \right] \quad (1.19)$$

In this case it is $\alpha_{s,5}(Q)$ which is defined with a constant term in complete analogy with Eq. (1.16). In the range of Q of practical interest the values of Λ_4 and Λ_5 are quite different being approximately related by:

$$\Lambda_5 \approx 0.65 \Lambda_4 \quad (1.20)$$

One sees that for practically the same $\alpha_s(Q)$ the corresponding values of Λ_4 and Λ_5 are quite different. On the other hand, it turns out that there is little numerical difference when the matching parameter a is moved in the interval $1 < a < 2$, or when other similar matching procedures are applied.

In conclusion, when a measurement of $\alpha_s(Q)$ is translated into a value of Λ or vice versa, it is essential that the exact functional form of $\alpha_s(Q)$ in terms of Λ is specified (whether one-loop or two-loop accuracy was assumed, which of the possible forms differing at $O(\alpha_s^3)$ has been adopted, the relevant number of excited flavours, the procedure for making α_s continuous at thresholds).

2. - EXPERIMENTAL DETERMINATIONS OF $\alpha_s(Q)$

The most direct set of quantitative tests of QCD consists in the comparison of several measurements of α_s in different processes. As already stated, for a meaningful determination of $\alpha_s(Q)$ at a well-defined scale Q , or equivalently for a measurement of Λ , in a given process, one needs a perturbative calculation of the corresponding observable at least at next-to-leading accuracy in α_s . In fact a change of scale Q or a

change of definition of α_s only affects the result at the level of sub-leading terms. In the following we shall review and discuss the most significant determinations of α_s from the available data [for a previous review, see for example Ref. (32)].

2.1 Total Hadronic Cross-Section in e^+e^- Annihilation

Neglecting quark masses $\sigma \equiv \sigma(e^+e^- \rightarrow \text{hadrons})$ is given by (9):

$$\sigma = \frac{4\pi\alpha^2}{3Q^2} (1+Z) R_{e^+e^-} \quad (2.1)$$

where $Q = \sqrt{s}$, Z is the effect of the weak neutral gauge boson, which can be explicitly computed in terms of $\sin^2\theta_W$ and M_Z , and $R_{e^+e^-}$ is given by (33,34,35):

$$\begin{aligned} R_{e^+e^-} = & 3 \sum_f Q_f^2 \left\{ 1 + \frac{\alpha_s(Q)}{\pi} + \left(1.986 - 0.115 f \right) \left[\frac{\alpha_s(Q)}{\pi} \right]^2 + \right. \\ & \left. + \left(70.985 - 1.200 f - 0.005 f^2 \right) \left[\frac{\alpha_s(Q)}{\pi} \right]^3 - \left(\sum_f Q_f \right)^2 \frac{1.619 \left[\frac{\alpha_s(Q)}{\pi} \right]^3}{\pi} \right\} = \\ = & \frac{11}{3} \left\{ 1 + \frac{\alpha_s(Q)}{\pi} + 1.411 \left[\frac{\alpha_s(Q)}{\pi} \right]^2 + 64.81 \left[\frac{\alpha_s(Q)}{\pi} \right]^3 \right\} \end{aligned} \quad (2.2)$$

where in the last line the small term proportional to $(\sum_f Q_f)^2$ has been reabsorbed for $f = 5$ into the main correction of order α_s^2 . Note that $R_{e^+e^-}$ is (essentially) proportional to N_c , so that the value of this quantity is an important direct test of the existence of three colour replicas of quarks. The correction of order α_s was computed long ago (33) and is clearly independent of the renormalization scheme, that of order α_s^2 was computed in the $\overline{MS}$ scheme by three independent collaborations (34). Finally the calculation of $R_{e^+e^-}$ at three loops in the $\overline{MS}$ scheme

was recently completed in Ref. (35). The precise expression of R given in Eq. (2.2) only applies to the photon exchange term. Differences due to the axial Z couplings are neglected in Eqs. (2.1) and (2.2).

Two different groups have analyzed all available data in the range 7 GeV < Q < 56 GeV in order to extract α_s . De Boer et al. (36) fix $\sin^2\theta_W = 0.23$ (the world average value) and obtain

$$\alpha_s(Q=34 \text{ GeV}) = 0.140 \pm 0.016 \quad (2.3)$$

Marshall (37) presents a combined fit of α_s and $\sin^2\theta_W$. Together with the fitted result $\sin^2\theta_W = 0.242 \pm 0.017$ the corresponding value of α_s is given by:

$$\alpha_s(Q=34 \text{ GeV}) = 0.135 \pm 0.016 \quad (2.4)$$

Note that the effect of the order α_s^3 correction to $R_{e^+e^-}$ is large enough to change the value of α_s by $\sim 10\%$ (the central value decreases from 0.155 down to 0.140). Although this variation of α_s is within the overall error still the size of the coefficient of $(\alpha_s/\pi)^3$ in the expansion of $R_{e^+e^-}$ is somewhat disturbing. In fact with the value of α_s given by Eq. (2.3) the expansion of $R_{e^+e^-}$ reads:

$$R_{e^+e^-} = \frac{11}{3} \left[1 + r_1 \frac{\alpha_s}{\pi} + r_2 \left(\frac{\alpha_s}{\pi} \right)^2 + r_3 \left(\frac{\alpha_s}{\pi} \right)^3 + \dots \right] \approx \frac{11}{3} [1 + 0.0446 + 0.0026 + 0.0057 + \dots] \quad (2.5)$$

While it is true that, strictly speaking, beyond one loop the coefficients of the expansion depend both on the renormalization scheme and the choice of scale Q , however, the large ratio of r_3 with respect to r_1 , r_2 makes it hopeless to improve the situation substantially by a reasonable reparametrization. Note that as far as one can tell from the

three-loop result of Eq. (1.9), the β -function expansion in the $\overline{MS}$ prescriptions is well behaved. The computation of the $O(\alpha_s^3)$ corrections to $R_{e^+e^-}$ dramatically shows the limitations of all the so-called optimization procedures (38). For whatever choice made at the two-loop level among these procedures, the result at three loops is always about as bad as in Eq. (2.5). This clearly shows that the optimization choices cannot pretend to reduce the theoretical error.

In conclusion the determination of α_s through $R_{e^+e^-}$ is in principle very clear. Unfortunately this method has a limited sensitivity because the QCD correction is quite small. In spite of that the result obtained by combining all experiments together is amazingly precise. Rounding off the errors we can summarize the results as follows

$$\alpha_s(Q=34 \text{ GeV}) = 0.14 \pm 0.02 \quad (2.6)$$

$$\Lambda_{\overline{MS}}^{(5)} \approx 0.65 \Lambda_{\overline{MS}}^{(4)} \approx (240 \pm 230) \text{ MeV} \quad (2.7)$$

2.2 Scaling Violations in Deep Inelastic Leptonproduction

In principle this is the most solid and powerful method for testing perturbative QCD and measuring α_s . As for the total hadronic cross-section in e^+e^- annihilation also in this case the underlying theory is very well founded (9) [for example in terms of the light-cone operator expansion (39)] and, the process being completely inclusive, there are no problems associated with the experimental definition of jets and its relation with theoretical partonic cross-sections. But with respect to the case of $\sigma(e^+e^- \rightarrow \text{hadrons})$ there are essential advantages. First, there are many independent structure functions (40) and all of them can be measured at different values of the Bjorken variable x for each given

$Q^2 = -q^2$ (where q_μ is the virtual γ or W^+ or Z four momentum). Thus in this case one actually deals with a system of tests (typically, after a suitable binning, one can compare with theory a number of $\ln Q^2$ slopes at different values of x). Second, for the structure functions the scaling violations are quantitatively more important than the small $O(\alpha_s)$ corrections to $\sigma(e^+e^- \rightarrow \text{hadrons})$ because they arise from the resummation of a series of logarithmically enhanced terms.

Over the last years an imposing experimental effort has been devoted to the measurement of scaling violations in deep inelastic scattering with electron or muon, neutrino or antineutrino beams (41) on hydrogen, deuterium and heavy nuclei. Recently a new generation of high precision experiments has been completed [EMC (42), BCDMS (43), CDHSW (44)]. Many important predictions of the theory have been confirmed. The existence of scaling violations is definitely established at Q^2 values large enough to support the prediction that their asymptotic decrease is only logarithmic. The observed pattern and magnitude of the scale breaking effects are in good agreement with the theoretical expectations. The values of Λ extracted from the data are consistent with other experimental derivations. However, at a closer look several serious problems remain in the comparison of the data among themselves and with the theory, so that unfortunately after so much theoretical and experimental work the situation is still not yet clear and satisfactory. There are experimental discrepancies that are certainly beyond the declared systematic and statistical errors among different measurements of the same observable. The most disturbing of these cases is perhaps the difference (45) between the BCDMS and the EMC measurements of the structure function $F_2(x, Q^2)$ with muon beams on hydrogen (see Fig. 1) (a priori the most accurate and comprehensive sets of hydrogen data). This discrepancy not only reflects

itself in a sizeable difference on the values of the proton parton densities useful for physics at pp, $\bar{p}p$ and ep colliders (which are preferentially extracted from hydrogen data to avoid possible nuclear effects) but also leads to serious doubts on the whole analysis of scaling violations. In fact it turns out a posteriori that the amount of uncontrolled systematics present in at least one of the two experiments is large enough to considerably affect the measurement of α_s and Λ (45).

A different problem has to do with experiments on iron target. In this case (45) there is reasonable consistency among different experiments but the data show deviations from the expected behaviour or at least complete agreement with the theory is only obtained at particularly large values of Q and W (the invariant mass of the produced hadronic system) where the statistical accuracy is however small.

We shall now concisely summarize the status of QCD tests based on scaling violations in leptonproduction. The Q^2 dependence of structure functions is dictated by the QCD evolution equations (46). In the non-singlet case these equations are simplest:

$$Q^2 \frac{\partial}{\partial Q^2} F_a^{NS}(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{F_a^{NS}(x, Q^2)} \int_0^1 \frac{dy}{y} F_a^{NS}(y, Q^2) \int_0^1 \frac{dz}{z} \gamma_{qq}^{NS(a)} \quad (2.8)$$

where in leading approximation the kernel $\gamma_{qq}^{NS(a)}$ coincides with the simple distribution (46)

$$P_{qq}(z) = \frac{4}{3} \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right] \quad (2.9)$$

Note that the lowest-order kernel P_{qq} is independent of the index a , i.e., it is the same for all non-singlet structure functions. The next-to-leading corrections to $\gamma_{qq}^{NS(a)}$ have been computed in Ref. (47).

$$Q^2 \frac{\partial}{\partial Q^2} g(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[\mathcal{F}_2^S(y, Q^2) \mathcal{G}_{qq}^{(2)}\left(\frac{x}{y}, \alpha_s(Q^2)\right) + \right. \\ \left. + g(y, Q^2) \mathcal{G}_{gg}^{(2)}\left(\frac{x}{y}, \alpha_s(Q^2)\right) \right] \quad (2.13)$$

The next-to-leading corrections to the splitting functions $\mathcal{G}$ for the singlet case have been computed in Refs. (49). While the lowest-order kernels are totally unambiguous the corresponding corrections of order α_s depend on the exact definition of quark and gluon densities beyond the leading order. For example, Eq. (2.10) provides a possible definition of quark densities to all orders (48).

An important feature of the QCD evolution equations, evident from Eqs. (2.8, 2.12, 2.13) is that the Q^2 derivative at x of a given structure function only depends on the quark and gluon densities at $y > x$. This allows us to predict the Q^2 evolution from the values of x actually measured. In fact, at fixed Q^2 , in practice it is not possible to reach too small values of x . Furthermore as is empirically true and theoretically reasonable that glue and sea densities are negligible with respect to valence quark densities at sufficiently large x , the gluon term in the singlet equation can correspondingly be omitted. Since the singlet kernel $\mathcal{G}_{qq}^S$ also approaches $\mathcal{G}_{qq}^{NS}$ at large x [$|\mathcal{G}_{qq}^S - \mathcal{G}_{qq}^{NS}| \sim (1-x)^5$ near $x \rightarrow 1$] (9) one can approximately reobtain the much simpler non-singlet equation at $x > x_0$, with a suitable value of x_0 . In practice, as we shall see, $x_0 = 0.25-0.30$ is normally adopted.

In the evolution equations there are two variables x and Q^2 . The shape in x of the structure functions at fixed $Q^2 = Q_0^2$ is not a prediction of perturbative QCD. But given the x dependence of the structure function at $Q^2 = Q_0^2$, then the evolution equations predict it at all Q^2

The non-singlet evolution equations are exactly valid for the difference of any given structure function measured on proton and neutron targets, i.e., $F^{NS} = F^p - F^n$, or for the structure function F_3 which is given by the difference of neutrino and antineutrino scattering on a given target. Thus the analysis based on F_3 (or $F^p - F^n$) is particularly clear in principle but it suffers from the relatively large errors arising from taking a difference of cross-sections (in any case F_3 is not accessible to experiments with muon beams).

For a general structure function one can separate a non-singlet and a singlet part. For example, for $\mathcal{F}_2 = F_2/x$ one starts from:

$$\mathcal{F}_2(x, Q^2) = \sum_{i=1}^{2f} c_i q_i(x, Q^2) \quad (2.10)$$

where the sum runs over all active flavours of quarks and antiquarks and c_i are the relevant electroweak charges. Equation (2.10) is certainly valid to lowest order and can be taken as a redefinition of quark densities beyond leading order (48). Then

$$q_i = \mathcal{F}_2^{NS} + \mathcal{F}_2^S = \sum_{i=1}^{2f} (c_i - \langle c \rangle) q_i + \langle c \rangle \sum_{i=1}^{2f} q_i \quad (2.11)$$

where $\langle c \rangle = 1/2f (\sum_{i=1}^{2f} c_i)$ and $\Sigma = \sum_{i=1}^{2f} q_i$. A gluon term is also present in the singlet evolution equation (46):

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{F}_2^S(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[\mathcal{F}_2^S(y, Q^2) \mathcal{G}_{qq}^{S(2)}\left(\frac{x}{y}, \alpha_s(Q^2)\right) + \right. \\ \left. + 2f \langle c \rangle g(y, Q^2) \mathcal{G}_{qg}^{(2)}\left(\frac{x}{y}, \alpha_s(Q^2)\right) \right] \quad (2.12)$$

The system is then closed by an analogous equation for the gluon density:

(in the singlet case the shape of the gluon density is also needed at $Q^2 = Q_0^2$). Although the x and Q^2 dependence are coupled by the evolution equations it is clear that for QCD tests what matters most is the Q^2 variation at fixed x rather than the x variation at fixed Q^2 (50). When the limited range in Q^2 and the experimental errors are taken into account one realizes that the QCD test in the non-singlet case essentially consists in checking that a single value of Λ can accommodate the measured logarithmic slopes $d \ln F / d \ln Q^2$ at a number of fixed values of x . It is in fact beyond the present possibilities to measure (within a single experiment) significant deviations from a straight line behaviour in $\ln Q^2$. In the singlet case the gluon density, in addition to Λ , is also to be determined from the logarithmic slopes. In fact the gluons are not directly coupled and their distribution is also to be inferred from the scaling violations (or from processes other than lepton production, e.g., large p_T photons in $p\bar{p}$ collisions).

After this concise theoretical summary we now review the data and the corresponding QCD analysis at next-to-leading accuracy.

The BCDMS collaboration (43) has measured F_2 with muon beams on carbon and hydrogen. This experiment has the largest statistics at large Q^2 . For the carbon data $Q^2 > 25 \text{ GeV}^2$ in the range $0.275 < x < 0.75$. The extrapolation to $x > 0.75$ does not introduce an important error because the structure functions are very small in this range. The data are analyzed in the non-singlet approximation. The results for the logarithmic slopes are shown in Fig. 2. The corresponding determination of $\Lambda_{\overline{MS}}^{(4)}$ for four flavours leads to the result

$$\Lambda_{\overline{MS}}^{(4)} = (230 \pm 20 \pm 60) \text{ MeV} \quad (2.14)$$

which corresponds to:

$$\alpha_s(Q=10 \text{ GeV}) = 0.160 \pm 0.003 \pm 0.010 \quad (2.15)$$

The BCDMS collaboration (43) has also analyzed the data on hydrogen in the non-singlet approximation for $x > 0.275$ with $Q^2 > 20 \text{ GeV}^2$. The logarithmic slopes on hydrogen are shown in Fig. 3 together with the QCD fit. In this case one finds:

$$\Lambda_{\overline{MS}}^{(4)} = (205 \pm 22 \pm 60) \text{ MeV} \quad (2.16)$$

or

$$\alpha_s(Q=10 \text{ GeV}) = 0.156 \pm 0.004 \pm 0.011 \quad (2.17)$$

The hydrogen fit is in perfect agreement with the results on carbon. They can be combined to give

$$\Lambda_{\overline{MS}}^{(4)} \simeq 1.54 \Lambda_{\overline{MS}}^{(5)} = (220 \pm 15 \pm 50) \text{ MeV} \quad (2.18)$$

The hydrogen data on F_2 by BCDMS are also available in the range $0.07 < x < 0.275$ (with $Q^2 > 8 \text{ GeV}^2$ for $x < 0.16$, $Q^2 > 14 \text{ GeV}^2$ for $0.16 < x < 0.25$) so that a singlet fit can be performed at small x (Fig. 4). In this region of x the logarithmic slopes require a sizeable gluon density. The resulting gluon density is shown in Fig. 13. The gluon distribution is indeed concentrated at small values of x as demanded by consistency as the gluon term in the evolution equations was neglected for $x > 0.275$ in the non-singlet analysis.

In conclusion the BCDMS analysis presents a remarkable consistency among carbon data, hydrogen data at large x and hydrogen data at small x and a beautiful agreement with perturbative QCD predictions.

Unfortunately this idyllic picture is somewhat spoiled by the results from other experiments of a priori comparable precision. As already mentioned, there is a severe disagreement (45) (about three times

larger than that allowed by the quoted systematic errors) between the BCDMS and the EMC data on F_2 for proton targets, shown in Fig. 1. Previous SLAC data (51) on F_2^p (up to $Q^2 \sim 20 \text{ GeV}^2$) cannot resolve this discrepancy because there is essentially no overlap in x and Q^2 . It is true that the discrepancy is mainly on the normalization of F_2 at different x and not on the logarithmic slopes (Fig. 5). The EMC data are indeed also consistent within errors with QCD. The non-singlet fit to F_2^p by EMC at $x > 0.35$ and $Q^2 > 8 \text{ GeV}^2$ (with $Q^2 > 22.5 \text{ GeV}^2$) leads to:

$$\Lambda_{\overline{MS}}^{(4)} = (105 \pm 55 \pm 45) \text{ MeV} \quad (2.19)$$

This value of $\Lambda_{\overline{MS}}^{(4)}$ is consistent with the corresponding BCDMS result in Eq. (2.16) although the EMC central value is smaller by a factor of two. The real problem is that the discrepancy indicates an uncontrolled systematic of large enough size (45) to make the agreement with QCD and the consistency of the fitted values of $\Lambda_{\overline{MS}}^{(4)}$ to some extent accidental.

The BCDMS results for $\Lambda_{\overline{MS}}^{(4)}$ obtained from the data on carbon are compatible with the CHARM collaboration (52) results obtained from neutrino scattering on marble (CaCO_3), a target not too heavier than carbon ($A \sim 20$ vs. $A \sim 12$). The CHARM result, obtained from the non-singlet structure function F_3 at next-to-leading accuracy for $Q^2 = 3\text{--}78 \text{ GeV}^2$, is given by

$$\Lambda_{\overline{MS}}^{(4)} = (310 \pm 140 \pm 70) \text{ MeV} \quad (2.20)$$

There are many high statistics experiments on the iron structure functions ($A \sim 56$). F_2^{Fe} has been measured by CCFRR (53), CDHSW (44) with ν beams and by BFP (54) and EMC (42) with muon beams. The data are in reasonable agreement among them, within the stated uncertainties, although the EMC data are 5–10 % below the other data sets. Similarly the

xF_3 measurements from CCFRR and CDHSW are also consistent. For iron structure functions the problem is that the logarithmic slopes in general show a steeper x dependence than expected from QCD with values of $\Lambda_{\overline{MS}}^{(4)}$ compatible with Eqs. (2.18, 2.19). For example, a comparison of the EMC data on F_2^{Fe} with the non-singlet QCD fit obtained at next-to-leading accuracy using the value of $\Lambda_{\overline{MS}}^{(4)}$ measured by BCDMS on carbon is shown (45) in Fig. 6 (together with possible modifications induced by a model of higher twist effects and target mass corrections). A quite similar behaviour is also observed in the logarithmic slopes of xF_3 recently measured by CDHSW (44). In this case agreement with QCD is reobtained for $Q^2 > 20 \text{ GeV}$, within a more limited accuracy determined by the smaller statistics. From the iron data one derives the indication that pre-asymptotic or non-perturbative effects could be relatively more important in the case of heavy targets.

In conclusion, the BCDMS data on H and C have the highest statistics at the largest values of Q^2 . These data are in beautiful agreement with QCD, and lead to the value of $\Lambda_{\overline{MS}}^{(4)}$ quoted in Eq. (2.18). This value of $\Lambda_{\overline{MS}}^{(4)}$ is in agreement with the values of $\Lambda_{\overline{MS}}^{(4)}$ quoted by EMC on H and CHARM on CaCO_3 . However, the agreement with EMC is somewhat illusory in view of the large discrepancies between BCDMS and EMC on F_2^p . Finally, the data on Fe are unfortunately not very conclusive as tests of QCD and measurements of α_s .

2.3 Quarkonium Decays

The rates of quarkonium (especially the T) provide a nominally rather precise experimental determination of α_s and $\Lambda_{\overline{MS}}^{(4)}$. The problem in this case is the theoretical error. In the non-relativistic approximation the decay rates are proportional to the absolute square of the wave

function at the origin. Thus, within the limits of this approximation, ratios of decay rates are independent of the wave function which is unknown. The most convenient ratios for the determination of α_s are $\Gamma_{\gamma\gamma g g g} / \Gamma_{\mu\mu g g g}$ and $\Gamma_{\gamma\gamma g g g} / \Gamma_{\mu\mu g g g}$. Next-to-leading calculations of these ratios have been performed in the $\overline{MS}$ scheme (55). In the τ case the results are:

$$\frac{\Gamma_{\mu\mu}}{\Gamma_{ggg}} = \frac{9\pi}{10(\pi^2-9)} \frac{\alpha^2}{\alpha_s^2(\mu)} \left\{ 1 - \frac{\alpha_s(\mu)}{\pi} \left(3\pi b_4 \ln \frac{\mu^2}{m_Q^2} + 0.43 \right) + \dots \right\} \quad (2.21)$$

$$\frac{\Gamma_{ggg}}{\Gamma_{ggg}} = \frac{4}{5} \frac{\alpha}{\alpha_s(\mu)} \left\{ 1 - \frac{\alpha_s(\mu)}{\pi} \left(\pi b_4 \ln \frac{\mu^2}{m_Q^2} + 2.6 \right) + \dots \right\} \quad (2.22)$$

where $b_4 = (33-2f)/12\pi$ ($f=4$) and μ is an arbitrary scale of the order of the heavy quark mass $m_Q \sim (m_\tau/2)$.

The experimental value of $\Gamma_{\mu\mu}/\Gamma_{ggg}$ is known for $\tau(1S)$, $\tau(2S)$ and $\tau(3S)$. It is obtained by the relation $\Gamma_{ggg}/\Gamma_{\mu\mu} = [1 - B_{\gamma\gamma g g} - B_{\pi\pi} - B_{E1}] / B_{\mu\mu} - 3-R$ where $R \approx 3.48$ and $B_{\pi\pi}$, B_{E1} are absent in the case of $\tau(1S)$. Recent precise measurements (56) by CUSB and CLEO when combined lead to $B_{\mu\mu} = 2.57 \pm 0.09$, 1.38 ± 0.29 , 1.73 ± 0.16 for $\tau(1S)$, $\tau(2S)$ and $\tau(3S)$ respectively. These values lead to $\Gamma_{ggg}/\Gamma_{\mu\mu} = 31.3 \pm 1.3$, 32.5 ± 8.3 , 31.0 ± 2.7 respectively. For $\mu = m_b \sim 4.9$ GeV one obtains:

$$1S: \alpha_s(m_b) = 0.1743 \pm 0.0024, \quad \Lambda_{\overline{MS}}^{(4)} = (159 \pm 9) \text{ MeV} \quad (2.23)$$

$$2S: \alpha_s(m_b) = 0.175 \pm 0.015, \quad \Lambda_{\overline{MS}}^{(4)} = (162 \pm 54) \text{ MeV} \quad (2.24)$$

$$3S: \alpha_s(m_b) = 0.1742 \pm 0.0021, \quad \Lambda_{\overline{MS}}^{(4)} = (155 \pm 18) \text{ MeV} \quad (2.25)$$

Similarly the world average for $\Gamma_{\gamma\gamma g g g} / \Gamma_{ggg}$ measured by CUSB, CLEO, ARGUS and CRYSTAL BALL is $(2.78 \pm 0.15)\%$ (57). From this value and Eq. (2.22) one obtains

$$\alpha_s(m_b) = 0.181 \pm 0.009 \quad (2.26)$$

The agreement between the values of α_s derived from $\Gamma_{\mu\mu g g g} / \Gamma_{ggg}$ for $\tau(1S)$, $\tau(2S)$ and $\tau(3S)$ and from $\Gamma_{\gamma\gamma g g g} / \Gamma_{ggg}$ for $\tau(1S)$ is remarkable and is an experimental check of the wave function factorization. Actually consistent values of α_s are also obtained with more uncertainty from charmonium decays. An overall fit of the available data, also including a crude estimate of relativistic corrections was performed in Ref. (58) with the results $\alpha_s(m_c) = 0.278 \pm 0.014$, $\alpha_s(m_b) = 0.185 \pm 0.006$ which correspond to $\Lambda_{\overline{MS}}^{(4)} = (199 \pm 22) \text{ MeV}$.

In the stated results for α_s and $\Lambda_{\overline{MS}}$ the error shown does not clearly include the theoretical error. This is certainly the largest source of uncertainty. Corrections to the non-relativistic approximation can still be sizeable in spite of the experimental success of factorization. The order of magnitude of v^2/c^2 is in fact ~ 0.25 for charmonium and ~ 0.10 for the τ system. The effects of higher perturbative orders and of non-perturbative terms could be large because the energy scale is relatively small. In the case of $\Gamma_{\gamma\gamma g g g} / \Gamma_{ggg}$ I see a further problem in the fact that the observed photon spectrum is not well understood in perturbation theory. The lowest-order spectrum is definitely too hard to accommodate the data (57). In Ref. (59) the observed soft γ spectrum is very convincingly explained as due to an effective mass of gluon jets. While the parton gluon is massless the physical gluon jet has a non-vanishing invariant mass. By a Monte Carlo simulation, Field (59) has shown that an average mass $\langle M \rangle \sim 1.6$ GeV should be attributed to the gluon jet in order to reproduce the data. Perturbative effects (gluon splitting into $q\bar{q}$ or $g g$) should indeed induce an invariant mass of order $\alpha_s M_{\tau}$ which is not far

from the observed value of $\langle M \rangle$. However, the only existing calculation of the perturbative corrections (60) to the normalized spectrum gives a negligible improvement. This calculation could be wrong and in fact it is somewhat obscure in many respects. It is important to clarify this point because if the spectrum is indeed dominated by non-perturbative (or higher order) effects, then also the perturbative evaluation of the total width could be to some extent affected, even if inclusive quantities are usually more protected.

In conclusion, it appears difficult to me to compress the total theoretical error below the 10%-20% level. Actually, the theoretical error is so relatively small only because the different measurements on the τ system are remarkably consistent among them. Thus I would tentatively conclude that:

$$\alpha_s(m_b) \approx 0.175 \quad (1 \pm 15\%) \quad (2.27)$$

Even with this enlarged error the resulting determination of $\Lambda_{\overline{MS}}^{(4)}$ is comparatively quite good:

$$\Lambda_{\overline{MS}}^{(4)} \approx 1.54 \Lambda_{\overline{MS}}^{(5)} = (180 \pm 80) \text{ MeV} \quad (2.28)$$

2.4 $e^+e^- \rightarrow \text{Jets}$

All methods of measuring α_s described in the previous sections are based on totally inclusive processes. We now consider the determination of α_s from the observed properties of jets in the final state. The study of jets in e^+e^- annihilation has provided a formidable laboratory for QCD testing for about a decade. Many striking confirmations of the theory have been obtained (61): the observation of the predicted jet structure and the expected hierarchy of two, three, four... jets, an evidence for

gluons and their vector nature, the quantitative correspondence between the observed distributions in energy and angles and the QCD matrix elements. While the more general aspects of jet physics will be discussed in the next section we concentrate here on the measurement of α_s from jets in e^+e^- annihilation.

The principle of the method is to measure a quantity which is zero in lowest order (corresponding only to a quark-antiquark pair in the final state) and starts at order α_s (quark-antiquark-gluon). For a meaningful determination of α_s it is necessary to know the same quantity at order α_s^2 . This implies computing virtual corrections to three-parton amplitudes and real four-parton matrix elements. As is well known, the contribution to the rate of virtual and real diagrams is separately divergent while only the sum is finite for well-defined physical observables. The additional difficulty of jet physics consists in the obvious fact that the theory deals with partons and the physical observables are jets of hadrons. The relation between partons and the experimentally defined jets necessarily requires some model of non-perturbative fragmentation and hadronization. While non-perturbative effects should asymptotically become negligible their influence on the extracted value of α_s is still sizeable at PEP/PETRA energies and contributes the main source of error.

For example, assume that one wants to compute some three-jet distribution $d\sigma(\text{three-jets})$. As we have seen the perturbative calculation at order α_s^2 needs "jet-dressing" to become finite. One must add to the contribution of three partons the integral over unresolved configurations from final states with four partons:

$$d\sigma(\text{3-jets}) = d\sigma(\text{3-partons}) + \int_{NR} d\sigma(\text{4-partons}) \quad (2.29)$$

where NR indicates the non-resolved configurations, i.e., those where any two partons are too close to be separated so that the event is observed as a three-jet event. Clearly some jet resolution criterion is needed. Typically this is a cut on the invariant mass y_{ij} of a pair i, j of partons: below a given value of y_{ij} the pair is detected as a single jet. Evidently the presence of these cuts introduces a problem of cut dependence of the result. Not only that but the experimental jet identification criterion based on observed hadrons can only be translated into a cut on parton variables by a model of fragmentation and hadronization. With time there has been considerable progress (62) in the understanding of early discrepancies from different calculations (63,64) based on different four-parton resolution criteria. Also, for the determination of α_s , one now selects some global quantity [e.g., oblateness (9), energy-energy correlations and their asymmetry (65,66)] which are independent or less dependent on jet resolution criteria. Of course the dependence on fragmentation and hadronization effects always remains even if care is taken to concentrate on quantities which are invariant under collinear splitting of one into two massless particles. Finally one demands a good apparent convergence, i.e., that the resulting non-leading correction of order α_s^2 is not too large for a natural choice of the renormalization scale μ appearing in the leading term proportional to $\alpha_s(\mu)$.

At present the method for the determination of α_s which is used most is based on the asymmetry of energy-energy correlations (AEEC) (65,66). The energy-energy correlation (EEC) is defined by:

$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\chi} = \frac{1}{\sigma} \sum_{i,j} \int \frac{d\sigma}{dx_i dx_j d\cos\chi} x_i x_j dx_i dx_j \approx \frac{1}{N_{\text{events}}} \sum_{i,j} x_i x_j \delta(\cos\theta_{ij} - \cos\chi) \quad (2.30)$$

where χ is the fixed angle between two calorimeter cells, $x = 2E/\sqrt{s}$. Clearly the contribution at χ not too close to 0 and π arises from non-collinear events. The energy weights make EEC infrared safe, the linearity in x_i guarantees invariance under collinear splitting of particle i .

The asymmetry AEEC is defined as

$$\frac{1}{\sigma} \frac{d\sigma_{\text{AEEC}}}{d\cos\chi} = \frac{1}{\sigma} \frac{d\sigma}{d\cos\chi} (\pi - \chi) - \frac{1}{\sigma} \frac{d\sigma}{d\cos\chi} (\chi) \quad (2.31)$$

The asymmetry is different from zero because in a typical three-jet event there is a slim jet in one hemisphere and a fat di-jet in the other one.

A purely perturbative calculation leads to a result of the form:

$$\frac{1}{\sigma} \frac{d\sigma_{\text{AEEC}}}{d\cos\chi} = \frac{\alpha_s(Q)}{\pi} A(\cos\chi) \left[1 + \frac{\alpha_s(Q)}{\pi} R(\cos\chi) + \dots \right] \quad (2.32)$$

In the range $-0.95 < \cos\chi < 0.95$ the value of R (67) varies between 2.5 and 3.5, so that the expansion is apparently well behaved. For $|\cos\chi|$ near 1 the perturbative expansion should be improved by a resummation of the corresponding singularities.

Experimentally it is found that for $\chi > 30^\circ$ the purely perturbative evaluation of the AEEC distribution at order α_s^2 provides an excellent fit to the data. The results on α_s determined from the purely perturbative fit are reproduced in Table 1 taken from Ref. (67). By taking the average one obtains:

$$\alpha_s(Q=34 \text{ GeV}) = 0.121 \pm 0.003$$

$$\Lambda_{\overline{MS}}^{(5)} = (100 \pm 15) \text{ MeV} \quad (\text{perturbative}) \quad (2.33)$$

As usual, also in this case the problem is to estimate the theoretical error. The main source of error is expected to arise from fragmentation and hadronization effects. Thus the most natural thing to do is to compare the purely perturbative result with those obtained by including models of jet formation. Following Ref. (67) where a more complete discussion is given, we report the results based on two different Monte Carlo analyses including the perturbative calculations of order α_s^2 and a model of fragmentation: a model by Ali et al. (68) and a version of the Lund model (69). The results from these analyses are reported in Table 2 (67). The average values are:

$$\alpha_s'(Q=34 \text{ GeV}) = 0.128 \pm 0.003$$

$$\Lambda_{\overline{MS}}^{(5)} = (144 \pm 16) \text{ MeV} \quad (\text{Ali et al.}) \quad (2.34)$$

$$\alpha_s(Q=34 \text{ GeV}) = 0.148 \pm 0.002$$

$$\Lambda_{\overline{MS}}^{(5)} = (325 \pm 20) \text{ MeV} \quad (\text{Lund}) \quad (2.35)$$

These results are in agreement with some other existing measurements (61,67) based on oblateness or the planar triple energy correlation (70). The conclusion is that there is indeed a systematic difference in the results from the Ali et al. and the Lund model. Both models lead to an increase of α_s . But in the case of Ali et al. the change with respect to the perturbative result is much smaller than in the Lund case. The dispersion of the results can be taken as an indication of the theoretical error. Thus one concludes that from $e^+e^- \rightarrow \text{jets}$ the present result is something like

$$\alpha_s(Q=34 \text{ GeV}) = 0.135 \pm 0.015$$

$$\Lambda_{\overline{MS}}^{(5)} = (215 \pm 130) \text{ MeV} \quad (2.36)$$

2.5 Other Processes

The main source of additional information on α_s is obtained from $\gamma\gamma$ reactions (71). The photon structure function F_2^γ measured in $\gamma\gamma$ collisions (one tagged photon of virtual squared mass $-Q^2$ on a quasi-real photon) is special because it is predicted to grow as $\ln Q^2$ (72). The logarithmic increase of F_2^γ is well supported by the data and is a nice confirmation of asymptotic freedom that preserves this prediction also in presence of QCD corrections that, however, very markedly modify the shape of the structure function (making it considerably softer).

The leading pointlike component is to a large extent computable (especially at relatively large x). However, early hopes of measuring α_s , free from hadronic non-perturbative unknowns, directly from the observed value of F_2^γ at sufficiently large Q^2 in some range of not too small x cannot be really fulfilled. It is by now generally recognized that the determination of $\Lambda_{\overline{MS}}^\gamma$ from F_2^γ requires data at different values of Q^2 (71). In fact it is clear that in order to measure $\Lambda_{\overline{MS}}^\gamma$ complete control of the next-to-leading terms is necessary. At that level (73) the complete separation of the computable pointlike-photon contribution from the hadronic terms becomes impossible. These terms arise from the hadronic component of the photon as visualized for example by vector-meson dominance (the virtual photon scatters on a ρ , ω , ϕ , ... in the quasi-real photon). Actually spurious singularities in the space of moments (which affect the behaviour of F_2^γ at small x) are generated in

the singlet sector if hadronic terms are not properly taken into account. It is therefore necessary to introduce a parametrization of the non-perturbative hadronic terms and to determine from the data the corresponding parameters together with $\Lambda_{\overline{MS}}$. This evidently generates some ambiguity on $\Lambda_{\overline{MS}}$ that the limitations of present data cannot eliminate.

Several theoretical approaches (74) have been advocated for a treatment of the hadronic component. Values of $\Lambda_{\overline{MS}}$ obtained by different experiments and procedures are listed in Table 3 (71,75). Note that most of the listed data were analyzed in terms of formulae for $f = 3$. Thus the reported values of $\Lambda_{\overline{MS}}$ should be mainly identified with $\Lambda_{\overline{MS}}^{(3)}$ which differs from $\Lambda_{\overline{MS}}^{(4)}$ by about 30% (because $\Lambda_{\overline{MS}}^{(3)} \approx 1.3 \Lambda_{\overline{MS}}^{(4)}$).

In conclusion, when translated in terms of $\Lambda_{\overline{MS}}^{(4)}$ the results on photon structure function lead to values of $\Lambda_{\overline{MS}}^{(4)}$ in the range 50-300 MeV which are perfectly consistent with other experiments.

There are many less precise or more ambiguous determinations of α_s from other experimental sources. For example, I can quote the determination of α_s at $\mu \approx m_\tau$ from the leptonic branching ratio of the τ lepton. This branching ratio is $B_e = (18.3 \pm 0.3)\%$. The fact that it is close to 1/5 rather than to 1/3 is a proof that $N_c = 3$, because $B_c \approx 1/(2+N_c)$. From the deviations from the value 1/5 a value of $\alpha_s(m_\tau)$ can be extracted (76) with some model dependence from the treatment of non-perturbative effects. On the other extreme a measurement of α_s at $\mu \approx M_W$ has been attempted (77) by UA2 from W -jet production at the CERN $p\bar{p}$ collider. The results are in both cases consistent with the more precise methods already described.

2.6 Summary and Conclusion on α_s

In this chapter we have tried to review and interpret the large amount of experimental information on α_s . A sample of the most significant determinations of α_s is reported in Table 4. It turns, each entry is a combination of different measurements. It is clearly remarkable that the final results on α_s from so many completely different sources are in very good agreement among them. This is one of the most important quantitative tests of QCD. A plot of a more extended set of data is shown in Fig. 7. I do not think that it would be appropriate to combine the errors according to Gaussian statistics in order to derive an average value of Λ . In fact, the data on deep inelastic scattering are in part contradictory or not always in agreement with QCD; the error on the T entry is a personal estimate; the $e^+e^- \rightarrow \text{jet}$ result is obtained by a particular combination of the existing data and so on. However, I cannot evade the task of providing the readers with a suggested set of values. In this spirit I propose:

$$\Lambda_{\overline{MS}}^{(4)} = (220 \pm 90) \text{ MeV} \quad (2.37)$$

or, equivalently:

$$\Lambda_{\overline{MS}}^{(5)} = (140 \pm 60) \text{ MeV} \quad (2.38)$$

where the reported error is about twice the value that would be obtained by combining the errors shown in Table 4.

An objection which is often made is that no experiment has really detected the running of α_s . Even from the whole set of data we have discussed, once the errors are taken into account, the running of α_s cannot be clearly established in the range $Q \sim 3\text{-}50$ GeV. This fact is simply a consequence of the slow logarithmic decrease of $\alpha_s(Q)$. Actually there are experiments that claim to have observed the decrease of α_s with

Q [see, for example, Ref. (78)] but, in my opinion, they are not convincing.

However, the fact that the values of α_s which are measured at large Q are so small, as shown in Table 4, is a very strong proof of the running of α_s , because such a relatively feeble strong force could not provide hadrons with the observed tight binding. A more significant test of QCD is evident from Fig. 7. A relatively loose determination of $\alpha_s(Q)$ at $Q \sim 1$ GeV leads to a very tight determination of $\alpha_s(Q)$ at large Q. For example, from the resulting value of $\Lambda_{\overline{MS}}^{(5)}$ given in Eq. (2.38) the prediction for the value of α_s to be measured at LEP (and HERA) is very precise:

$$\alpha_s(Q \approx M_Z) \approx 0.11 \pm 0.01 \quad (2.39)$$

Establishing that this prediction is experimentally true would be a very quantitative and accurate test of QCD conceptually equivalent but more reasonable than trying to see the running in a given experiment.

3. - THE QCD THEORY OF HARD PROCESSES

In this chapter we briefly discuss the additional experimental evidence for QCD which is derived from the phenomenology of hard processes. The property of asymptotic freedom provides the theoretical framework for a consistent and systematic formulation of the parton model in QCD (renormalization group, factorization theorem, etc.). Thus the successes of the naive parton model are directly inherited by QCD. Clearly we are mostly interested in those predictions of the parton model in QCD that go beyond a generic and naive formulation of the parton

picture thus providing us with specific dynamical tests of the underlying QCD theory.

An enormous amount of theoretical and experimental work on hard processes in QCD has been accumulated over the years. A systematic review of the phenomenology of high momentum transfer reactions would by far exceed the limits of the present relatively concise article. In the following only a number of important examples will be mentioned that can be considered among the most significant experimental facts in support of QCD.

3.1 Jets in e^+e^- Annihilation

Experiments on e^+e^- annihilation at high energy (61) have provided a wonderful laboratory for systematically testing the distinct signatures predicted by QCD for the structure of the final state averaged over a large number of events. In the following we discuss the predictions of QCD concerning the properties of the final state.

Typical of asymptotic freedom is the hierarchy of configurations which emerges from the smallness of $\alpha_s(Q)$ at high energies. Each configuration starts at a given order in $\alpha_s(Q)$ and is characterized by a specified topology. When all corrections of order $\alpha_s(Q)$ are neglected one recovers the naive parton model prediction for the final state: almost collinear events with two back-to-back jets with limited transverse momentum and an angular distribution as $(1+\cos^2\theta)$ with respect to the beam axis. The two-jet structure of the majority of the events and the angular distribution of jets typical of spin- $\frac{1}{2}$ quarks (scalar particles would lead to a $\sin^2\theta$ distribution) were first established at SPEAR (79) and later confirmed and extensively studied especially at the high-energy e^+e^- colliders PEP, PETRA and more recently also at TRISTAN. For example, the angular distribution of jets in e^+e^- annihilation measured

by TASSO (61) at PETRA is shown in Fig. 8. At order $\alpha_s(Q)$ a tail of events is predicted to appear with large transverse momentum $p_{\perp} \sim Q/2$ with respect to the thrust axis (the axis that maximizes the sum of the absolute values of the longitudinal momenta). The small fraction of events with large $p_{\perp}$ mostly consists of three-jet events with an almost planar topology. The skeleton of a three-jet event at leading order in $\alpha_s(Q)$ is formed by three hard partons, the third being a gluon emitted by a q or a $\bar{q}$ line. The first observation of three-jet events at PETRA gave a relatively direct experimental support to gluons. At order $\alpha_s(Q)$ the transverse momentum in the event plane $\langle p_{\perp} \rangle_{\text{in}}$ with respect to the thrust axis is predicted to increase linearly with Q (apart from logarithms) while $\langle p_{\perp} \rangle_{\text{out}}$ is still fixed in this approximation. Similarly the most energetic jet, called the slim jet, should look like a jet of a two-jet event (at somewhat scaled down energy) and correspondingly $\langle p_{\perp} \rangle_{\text{slim}}$ is fixed, while $\langle p_{\perp} \rangle_{\text{fat}}$ increases with Q . At order $\alpha_s^2(Q)$ a hard perturbative non-planar component starts to build up and some small fraction of four-jet events is predicted to appear: both $\langle p_{\perp} \rangle_{\text{out}}$ and $\langle p_{\perp} \rangle_{\text{slim}}$ start increasing.

The topological signatures which have just been described in a qualitative way are quite well supported by the available data. For example, we reproduce in Fig. 9 the data from TASSO (61) that clearly show the increase of $\langle p_{\perp}^2 \rangle_{\text{in}}$ with the centre-of-mass energy $W = 2E_{\text{beam}}$ = Q . Even more impressive is Fig. 10 taken from MARK J (61) that compares the observed energy flow diagrams with the predictions of QCD and of some other (rather artificial) models. In (a) F_{minor} is a measure of acoplanarity. The data support the QCD prediction which is less spherical than phase space and more acoplanar than two-jet events. In (b) and (c) the energy flow polar angle diagrams for non-collinear events are shown for two different cuts in thrust and jet angles. Finally in (d) the unfolded

energy flow diagram is compared with different detailed model predictions and shows very good agreement with QCD (also including a model treatment of fragmentation effects).

The precise form of the QCD matrix element for three or four partons in the final state can be confronted with experiment although some model of the relation between computed partons and observable jets must be superimposed. The determination of α_s from the energy-energy correlation distributions is the most quantitative of these comparisons of e^+e^- annihilation data with QCD matrix elements. The determination of the gluon spin was also attempted (66,67) from the study of three-jet distributions in continuum e^+e^- annihilation and in τ hadronic decays ($\tau \rightarrow ggg, \gamma g g$). In Fig. 11 we show, as an example, the results obtained by TASSO (61) on the observed distribution in $\cos\tilde{\theta}$, $\tilde{\theta}$ being the angle between the thrust axis (roughly aligned with the most energetic jet called jet 1) and the line of jets 2 and 3 in their own centre-of-mass frame (Ellis-Karliner test) (80). The distribution for vector gluons is clearly preferred with respect to the analogous matrix element computed for scalar gluons (although such a case does not really correspond to a sensible theory).

More recently, some more detailed aspects of the QCD predictions for the parton branching and cascade have been discussed and in part tested. An important example is the class of coherence effects (81,82) and the tested difference between $\gamma q \bar{q}$ and $g q \bar{q}$ final states (83).

In conclusion the wealth of experimental results on jet physics in e^+e^- annihilation and their successful comparison with the theory has much contributed to establish a solid observational basis for QCD.

3.2 Deep Inelastic Leptonproduction and the Nucleon Parton Densities

In Section 2.2 we have summarized the determination of α_s from the observed scaling violation in deep inelastic leptonproduction. In this section we discuss the additional very important information concerning the QCD-improved parton model that can be obtained from the data on deep inelastic scattering with muon and neutrino beams. This includes tests of the parton-model predictions and the experimental determination of the quark and gluon parton densities in the proton (or the isoscalar nucleon). Once the parton-densities have been measured at some Q^2 , they can be evolved at all Q^2 and used to predict all sorts of other processes, some of them (Drell-Yan, W/Z production, jets in $\bar{p}p$ collisions, photons at large P_T , etc.) will be discussed in the following sections.

In the naive parton model, for spin- $\frac{1}{2}$ quarks, the longitudinal structure function $F_L = F_2 - 2xf_1$ is predicted to vanish asymptotically as $1/Q^2$ [Callan-Gross relation (84)]. In QCD, $F_L(x, Q^2)$ is instead of order $\alpha_s(Q)$ and therefore vanishes more slowly as $\ln Q^2$. The leading QCD expression for F_L is given by (85):

$$F_L(x, Q^2) = \frac{\alpha_s(Q)}{2\pi} x^2 \int_0^1 \frac{dy}{y} \left[\frac{8}{3} F_2(y, Q^2) + 2 \sum_{i=1}^{24} e_i^2 (y-x) g(y, Q^2) \right] + \dots \quad (3.1)$$

where $(\sum_{i=1}^{24} e_i^2)$ is the sum of all coefficients of q and $\bar{q}$ in the naive parton model expression of F_2/x (for $f=4$ it is 20/9 in electroproduction and 8 for ν or $\bar{\nu}$ scattering from charged currents). In Fig. 12a,b, we report some recent data on F_L at large Q^2 for (a) muon production on protons (42,43) ($\langle Q^2 \rangle \approx (15-60) \text{ GeV}^2$) and (b) for neutrino-production on iron (44) ($\langle Q^2 \rangle \approx 1.4-69 \text{ GeV}^2$). One sees that the longitudinal structure function is indeed small at large Q^2 , once again confirming that the

charged partons have spin- $\frac{1}{2}$. The data are perfectly consistent with the QCD prediction in Eq. (3.1). The expected rise at small x of F_L due to the increasing sea and gluon contributions is indicated by the CDRSW and BCDMS data.

The QCD corrected parton sum rules, in particular the Adler sum rule

$$(86) \quad \int_0^1 \frac{dx}{x} \left[F_2^{\nu N}(x, Q^2) - F_2^{\nu p}(x, Q^2) \right] = 2 \quad (3.2)$$

and the Gross-Llewellyn Smith (87) sum rule

$$\int_0^1 dx \, F_3^{\nu N}(x, Q^2) = 3 \left[1 - \frac{\alpha_s(Q)}{\pi} + \dots \right] \quad (3.3)$$

(N = isoscalar nucleon target) are well supported by experiment [see, for example, Fig. 13 with the data by the BEBC collaboration, Ref. (88)]. Similarly the approximate prediction of a ratio of 5/18 between F_2 for electroproduction and neutrino-production on isoscalar targets is in agreement with the data (89).

The integral $\int_0^1 dx F_2^{\nu N}(x, Q^2)$ where N is an isoscalar nucleon target is a good measure of the total momentum fraction carried by quark and antiquarks in the proton. It is well known (41) that this quantity is about 0.45-0.50 at $Q^2 \sim 10 \text{ GeV}^2$ and nearly constant. The remaining fraction of momentum is attributed to gluons.

In general it is fair to say that the great wealth of accumulated data on deep inelastic leptonproduction is in very good agreement with the QCD-improved parton model. It is therefore possible to extract relatively reliable quark and antiquark distributions from the existing data. More difficult is the measurement of the gluon density because gluons are not directly coupled to electroweak currents.

The sum $u_v + \bar{d}_v$ of valence densities $u_v = u - \bar{u}$ and $\bar{d}_v = \bar{d} - d$ is obtained from the structure function F_3 measured in neutrino scattering on isoscalar targets. The u_v and $\bar{d}_v$ distributions can be separated by using the charged-current cross-sections for hydrogen and deuterium targets, which indicate that $\bar{d}_v/u_v \sim 0.57(1-x)$ at $Q^2 \sim \text{few GeV}^2$. Additional information on the ratio $\bar{d}_v/u_v$ is derived from the measurements of $F_2^{n,p}/F_2^{p,p}$ by BCDMS (43) and EMC (42) which are mutually consistent for this ratio. The sea densities can be constrained by using the data on F_2 on protons or isoscalar targets. The information on the flavour dependence of the sea densities is not very rich. One usually assumes $\bar{u} = \bar{d}$. The amount of strange sea $\bar{s}$ (presumably about equal to s) can be measured from anti-neutrino induced dimuon production (explained by charm production) and its shape is consistent with that for $\bar{u}$ and $\bar{d}$ which is obtained from F_2 . Roughly one finds $(90) \bar{s} \sim 0.4 \bar{u}$ at $Q^2 \sim 5 \text{ GeV}^2$. Actually the data on Drell-Yan muon pair production in $p\bar{p}$ collisions are also often used to further constrain the shape in x of the sea distributions.

Some uncertainty on the quark densities is introduced by the observed dependence of the structure functions per nucleon measured on nuclei on the nuclear size [the so-called EMC effect (42)]. This is to be taken into account when the data on heavy isoscalar targets are used to derive information on the parton densities in the proton. However, in the last few years the amount of nuclear effects at small and intermediate x measured in several different experiments has settled down to a tolerable size (45). Actually somewhat paradoxically the main problem is now represented by the already mentioned experimental discrepancy between the BCDMS and the EMC data on $F_2^{p,p}$ (45).

Apart from evident practical reasons the determination of the gluon density in the proton is clearly also very important from the point of

view of testing the theory. In fact the physical reality of quarks was first established by the study of the spectroscopy of hadrons and later confirmed by the parton model description of leptonproduction as the quarks are directly coupled to the electroweak currents. For the gluons it is certainly more difficult to obtain a solid basis of experimental evidence. We have seen in the last section that good evidence for gluon jets has been obtained from the study of the final state in e^+e^- annihilation. We have also seen that about half of the proton momentum is carried by gluons. Here we consider the experimental information on the gluon parton density in the nucleon.

The main input on the gluon parton density in the proton is obtained from the study of scaling violations at small x in leptonproduction. Accurate analyses of the scaling violations in the singlet sector have been performed by CDHS (91), CHARM (92), EMC (42) and BCDMS (43). $\Lambda_{\overline{MS}}$ and the gluon density have to be separately determined from a fit of the observed scaling violations (Fig. 4). As only a few logarithmic slopes at small x are important for the fit the trial parametrization of the gluon density can only be a very crude one. The problem is further complicated by the fact that at small x the average values of Q^2 are in general smaller and also the effects associated with the charm threshold can simulate scaling violations.

In spite of these difficulties there is a reasonable agreement on the shape of the gluon density at $Q^2 = (5-10) \text{ GeV}^2$ as determined by different experiments (Fig. 14). The gluon density is concentrated at small x and its effects on scaling violations are indeed found to be negligible above $x \approx 0.25-0.3$. The more recent data show a somewhat softer gluon distribution than that first obtained by CDHS (91). On the other hand, the new CDHSW data (44), based on larger statistics, show marked differences at low x with respect to the previous structure functions. A soft

gluon is also supported by the data on large p_T photons in $p\bar{p}$ collisions (93,94) and on J/ψ production (95). The CDHS gluon served as an input to the widely used parametrizations of parton densities by Duke-Owens (96), Eichten et al. (97), etc. More modern sets of parton densities by Diemoz et al. (98) and Martin et al. (95) are instead based on the now preferred soft-gluon density.

It is remarkable that the gluon density obtained from the scaling violations in deep inelastic scattering, evolved at much larger values of Q^2 , is found to be a necessary contribution for the interpretation of ISR and $p\bar{p}$ collider data, for example on jet or photon production at large p_T , on heavy flavour hadro- or photoproduction, etc., as will be discussed later in this chapter.

3.3 Drell-Yan Processes and W/Z Production

The production of lepton pairs in hadron-hadron collisions, via virtual photon or intermediate weak boson exchange, is a process of great importance for QCD. Drell-Yan processes (9) in fact provide a crucial and quite non-trivial test of the validity of the parton approach and of its implementation in QCD through the factorization theorem (99). This theorem predicts that the Q^2 dependent quark and gluon densities measured in leptonproduction on a given hadronic target, evolved in Q^2 by the QCD evolution equations, are directly relevant to predict cross-sections for other hard processes involving the same hadron. The key point of the factorization theorem is that the Q^2 dependent parton densities are universal, i.e., process independent. The dependence on the particular process only enters at the level of the partonic subprocesses. The resulting prediction for hadron-hadron collisions is a double convolution of the Q^2 dependent parton densities with the parton cross-section which

is perturbative and can be computed as an expansion in $\alpha_s(Q)$. For an inclusive process $A+B \rightarrow X$ one has:

$$\frac{d\sigma}{d^3p} = \sum_{i,j} \int dx_A dx_B F_{i/A}(x_A, Q^2) F_{j/B}(x_B, Q^2) \cdot E \frac{d\hat{\sigma}}{d^3p}(x_A p_A, x_B p_B, p, \dots, \alpha_s(Q)) \quad (3.4)$$

where $\sigma(\hat{\sigma})$ is the hadronic (partonic) cross-section, $F_{i/A}$ is the density of parton i in hadron A and Q is the large energy scale in the process (typically the lepton-pair mass in the Drell-Yan case). For simplicity we have here identified the factorization scale Q , which appears in the parton densities, with the renormalization scale, which enters in $\alpha_s(Q)$. Q can be chosen, with relatively wide freedom, around the natural physical scale for a given process. A change in Q is compensated by the corresponding change in $\hat{\sigma}$. Of course the compensation is exact only for the complete expression of $\hat{\sigma}$. At any fixed order in α_s a scale change produces a variation of the cross-section by terms of higher order in α_s .

The class of Drell-Yan reactions (also including W/Z production) are simplest among hard hadron-hadron processes in that the final state can be totally inclusive and the observed particles (the lepton pair) are non-strongly interacting. The fact that the cross-sections are quadratic in the parton densities implies testing the parton model in a dynamical configuration far more complex than in leptonproduction. In fact the validity of the factorization theorem in the case of Drell-Yan processes has been the subject of a long debate (100). Finally the conclusion was in favour of the validity of the parton-model prediction (101), but the arguments are certainly not as simple and clear as for processes where the light-cone operator expansion can be applied.

The precise measurements of muon-pair production cross-sections at fixed target experiments and at ISR have been very important for establishing the signatures of the parton-model approach to Drell-Yan processes. These characteristic properties are: (a) linear A dependence for experiments on nuclei with atomic number A (a consequence of the incoherent sum of the parton contributions in the target); (b) the angular distribution of the lepton pair predominantly as $1+\cos^2\theta$ in their centre-of-mass; (c) the approximate scaling of $Q^2(d\sigma/dQ^2 dy)$ and other similar adimensional quantities (which follows from the fact that in QCD the scaling breaking effects are only logarithmic); (d) intensity rules. For example, the dominance of valence-valence (π^+N , K^+N , $\bar{p}N$) over valence-sea cross-sections (K^+N , $\bar{p}N$). The available data at sufficiently large energies, for masses of the pair above the J/ψ and not too close to the phase space boundary (e.g., for $\tau = Q^2/s$ not too close to 1) neatly support (102) all the previous distinctive predictions. The study of Drell-Yan processes has also produced important information on the x-behaviour of the sea densities in the nucleon and on the otherwise inaccessible quark densities in pions and kaons.

The QCD corrected parton model leads to an absolute prediction for the total cross-section (9). The value of the lowest-order cross-section is inversely proportional to N_c , the number of colour replicas for quarks, because a given quark can only annihilate with an antiquark of the same colour to produce a colourless lepton pair. The order $\alpha_s(Q)$ corrections to the cross-section have been computed long ago (103) and found to be large (when the parton densities are defined from the structure function F_2 measured in leptonproduction at $q^2=Q^2$). The ratio $\sigma_{\text{corr}}/\sigma_{\text{LO}}$ of the corrected and the lowest-order cross-sections, called the K-factor, is slowly varying in Q^2 and y . Recently the calculation of an important part of the two-loop K-factor has also been completed (104).

What has been computed at order $\alpha_s^2(Q)$ is the contribution of the quark-antiquark annihilation channel to the partonic cross-section limited to the terms which are singular near $z = (\tau/x_A x_B) = 1$ with $\tau = Q^2/s$ [in lowest order the whole contribution arises from $z=1$; at first order, and presumably in higher orders as well, the bulk of the correction arises from the singular terms of the form $\delta(1-z)$, $\ln(1-z)/(1-z)_+$ and $1/(1-z)_+$. The result is that the terms of order $\alpha_s^2(Q)$ at small τ are of the expected size relative to the large $O(\alpha_s)$ terms, have the same sign of the first-order correction and somewhat exceed the estimate obtained by a simple exponentiation of the first order. For example for W/Z production at 630 GeV the estimates of Ref. (104) are that the K factor at one loop is 1.39 with $\alpha_s \equiv \alpha_s(M_W)$ and becomes 1.57 with the addition of the computed two-loop effects.

The QCD predictions for Drell-Yan processes can best be tested for W/Z production. W/Z production at CERN and Tevatron energies is ideal because $Q \sim M_{W/Z}$ is large so that the K factor [which decreases with $\alpha_s(Q)$] is acceptable, Q and $\sqrt{s}$ are not too unbalanced: $\sqrt{\tau} = Q/\sqrt{s} = 0.13-0.15$ at $\sqrt{s} = 630$ GeV and $\sqrt{\tau} = 0.046 \pm 0.052$ at the Tevatron and the parton densities are reasonably well known in the relevant region of $x \sim O(\sqrt{\tau})$. Note in this respect that a precise calculation of W/Z production at supercollider energies is a more difficult problem. The predicted W and Z cross-sections are reported in Table 5 (105), where only the first-order K factor was taken into account, while an estimate of the higher orders assuming exponentiation was included in the stated errors. The error is asymmetric because the "central" value was computed by choosing $Q = M_W$ in the corrective terms, while the p_T distribution with $\langle p_T \rangle \ll M_W$ would perhaps suggest a lower scale, hence a larger α_s . [This

suggestion is apparently confirmed by the approximate calculation of the two-loop K factor of Ref. (104).] What is actually measured is the cross-section times branching ratio for $W \rightarrow e\nu$ or $Z \rightarrow e^+e^-$. Especially $B(W \rightarrow e\nu)$ depends on m_t : $B(W \rightarrow e\nu) \sim 0.089$ for $m_t = 40$ GeV, ~ 0.100 for $m_t = 60$ GeV and ~ 0.109 for $m_t > 80$ GeV. The comparison with the data is shown in Fig. 15 for $m_t = 60$ GeV (105). The agreement between theory and experiment is quite good. To better appreciate this point, note that the presence of colour introduces a factor of $1/N_c \sim 1/3$ in σ and a factor $\sim 5/(3+2N_c) \sim 5/9$ in the branching ratio $B(W \rightarrow e\nu)$ (for large m_t). The fact that the data appear to favour the upper side of the error bars is partly explained by the now largely computed two-loop K factor and could also be an indication for $m_t > 60$ GeV. Finally recall that if the already-mentioned discrepancy between BCDMS and EMC on F_2^p (45) is eventually solved in favour of BCDMS then the quark densities would be increased by 10-15% in the relevant region of x . In conclusion, within the present uncertainties there is very good agreement between theory and experiment on the W/Z production cross-sections. The agreement is a significant test of QCD and even more significant it will soon become when the results from the recent high luminosity runs at CERN and at the Tevatron will be available.

The prediction of the transverse momentum distribution of the W or Z has an even deeper dynamical significance. The purely perturbative calculations are only valid at $P_T \approx M_{W/Z}$. At smaller P_T values, in the region $\Lambda \ll P_T \ll M_{W/Z}$, where the bulk of the data on $d\sigma/dp_T$ is concentrated, the sequence of logarithmic terms of all orders which arises from the soft gluon radiation from the initial parton legs must be resummed (106). The corresponding Sudakov exponent is typical of vector gluons. The resulting (107) prediction of the P_T distribution with the correct

perturbative limit, the soft gluon resummation and the exact integral under the curve to reproduce the corrected total cross-section [with the $O(\alpha_s)$ K-factor included], is compared with the data in Fig. 16 (108). The observed P_T distribution is correctly reproduced in terms of a commonly adopted parametrization of parton densities (96) and with a reasonable value of $\Lambda_{\overline{MS}}$. This is very important because the P_T distribution has no analogue in the naive parton model. The average P_T for W/Z production is quite large ($\langle p_T \rangle \approx 8$ GeV at $\sqrt{s} \approx 0.63$ TeV) in comparison to all possible hadronic scales and is entirely produced by QCD radiation.

3.4 Hard Processes in $\bar{p}p$ Collisions

In recent years the contribution of $\bar{p}p$ collider experiments (also including the ISR) has been extremely relevant for considerably broadening the observational support of QCD. The study of the production properties of weak bosons, discussed in the previous section, is a first beautiful example of the importance of collider physics for QCD. In this section we briefly consider the most prominent experimental tests of QCD at $\bar{p}p$ colliders including jet physics, photons at large P_T , heavy flavour production.

The study of jet production at $\bar{p}p$ colliders has opened an entire new territory for probing the validity of the parton model. In $\bar{p}p$ reactions the amplitudes for 2+2 or 2+3 quarks/gluons are made accessible to experimental study. The relative simplicity of semileptonic weak interactions, as compared to the intricacies of weak non-leptonic processes, is a reminder that the strong corrections to a single electro-weak current vertex are in principle much simpler than gluon exchange or radiation in presence of four or five coloured legs. Finally, the scale

Q of energy at $\bar{p}p$ colliders is the largest accessible to experiment at present. Thus the successful predictions by QCD (109) of the two-jet production rate at large p_T , of their angular distribution, of the ratio of three to two jets and the three-jet distributions are certainly very impressive tests of the theory. One may object that these predictions are obtained in the leading logarithmic approximation. The complex work for a complete calculation of jet production in next-to-leading approximation (110,111) is under way and about to produce results relevant for actual experiments. The uncertainties connected with the choice of the scale Q plus the ambiguities on the value of Λ , the errors connected with our ignorance on some details of parton densities and so on, only lead in practice to predictions within a factor of two or so. While this is true, and the experimental errors are also of about the same size, still predictions within a factor of two are extremely significant when applied to steep functions of several variables, i.e., functions that vary by many orders of magnitude in the explored domain. This is the case of the jet production rate that varies by orders of magnitude both in $\sqrt{s}$ (from the ISR to the SpS collider and more recently to the Tevatron) and in p_T (Fig. 17) (112). Thus the physics of large p_T jets in $\bar{p}p$ reactions is a marvellous success of the QCD improved parton model. For example, it is well established (109) that the agreement at relatively small p_T values (but large enough for the perturbative calculation to hold) would not be possible without gluons. Even more striking is the effect of gluons in determining the angular distribution of jets with respect to the beam axis. At large $\cos\theta$ values the dominant behaviour is determined by the Rutherford singularity as $(1-\cos\theta)^{-2}$ typical of the exchange of massless vector gluons. The expected angular distribution is exactly reproduced by the data, as shown in Fig. 18 (113).

The production of hard photons at large p_T has been observed in fixed target experiments (93,114), at the ISR (115) and by both UAL and UA2 (116) at the CERN $\bar{p}p$ collider. In this case, the calculation of QCD predictions at the next-to-leading accuracy have been completed (117). At all explored energies the agreement between theory and experiment is quite good. In fig. 19 the UAL and UA2 data are compared with the QCD predictions of Ref. (117). The data on large p_T photons, especially those at relatively low energy, are also useful to obtain information on the x distribution of the gluon density in the proton.

Recently great progress in the QCD theory of heavy flavour hadro- and photoproduction has been achieved with the calculation of next-to-leading corrections (118,119). The data on the photoproduction of charm, in particular the recent set of precise data by the experiment E691 at FNAL (120), are in good agreement with QCD predictions from photon-gluon fusion, for a reasonable value of the effective charm mass. The hadro-production of charm and beauty at fixed targets and at $\bar{p}p$ colliders is (121), in general, affected by larger theoretical errors with respect to photoproduction. For charm the next-to-leading corrections work in the right direction to make the effective mass of charm required by the data to come in closer agreement with the photoproduction data and with the mass expected from charm particle spectroscopy. For beauty the cross-section and p_T distributions observed by UAL at the CERN $\bar{p}p$ collider (122) are in agreement with the QCD prediction (with large theoretical errors). The theoretical predictions become particularly reliable for top production at present colliders because m_t is large and the ratio $m_t/\sqrt{s}$ is not too small.

For heavy flavour production gluons are also essential. Without gluons there would be neither photon-gluon or gluon-gluon fusion

diagrams. For example, the predicted cross-section for b production at colliders would be a factor 30 or 40 smaller. Of course, one could invoke ad hoc intrinsic charm and beauty quark densities in the proton to explain the up to now scarce data without gluons. While more data on more processes can in principle decide the issue, it remains true that QCD correctly predicts the observed amount of c and b production in terms of the gluon density measured from the observed scaling violations in leptonproduction.

4. - CONCLUSIONS

In this article the main results that form the experimental basis for perturbative QCD have been briefly discussed. It is a fact that a wide variety of observables related to hard processes are correctly predicted by the theory. The measured values of $\Lambda_{\overline{MS}}$ obtained from several different processes quantitatively coincide to a fairly good accuracy. To quantify the level of precision, we recall that the value of α_s at a scale close to the Z mass, relevant for QCD tests at LEP, can be at present predicted with a $\sim 9\%$ accuracy [see Eq. (2.39)]. The running coupling obtained from experiment, together with the parton densities measured in leptonproduction, computed at the relevant scale Q by the QCD evolution equations invariably produce correct predictions for all hard processes that with time become accessible to measurements. If it is true that each individual test cannot be pushed to a high level of precision, it is also true that the data accumulated over the last 15 years or so and the related theoretical work have produced a very large number of successful and increasingly precise tests of the theory. At present, it is fair to say that the experimental support of QCD is quite solid and quantitative. The forthcoming experiments at $p\bar{p}$ colliders, at

LEP, SLC and at HERA will certainly be very important with their great potential for considerably extending the experimental investigation of the validity of QCD.

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Table 1

Values (67) of $\alpha_s(Q)$ and $\Lambda_{\overline{MS}}^{(5)}$ derived from a purely perturbative treatment of the asymmetry of energy-energy correlations.

Q (GeV)	$\alpha_s(Q)$	$\Lambda_{\overline{MS}}^{(5)}$ MeV	Exp.
14	0.14 ± 0.01	100 ± 35	JADE
22	0.13 ± 0.01	100 ± 40	JADE
34	0.115 ± 0.005	70 ± 20	JADE
34.6	0.125 ± 0.005	125 ± 30	PLUTO
34.8	0.125 ± 0.005	125 ± 30	TASSO

Table 2

Values (67) of $\alpha_s(Q)$ and $\Lambda_{\overline{MS}}^{(5)}$ derived from the asymmetry of energy-energy correlations by adding to the perturbative treatment a model of fragmentation and hadronization.

Q (GeV)	Model	$\alpha_s(Q)$	$\Lambda_{\overline{MS}}^{(5)}$ (MeV)	Exp.
35	Ali et al. (68)	0.122 ± 0.004	108 ± 21	MARK J
44	"	$0.129 \pm 0.004 \pm 0.011$	190 ± 40	TASSO
5	"	0.133 ± 0.004	180 ± 30	PLUTO
35	Lund (69)	0.137 ± 0.005	217 ± 42	MARK J
44	"	$0.143 \pm 0.005 \pm 0.012$	340 ± 60	TASSO
35	"	0.142 ± 0.004	260 ± 40	PLUTO
35	"	$0.157 \pm 0.005 \pm 0.012$	440 ± 50	CELLO
29	"	$0.158 \pm 0.003 \pm 0.008$	380 ± 30	MARK II

Table 3

Results on $\Lambda_{\overline{\text{MS}}}$ obtained by different experiments on the photon structure function. Entries quoted with the label (a), (b), ... for one given experiment differ by the assumed model for the hadronic component. The quoted value of $\Lambda_{\overline{\text{MS}}}$ in most cases should be read as $\Lambda_{\overline{\text{MS}}}^{(3)}$ because it is obtained from a fit to the (charm-subtracted) data done in terms of QCD formulae with $f = 3$. The first five entries were taken from Ref. (71), the TPC results from Ref. (75).

Experiment	Q^2 (GeV^2)	$\Lambda_{\overline{\text{MS}}} \text{ (MeV)}$
PLUTO (a)	3 - 100	183 ± 80 54
PLUTO (b)	3 - 100	240 ± 90
PLUTO (c)	3 - 100	160 ± 60
JADE	10 - 220	250 ± 90
TASSO	7 - 70	190 140 ± 60
TPC/2 γ (a)	0.7 - 22	108 ± 32
TPC/2 γ (b)	0.7 - 22	232 ± 59

Table 4

Summary of the most significant determinations of α_s and $\Lambda_{\overline{\text{MS}}}$. The values and the errors quoted are discussed in the text.

	α_s (34 GeV)	$\Lambda_{\overline{\text{MS}}}^{(4)} \text{ (MeV)}$	$\Lambda_{\overline{\text{MS}}}^{(5)} \text{ (MeV)}$
$\text{R}^+\text{e}^-\text{e}^-$	0.14 ± 0.02	370 ± 350 220	240 ± 230 140
BCDMS	0.127 ± 0.006	220 ± 60	140 ± 40
T	0.123 ± 0.009	180 ± 80	120 ± 50
$\text{e}^+\text{e}^- \rightarrow \text{jets}$	0.135 ± 0.015	330 ± 200	215 ± 130
γ -structure function	0.120 ± 0.016	175 ± 125	115 ± 80

Table 5

Values of W ($= W^+W^-$) and Z production cross-sections in $p\bar{p}$ collisions for $\sin^2\theta_W = 0.229$, $m_W = 80.8 \text{ GeV}/c^2$, $m_Z = 92.0 \text{ GeV}/c^2$ (105).

$\sqrt{s}$ (TeV)	σ^W (nb)	σ^Z (nb)
0.54	4.3 ± 1.3 $- 0.6$	1.4 ± 0.4 $- 0.2$
0.63	5.4 ± 1.6 $- 0.9$	1.7 ± 0.5 $- 0.3$
1.6	17 ± 4.0 $- 2.5$	5.1 ± 1.2 $- 0.8$
1.8	19 ± 5.0 $- 3.3$	5.8 ± 1.6 $- 1.0$
2.0	21 ± 6.0 $- 4.0$	6.4 ± 1.9 $- 1.2$

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FIGURE CAPTIONS

Fig. 1 : Ratio of F_2^p measured by BCDMS and EMC as function of x [Ref.(45)]. Only statistical errors are shown.

Fig. 2 : The logarithmic slopes of F_2 measured on carbon by BCDMS (43) compared to the next-to-leading non-singlet QCD evolution for the indicated values of $\Lambda \equiv \Lambda_{\overline{MS}}^{(4)}$.

Fig. 3 : The logarithmic slopes of F_2 measured on hydrogen by BCDMS (43) compared to the next-to-leading non-singlet QCD evolution for the indicated values of $\Lambda \equiv \Lambda_{\overline{MS}}^{(4)}$.

Fig. 4 : The logarithmic slopes of F_2 measured by BCDMS (43) on hydrogen compared to the next-to-leading singlet QCD evolution with $\Lambda_{\overline{MS}}^{(4)} \approx 220$ MeV and a gluon density $xg(x, Q^2) \sim \Lambda(1-x)^\eta$ for $Q^2 = 5 \text{ GeV}^2$.

Fig. 5 : Comparison (45) of EMC (42) and BCDMS (43) logarithmic slopes of F_2 on hydrogen.

Fig. 6 : Comparison of data by EMC (42) on F_2 on iron and by BCDMS (43) on F_2 on carbon. The slopes on iron are apparently steeper than those on carbon.

Fig. 7 : A summary of the determinations of α_s discussed in the text. The curves for $\Lambda_{\overline{MS}}^{(5)} = (140 \pm 60) \text{ MeV}$ are obtained following the matching procedure at the b threshold explained in Eqs. (1.14)-(1.19) (with $a = 1$).

Fig. 8 : Angular distributions of jets in e^+e^- annihilation measured by TASSO at PETRA, compared with the expected angular distribution for spin-1/2 quarks ($1+\cos^2\theta$).

Fig. 9 : Increase of $\langle p_T^2 \rangle_{\text{in}}$ with the centre-of-mass energy W (the transverse momentum squared in the event plane, with respect to the thrust axis) measured by TASSO at PETRA.

Fig. 10 : (a) The distribution $d\sigma/dF_{\text{minor}}$ in the fraction of the visible energy flow of the entire event which is projected along the minor axis (perpendicular to the event plane); (b) Comparing the data with QCD, and $q\bar{q}$ models, using energy flow diagrams in the thrust-major event plane for events with $0_B > 0.3$, $T_N > 0.98$ or $\theta_{\text{minor}} > 60^\circ$; (c) Same as (b) but with $T_N < 0.98$ and $\theta_{\text{minor}} < 60^\circ$; (d) The unfolded energy flow diagram of (c) compared with the models of QCD, $q\bar{q}$, phase space, and a $q\bar{q}$ model with $\exp(-p_T/650)$ fragmentation distribution [MARK J (61)].

Fig. 11 : Ellis-Karliner test (80) on three-jet events in e^+e^- annihilation (data by TASSO at PETRA). The prediction of the QCD matrix element, with vector gluons, is strongly preferred by the data with respect to a model calculation with scalar gluons.

Fig. 12 : The longitudinal structure function for muon (a) and neutrino (b) deep inelastic scattering on nucleons. In (a) the data by EMC (42) and BCDMS (43) on hydrogen are shown. In (b) the ν data on iron by CDHSW (44) are displayed (with the Q^2 value for each point explicitly shown). The data are compared with the QCD expectation based on Eq. (3.1) (85).

Fig. 13 : Tests of the Gross-Llewellyn Smith and Adler sum rules, [Eqs. (3.3) and (3.2)] by the BEBC Collaboration (88).

Fig. 14 : A collection of gluon densities at $Q^2 = 5 \text{ GeV}^2$ obtained from scaling violations in deep inelastic scattering. The curves a, b, c are taken from Ref. (43), while I am grateful to Dr. M. Diemoz for the addition of curves d and e.

Fig. 15 : Theory and experiment on W/Z production cross-sections, assuming $m_t = 60 \text{ GeV}$ (105).

Fig. 16 : The UAL data (108) on the W P_T -distribution are compared with the QCD theory (107). The Duke-Owens structure functions and the corresponding values of $\Lambda_{\overline{MS}}$ were used.

Fig. 17 : Data (112) on inclusive jet production at large transverse energy E_T measured at the ISR, the SpP5 and the Tevatron compared with the QCD predictions.

Fig. 18 : Angular dependence of jet-jet events in $p\bar{p}$ collisions measured by UAL (113) displaying the Rutherford singularity as $(1-\cos\theta)^2$ expected from gluon exchange.

Fig. 19 : The inclusive cross-section for direct photon production measured by UAL and UA2 compared with the QCD prediction of Ref. (117).

FIGURE 1

FIGURE 2

FIGURE 3

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FIGURE 9

FIGURE 10

FIGURE 12a

FIGURE 12b

FIGURE 11

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FIGURE 15

FIGURE 13

FIGURE 16

FIGURE 18

FIGURE 19

FIGURE 17