UWThPh-1989-15
CERN-TH.5352/89

Radiative Neutrino Masses in an $SU(2) \times U(1)$ Model

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Abstract

We extend the standard model with n_L lepton doublets and n_H Higgs doublets by adding n_R right-handed neutrino singlets with a Yukawa term and a Majorana mass term. Assuming $n_L > n_R$ there are $n_L - n_R$ massless Majorana neutrinos at the tree level of which $n_L - n_R - \max(0, n_L - n_H n_R)$ states acquire a mass by neutral Higgs exchange at the one-loop level. With heavy Majorana masses of the neutrino singlets this model combines the seesaw mechanism and radiative mass production to obtain naturally light neutrino masses. The minimal case $n_L = 3$, $n_R = 1$, $n_H = 2$ is examined in detail including a phenomenological discussion.

CERN-TH.5352/89
April 1989

1 Introduction

Neutrinos interact in a weak and peculiar way. All reactions known so far are compatible with neutrinos existing solely as left-handed states. Therefore it is not yet known if neutrinos are massless or if they possess small Dirac or Majorana masses. In the standard model (SM) with gauge group $SU(2) \times U(1)$ [1] by definition neutrinos appear only as components in left-handed lepton doublets. Then, with the Higgs multiplet being an $SU(2)$ doublet no neutrino mass term can be built and the neutrinos remain massless to all orders in perturbation theory. Thus, in the SM the masslessness of the neutrinos is a consequence of the special choice of lepton and Higgs multiplets. However, there is no fundamental reason to keep them massless and in many extensions of the SM they indeed get a mass in a natural way. Therefore, in general the basic question is rather “Why are neutrino masses so small?” or strictly speaking “Why are neutrino masses so much smaller than the masses of the corresponding charged leptonic partners?” (for bounds on neutrino masses see Ref. [2]). To answer this question two major principles have been applied so far: the seesaw mechanism [3] and radiative production of neutrino masses [4]. In this paper we present a model where both principles are employed.

To construct our model we add right-handed $SU(2) \times U(1)$ neutrino singlets ν_R to the multiplets of the SM. Furthermore we do not demand lepton number conservation. Consequently, two terms not present in the SM have to be added to get the most general Lagrangian compatible with the gauge group:

- A) a Yukawa term with ν_R ,
- B) a ν_R Majorana mass term.

To work out the general features of the model we will keep the numbers of left-handed lepton doublets, right-handed neutrino singlets and Higgs doublets arbitrary for the time being. These numbers will be denoted by n_L , n_R and n_H , respectively. Term B enables us to invoke the seesaw mechanism by assuming that the masses corresponding to it are much higher than the mass of the heaviest charged lepton. Now we come to discuss the effect terms A and B have on neutrino masses thereby anticipating some results of this paper.

Tree level After spontaneous symmetry breaking of the gauge group one obtains a $(n_L + n_R) \times (n_L + n_R)$ Majorana mass matrix for the neutrinos. Now, if $n_L > n_R$ we can make a crucial observation. It has been shown in Ref. [5] that the tree level mass spectrum consists of the following Majorana neutrinos:

- i) n_R heavy states with masses determined by the ν_R mass term,
- ii) n_R light neutrinos because of the seesaw mechanism and
- iii) $n_L - n_R$ massless neutrinos.

The masslessness of the latter states is not protected by any symmetry. Therefore they will get non-zero masses through loop corrections. Furthermore, such masses must be calculable in terms of the tree-level parameters since no counter terms are available. This is the basic point in our discussion.

One-loop level As we will show in this paper category iii) of the massless neutrinos splits up into two parts at the one-loop level:

iiia) $n_L - n_R - n_0$ states with $n_0 = \max(0, n_L - n_H n_R)$ become massive,

iiib) n_0 states remain massless.

Note that in order to have one-loop masses at least $n_H = 2$ Higgs doublets are necessary.

Two-loop level At this order the remaining n_0 states get masses [6,7].

The subject of the present paper is the calculation of the above-mentioned one-loop neutrino masses. For a discussion of the phenomenological consequences we will take the special case with the most economic assumptions $n_L = 3$, $n_R = 1$, $n_H = 2$. It is noteworthy that this case has a natural hierarchy of Majorana neutrino masses: leaving aside the heavy state, among the three light neutrinos one mass is obtained by the seesaw mechanism, the next one appears at the one-loop level and the third one at the two-loop level. We want to emphasize that for this outcome neither exotic fermions nor sophisticated discrete symmetries are necessary. The point is just to have more left-handed lepton doublets than right-handed neutrino singlets.

The paper is organized as follows. In Sect. 2 the model is introduced and the Lagrangian is rewritten in terms of tree-level mass eigenstates. The general formulas for the tree-level and one-loop neutrino masses are discussed in Sect. 3 whereas in Sect. 4 the case $n_L = 3$, $n_R = 1$, $n_H = 2$ is treated in detail. Sect. 5 contains the phenomenological discussion of this case and we summarize our findings in Sect. 6.

2 Description of the Model

We consider an $SU(2) \times U(1)$ gauge theory with an arbitrary number n_H of Higgs doublets

$$\Phi_\alpha = \begin{bmatrix} \Phi_\alpha^+ \\ \Phi_\alpha^0 \end{bmatrix}, \quad 1 \leq \alpha \leq n_H. \quad (2.1)$$

The weak eigenfields of the leptons are denoted by

$$L_i = \begin{bmatrix} \nu_L \\ \ell_L \end{bmatrix}_i, \quad \ell_{Ri}, \quad 1 \leq i \leq n_L,$$

and

$$\nu_{Rj}, \quad 1 \leq j \leq n_R. \quad (2.2)$$

They transform as $L_i \sim (1/2, -1/2)$, $\ell_{Ri} \sim (0, -1)$, $\nu_{Rj} \sim (0, 0)$ with respect to weak isospin and weak hypercharge $(T, Y/2)$.

The Yukawa interaction of leptons and scalar fields is of the general form

$$\mathcal{L}_Y = - \sum_{\alpha=1}^{n_H} (\bar{L} \Gamma_{\alpha} \Phi_{\alpha} \ell_R + \bar{L} \Delta_{\alpha} \tilde{\Phi}_{\alpha} \nu_R) + h.c. \quad (2.3)$$

with $\tilde{\Phi}_{\alpha} = i\sigma_2 \Phi_{\alpha}^*$. Γ_{α} , Δ_{α} are $n_L \times n_L$ and $n_L \times n_R$ matrices, respectively.

The singlet field ν_R permits the construction of an explicit Majorana mass term

$$\mathcal{L}_M = \frac{1}{2} \nu_R^T C^{-1} M_R \nu_R + h.c. \quad (2.4)$$

where the n_R -dimensional matrix M_R is symmetric because of the antisymmetry of the charge conjugation matrix C and the anticommutativity of fermion fields.

The gauge group is broken down to $U(1)_{em}$ by the scalar vacuum expectation values

$$\langle \Phi_{\alpha} \rangle_{vac} = \begin{bmatrix} 0 \\ v_{\alpha}/\sqrt{2} \end{bmatrix}. \quad (2.5)$$

After spontaneous symmetry breaking the lowest order mass term in the neutral lepton sector is given by

$$\mathcal{L}_D + \mathcal{L}_M = -\bar{\nu}_L M_D \nu_R + \frac{1}{2} \nu_R^T C^{-1} M_R \nu_R + h.c. \quad (2.6)$$

with

$$M_D = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^{n_H} v_{\alpha}^* \Delta_{\alpha} \quad (2.7)$$

where $\mathcal{L}_D$ denotes the Dirac part of (2.6). Defining an $n_L + n_R$ component neutrino field

$$\omega_R = \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix} \begin{matrix} \} n_L \\ \} n_R \end{matrix} \quad (2.8)$$

one obtains finally

$$\mathcal{L}_D + \mathcal{L}_M = \frac{1}{2} \omega_R^T C^{-1} M_{\nu} \omega_R + h.c. \quad (2.9)$$

where the matrix $M_{\nu} = M_{\nu}^T$ is given by

$$M_{\nu} = \begin{bmatrix} \overbrace{0}^{n_L} & \overbrace{M_D}^{n_R} \\ \overbrace{M_D^T}^{n_L} & \overbrace{M_R}^{n_R} \end{bmatrix} \begin{matrix} \} n_L \\ \} n_R \end{matrix}. \quad (2.10)$$

To obtain the (tree-level) mass eigenfields $\hat{\omega}_R$ we perform a unitary transformation

$$\omega_R = U^* \hat{\omega}_R \quad (2.11)$$

such that $U^\dagger M_\nu U^* = \hat{M}_\nu$ is a diagonal, positive mass matrix and

$$\mathcal{L}_D + \mathcal{L}_M = \frac{1}{2} \hat{\omega}_R^T C^{-1} \hat{M}_\nu \hat{\omega}_R + h.c. = -\frac{1}{2} \overline{\hat{\omega}_R^c} \hat{M}_\nu \hat{\omega}_R + h.c. = -\frac{1}{2} \bar{N} \hat{M}_\nu N \quad (2.12)$$

with

$$N = \hat{\omega}_R + \hat{\omega}_R^c. \quad (2.13)$$

By an appropriate decomposition of the matrix U

$$U = \left[\begin{array}{c} \overbrace{U_L}^{n_L + n_R} \\ U_R^* \end{array} \right] \begin{array}{l} \} n_L \\ \} n_R \end{array} \quad (2.14)$$

the weak eigenfields $\nu_{L,R}$ can be expressed through the mass eigenfields

$$\begin{aligned} \nu_L &= U_L \hat{\omega}_R^c = U_L P_L N, \\ \nu_R &= U_R \hat{\omega}_R = U_R P_R N \end{aligned} \quad (2.15)$$

with $P_{L,R} = \frac{1 \mp \gamma_5}{2}$.

From (2.3) and (2.5) one obtains the mass matrix of the charged leptons

$$M_\ell = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^{n_H} v_\alpha \Gamma_\alpha. \quad (2.16)$$

Diagonalization of M_ℓ yields the physical lepton fields $\hat{\ell} = \hat{\ell}_L + \hat{\ell}_R$ via the relation

$$\ell_{L,R} = U_{L,R}^\ell \hat{\ell}_{L,R}, \quad (2.17)$$

with

$$U_L^{\ell\dagger} M_\ell U_R^\ell = \hat{M}_\ell$$

and with a diagonal, positive $\hat{M}_\ell$.

Now we can express the couplings of the model in terms of mass eigenfields. The charged current Lagrangian

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \ell_L W_\mu^+ + h.c. \quad (2.18)$$

is written as

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \overline{\hat{\omega}_R^c} \gamma^\mu K \hat{\ell}_L W_\mu^+ + h.c. = -\frac{g}{\sqrt{2}} \bar{N} \gamma^\mu K P_L \hat{\ell} W_\mu^+ + h.c., \quad (2.19)$$

where $K = U_L^\dagger U_L^\ell$ is the $(n_L + n_R) \times n_L$ lepton mixing matrix.

The neutral current Lagrangian

$$\mathcal{L}_{NC} = -\frac{g}{\cos \Theta_W} [(-\frac{1}{2} + \sin^2 \Theta_W) \bar{\ell}_L \gamma^\mu \ell_L + \sin^2 \Theta_W \bar{\ell}_R \gamma^\mu \ell_R + \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L] Z_\mu \quad (2.20)$$

is given by

$$\mathcal{L}_{NC} = -\frac{g}{\cos \Theta_W} [(-\frac{1}{2} + \sin^2 \Theta_W) \bar{\ell} \gamma^\mu P_L \hat{\ell} + \sin^2 \Theta_W \bar{\ell} \gamma^\mu P_R \hat{\ell} + \frac{1}{2} \bar{N} \gamma^\mu U_L^\dagger U_L P_L N] Z_\mu \quad (2.21)$$

in the physical basis.

A charged scalar mass eigenfield Φ_a^+ is a certain linear combination

$$\Phi_a^+ = \sum_{\alpha=1}^{n_H} a_\alpha^* \Phi_\alpha^+ = a^\dagger \Phi^+ \quad (2.22)$$

of the original fields Φ_α^+ . a is an n_H -component unit vector ($\sum_{\alpha=1}^{n_H} |a_\alpha|^2 = 1$). Different fields $\Phi_a, \Phi_{a'}$ are described by vectors a, a' with $a^\dagger a' = 0$. So, the whole set of mass eigenstates can be characterized by a certain orthonormal basis of $\mathbf{C}^{n_H}$. The charged would-be-Goldstone boson

$$\Phi_W^+ = -\frac{i}{v} \sum_{\alpha=1}^{n_H} v_\alpha^* \Phi_\alpha^+ \quad (2.23)$$

corresponds to the vector

$$\frac{i}{v} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n_H} \end{bmatrix} \quad (2.24)$$

with $v^2 = \sum_{\alpha=1}^{n_H} |v_\alpha|^2$. Physical Higgs particles are associated with vectors orthogonal to (2.24). From (2.3) we obtain the Yukawa couplings of Φ_a^+ :

$$\mathcal{L}_Y(\Phi_a^+) = \sum_{\alpha=1}^{n_H} (-\bar{\nu}_L \Gamma_\alpha \ell_R + \bar{\nu}_R \Delta_\alpha^\dagger \ell_L) a_\alpha \Phi_a^+ + h.c. = (-\bar{N} A_R^a P_R \hat{\ell} + \bar{N} A_L^a P_L \hat{\ell}) \Phi_a^+ + h.c. \quad (2.25)$$

with

$$\begin{aligned} A_R^a &= \sum_{\alpha=1}^{n_H} a_\alpha U_L^\dagger \Gamma_\alpha U_R^\ell, \\ A_L^a &= \sum_{\alpha=1}^{n_H} a_\alpha U_R^\dagger \Delta_\alpha^\dagger U_L^\ell. \end{aligned} \quad (2.26)$$

The mass eigenfields of the neutral scalars

$$\Phi_b^0 = \sqrt{2} \sum_{\alpha=1}^{n_H} \text{Re} (b_\alpha^* \Phi_\alpha^0) = \sqrt{2} \text{Re} (b^\dagger \Phi^0) \quad (2.27)$$

are characterized by $2n_H$ unit vectors $b \in \mathbf{C}^{n_H}$. For two different states $\Phi_b^0, \Phi_{b'}^0$, the vectors b, b' have to fulfill the orthogonality condition $\text{Re} (b^\dagger b') = \sum_{\alpha=1}^{n_H} \text{Re} (b_\alpha^* b'_\alpha) = 0$. The neutral would-be-Goldstone boson

$$\Phi_Z = \frac{\sqrt{2}}{v} \sum_{\alpha=1}^{n_H} \text{Im} (v_\alpha^* \Phi_\alpha^0) = \sqrt{2} \sum_{\alpha=1}^{n_H} \text{Re} \left(-\frac{i v_\alpha^*}{v} \Phi_\alpha^0 \right) \quad (2.28)$$

also corresponds to the vector (2.24). Physical Higgs fields Φ_b^0 must be orthogonal to Φ_Z which is expressed by the condition

$$\sum_{\alpha=1}^{n_H} \text{Re} \left(-\frac{iv_\alpha^*}{v} b_\alpha \right) = \frac{1}{v} \sum_{\alpha=1}^{n_H} \text{Im} (v_\alpha^* b_\alpha) = 0. \quad (2.29)$$

The Yukawa interaction of Φ_b^0 is given by

$$\mathcal{L}_Y(\Phi_b^0) = -\frac{1}{\sqrt{2}} \sum_{\alpha=1}^{n_H} (\bar{\ell}_L \Gamma_\alpha \ell_R b_\alpha + \bar{\nu}_L \Delta_\alpha \nu_R b_\alpha^*) \Phi_b^0 + h.c. = -(\bar{\ell} B_\ell^b P_R \hat{\ell} + \bar{N} B_\nu^b P_R N) \Phi_b^0 + h.c. \quad (2.30)$$

with

$$\begin{aligned} B_\ell^b &= \sum_{\alpha=1}^{n_H} U_L^{\ell\dagger} \Gamma_\alpha U_R^\ell \frac{b_\alpha}{\sqrt{2}}, \\ B_\nu^b &= \sum_{\alpha=1}^{n_H} U_L^\dagger \Delta_\alpha U_R \frac{b_\alpha^*}{\sqrt{2}}. \end{aligned} \quad (2.31)$$

3 Neutrino Masses at the Tree and the One-Loop Level

Let us first diagonalize the lowest-order neutrino mass matrix (2.10). We will assume $n_L > n_R$. The $n_R \times n_L$ matrix M_D^T can be regarded as linear mapping $M_D^T : \mathbb{C}^{n_L} \rightarrow \mathbb{C}^{n_R}$. Because of

$$\dim \ker M_D^T = \dim \mathbb{C}^{n_L} - \dim \text{im } M_D^T \geq n_L - n_R, \quad (3.1)$$

one can find in general $n_L - n_R$ orthonormal vectors $u'_i \in \mathbb{C}^{n_L}$ ($1 \leq i \leq n_L - n_R$) such that $M_D^T u'_i = 0$. Then the $(n_L + n_R) \times (n_L + n_R)$ unitary matrix U of (2.11) can be written in the form

$$U = [u_1, \dots, u_{n_L+n_R}] \quad (3.2)$$

with

$$u_i = \begin{bmatrix} u'_i \\ 0 \end{bmatrix} \begin{matrix} \} n_L \\ \} n_R \end{matrix}, \quad 1 \leq i \leq n_L - n_R, \quad (3.3)$$

yielding [5]

$$U^\dagger M_\nu U^* = \begin{bmatrix} \overbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}^{n_L-n_R} \overbrace{\begin{bmatrix} 0 \\ \hat{M}' \end{bmatrix}}^{2n_R} \end{bmatrix} \begin{matrix} \} n_L - n_R \\ \} 2n_R \end{matrix} \quad (3.4)$$

where $\hat{M}'$ is diagonal and positive.

Thus, at the tree-level there are in general $n_L - n_R$ massless Majorana neutrinos and $2n_R$ massive ones. Assuming a heavy mass scale for the elements of M_R and demanding $\det M_R \neq 0$ [8] the seesaw mechanism leads to n_R light massive neutrinos and n_R heavy neutral particles.

By one-loop corrections the form of the neutrino mass matrix is changed to

$$M_\nu^{(1)} = \begin{bmatrix} \delta M & M_D \\ M_D^T & M_R \end{bmatrix} \quad (3.5)$$

with δM being a symmetric $n_L \times n_L$ matrix. In order to diagonalize $M_\nu^{(1)}$ it is convenient to decompose the matrix U of Equ. (3.2) in the following way:

$$U = \begin{bmatrix} \overbrace{U'_L}^{n_L - n_R} & \overbrace{U''_L}^{2n_R} \\ 0 & U''_R \end{bmatrix} \begin{matrix} \} n_L \\ \} n_R \end{matrix}. \quad (3.6)$$

Then we obtain

$$U^\dagger M_\nu^{(1)} U^* = \begin{bmatrix} 0 & 0 \\ 0 & \hat{M}' \end{bmatrix} + \begin{bmatrix} U'_L{}^\dagger \delta M U'_L{}^* & U'_L{}^\dagger \delta M U''_L{}^* \\ U''_L{}^\dagger \delta M U'_L{}^* & U''_L{}^\dagger \delta M U''_L{}^* \end{bmatrix}. \quad (3.7)$$

Up to now, the $n_L - n_R$ orthonormal vectors u'_i in $U'_L = [u'_1, \dots, u'_{n_L - n_R}]$ were only subject to the condition $M_D^T u'_i{}^* = 0$. At this point they can be chosen in such a way that $U'_L{}^\dagger \delta M U'_L{}^*$ is diagonal and positive. The remaining off-diagonal terms in (3.7) can be removed by a unitary transformation V with $V - 1$ being of one-loop order

$$V \simeq 1 + i\Omega, \quad \Omega = \Omega^\dagger \quad (3.8)$$

(we assume non-degeneracy of $\hat{M}'$). Finally we obtain the diagonal and positive matrix

$$\hat{M}_\nu^{(1)} = (UV)^\dagger M_\nu^{(1)} (UV)^* = \begin{bmatrix} U'_L{}^\dagger \delta M U'_L{}^* & 0 \\ 0 & \hat{M}' \end{bmatrix}. \quad (3.9)$$

The explicit form of δM is given by

$$\delta M = \frac{1}{(4\pi)^2} \sum_b \mathcal{M}_b M_R^* \frac{M_b^2 \ln \frac{M_b^2}{M_R M_R^*}}{M_b^2 - M_R M_R^*} \mathcal{M}_b^T + M_D \mathcal{A} M_D^T \quad (3.10)$$

with

$$\mathcal{M}_b = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^{n_H} b_\alpha^* \Delta_\alpha. \quad (3.11)$$

In deriving this formula we have neglected terms suppressed by a factor of order M_D/M_R . The first term in (3.10) is generated by neutral Higgs exchange. The sum in (3.10) runs over all physical states. The second term containing the matrix $\mathcal{A}$ cannot contribute to (3.9) because of $M_D^T U'_L{}^* = 0$. Such a term originates from Z^0 exchange and also from terms in neutral Higgs exchange. For terms which do not depend on M_b^2 one can apply the relation

$$\sum_b b_\alpha^* b_\beta^* = \frac{v_\alpha^* v_\beta^*}{v^2}. \quad (3.12)$$

Equ. (3.12) is thus responsible for the dropping out of the infinity in the one-loop masses of (3.9) and ensures in this way the calculability of these masses in terms of the tree-level parameters of the model. W and charged Higgs exchange give corrections to M_D in the neutrino mass matrix M_ν (2.10) and are not considered here since this would be a higher order effect in (3.9). Again as a consequence of (3.12) the one-loop masses in (3.9) vanish if all neutral scalar masses M_b are equal.

Examining (3.9) and (3.10) in detail one sees that in general (i.e. without any relations among the Yukawa couplings)

$$n_0 = \max(0, n_L - n_H n_R) \quad (3.13)$$

of the $n_L - n_R$ neutrinos with vanishing tree-level mass remain massless also at the one-loop level whereas the remaining $n_L - n_R - n_0$ ones receive radiative masses from (3.10). The validity of (3.13) can be seen as follows. To simplify the proof we can always perform a basis transformation on the Higgs doublets such that only $v_1 \neq 0$ whereas $v_\alpha = 0$ ($2 \leq \alpha \leq n_H$). In the new basis we have $M_D = v_1^* \Delta_1 / \sqrt{2}$ with

$$\dim \ker M_D^T = \dim \ker \Delta_1^T = n_L - n_R.$$

It is clear from (3.9), (3.10) and (3.11) that in the next step we have to study the restriction of Δ_2^T on $\ker \Delta_1^T$, $\Delta_2^T|_{\ker \Delta_1^T}$ with

$$\dim \ker \Delta_2^T|_{\ker \Delta_1^T} = n_L - 2n_R.$$

We continue this procedure until we either reach 0 or $n_L - n_H n_R$ which completes the proof of (3.13).

4 The Special Case $n_L = 3$, $n_R = 1$, $n_H = 2$

In this section we consider the minimal extension of the standard model with a one-loop radiative neutrino mass adding only one right-handed field ν_R to the three left-handed fields contained in ν_L . From (3.13) it is clear that we need at least two Higgs doublets in order to obtain a radiatively induced neutrino mass at the one-loop level. Taking $n_H = 2$ one neutrino will remain massless to this order according to (3.13).

Without loss of generality we are free to choose certain bases for the various fields. By a unitary rotation of the Higgs doublets we can achieve

$$\langle \Phi_1^0 \rangle_{vac} = \frac{v}{\sqrt{2}} > 0, \quad \langle \Phi_2^0 \rangle_{vac} = 0. \quad (4.1)$$

After a suitable phase transformation acting on ν_R we have

$$M_R > 0. \quad (4.2)$$

A basis transformation on ν_L can be performed in such a way that

$$\begin{aligned}\Delta_1 &= \frac{\sqrt{2}m_D}{v} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & m_D > 0, \\ \Delta_2 &= \begin{bmatrix} 0 \\ d \\ d' \end{bmatrix}, & d > 0.\end{aligned}\tag{4.3}$$

Finally, we have the freedom to choose $\ell_R = \hat{\ell}_R$ which implies $U_L^{\ell\dagger} M_\ell = \hat{M}_\ell$ for the diagonalization of the charged lepton mass matrix $M_\ell = v\Gamma_1/\sqrt{2}$.

The tree-level neutrino mass matrix (2.10) is now given by

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_D \\ 0 & 0 & m_D & M_R \end{bmatrix}\tag{4.4}$$

with mass eigenvalues

$$m_3 = \sqrt{\frac{M_R^2}{4} + m_D^2} - \frac{M_R}{2} \simeq \frac{m_D^2}{M_R} \quad \text{and} \quad m_4 = \sqrt{\frac{M_R^2}{4} + m_D^2} + \frac{M_R}{2} \simeq M_R\tag{4.5}$$

for $m_D \ll M_R$.

The one-loop neutrino mass matrix $M_\nu^{(1)}$ of (3.5) is determined by the 3×3 submatrix

$$\delta M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu d^2 & \mu' d \\ 0 & \mu' d & 0 \end{bmatrix} + M_D \mathcal{A} M_D^T\tag{4.6}$$

with

$$\begin{aligned}\mu &= \frac{M_R}{32\pi^2} \sum_b \frac{\ln(M_R^2/M_b^2)}{M_R^2/M_b^2 - 1} (b_2^*)^2, \\ \mu' &= \frac{M_R}{32\pi^2} \sum_b \frac{\ln(M_R^2/M_b^2)}{M_R^2/M_b^2 - 1} (b_1^* \frac{\sqrt{2}m_D}{v} + b_2^* d') b_2^*.\end{aligned}\tag{4.7}$$

We follow the diagonalization procedure described in Sect. 3. The unitary matrix U (3.2) is given by

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\varphi} & 0 & 0 \\ 0 & 0 & i \cos \vartheta & \sin \vartheta \\ 0 & 0 & -i \sin \vartheta & \cos \vartheta \end{bmatrix}\tag{4.8}$$

with $2\varphi = \arg(\mu)$ and $\sin \vartheta = \sqrt{\frac{m_3}{m_3 + m_4}} \simeq \frac{m_D}{M_R}$, $\cos \vartheta > 0$. After the application of U , a further transformation V of the form (3.8) finally yields

$$\hat{M}_\nu^{(1)} = \text{diag}(0, |\mu|d^2, m_3, m_4).\tag{4.9}$$

So we have four Majorana neutrinos N_i ($1 \leq i \leq 4$) with a natural mass hierarchy $m_1 < m_2 < m_3 < m_4$. The mass of N_4 originates from some large energy scale $\sim M_R$. N_3 gets a mass $m_3 \simeq m_D^2/M_R$ through the seesaw mechanism. N_2 obtains a radiatively induced mass $m_2 = |\mu|d^2$ at the one-loop level. N_1 , which is still massless at this stage, will in general receive a mass from two-loop corrections.

To conclude this section we want to list the coupling matrices at lowest order¹ which appear in the various parts of the Lagrangian as discussed in Sect. 2. The matrix U_L^ℓ is free for the time being but the order of magnitudes of its elements will be fixed by plausible arguments in the next section. The same will be the case for

$$U_L^{\ell\dagger}\Gamma_2 \equiv \Gamma'_2 \quad \text{and} \quad U_L^{\ell\dagger}\Delta_\alpha \equiv \Delta'_\alpha, \quad \alpha = 1, 2. \quad (4.10)$$

With (2.14) and (4.8) the lepton mixing matrix in (2.19) is given by

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\varphi} & 0 \\ 0 & 0 & -i \cos \vartheta \\ 0 & 0 & \sin \vartheta \end{bmatrix} U_L^\ell. \quad (4.11)$$

In the neutral current the neutrino couplings are determined by (2.21)

$$U_L^\dagger U_L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos^2 \vartheta & -i \cos \vartheta \sin \vartheta \\ 0 & 0 & i \cos \vartheta \sin \vartheta & \sin^2 \vartheta \end{bmatrix}. \quad (4.12)$$

With $a_1 = 0$, $a_2 = 1$ and using (4.8) and the definitions (4.10) the couplings (2.26) of the charged physical Higgs are given by

$$A_R = K\Gamma'_2, \quad A_L = \begin{bmatrix} 0 \\ 0 \\ -i \sin \vartheta \\ \cos \vartheta \end{bmatrix} \Delta_2'^\dagger. \quad (4.13)$$

Finally we obtain the couplings of the neutral physical scalars as

$$\begin{aligned} B_\ell^b &= \frac{1}{v} \hat{M}_\ell b_1 + \frac{\Gamma'_2 b_2}{\sqrt{2}} \\ B_\nu^b &= \sum_{\alpha=1}^2 K \Delta'_\alpha [0, 0, i \sin \vartheta, \cos \vartheta] \frac{b_\alpha^*}{\sqrt{2}}. \end{aligned} \quad (4.14)$$

5 Phenomenology

Here we discuss the most important phenomenological features of the special case $n_L = 3$, $n_R = 1$, $n_H = 2$ considered in Sect. 4. To this end we necessarily have to make assumptions

¹This means that we set $V = 1$.

about the orders of magnitude of the Yukawa couplings. The masses of the charged leptons are of very different orders of magnitude thus introducing a large hierarchy into those Yukawa couplings which are responsible for the mass matrix of the charged leptons:

$$U_L^{\ell\dagger}\Gamma_1 = \sqrt{2}\hat{M}_\ell/v. \quad (5.1)$$

It is therefore reasonable to assume that the rest of the Yukawa couplings conforms to the same hierarchy. This leads us to postulate that an order-of-magnitude estimate in the basis where the mass matrix of the charged leptons is diagonal is given by

$$\begin{aligned} (U_L^{\ell\dagger}\Gamma_2)_{ij} &= (\Gamma'_2)_{ij} \sim \frac{\sqrt{2m_{\ell i}m_{\ell j}}}{v}, \\ (U_L^{\ell\dagger}\Delta_\alpha)_i &= (\Delta'_\alpha)_i \sim \frac{\sqrt{2}m_{\ell i}}{v}, \quad \alpha = 1, 2. \end{aligned} \quad (5.2)$$

This will be the basis of the following phenomenological discussion.

From (4.3) it is easy to derive that the assumptions (5.2) fix the orders of magnitude of the elements of U_L^ℓ :

$$U_L^\ell \sim \begin{bmatrix} 1 & m_e/m_\mu & m_e/m_\tau \\ m_e/m_\mu & 1 & m_\mu/m_\tau \\ m_e/m_\tau & m_\mu/m_\tau & 1 \end{bmatrix}. \quad (5.3)$$

Let us first discuss the neutrino masses in the light of (5.2). We assume that the light neutrinos obey the cosmological bound [9] $\sum m_\nu < 100$ eV where the sum goes over the light degrees of freedom (see also Refs. in [10]). Using (5.3) it is evident that $m_D \sim m_\tau$ and therefore we get a lower bound on the right-handed scale of order

$$m_4 \simeq M_R \gtrsim 10^4 \text{ TeV} \quad (5.4)$$

corresponding to $m_3 \sim 100$ eV. With $d \sim \sqrt{2}m_\mu/v$ (see (5.2) and (5.3)) the one-loop neutrino mass is estimated by

$$m_2 \sim \frac{1}{8\pi^2} \left(\frac{M_b}{v}\right)^2 \frac{m_\mu^2}{M_R} \ln \frac{M_R}{M_b}. \quad (5.5)$$

In deriving (5.5) we have assumed $M_b \ll M_R$ which is reasonable in view of the symmetry breaking scale $v \simeq 246$ GeV. Taking $M_R = 10^4$ TeV and $M_b = v$ we obtain $m_2 \sim 0.1$ eV which can be regarded as a sort of upper bound on m_2 on the basis of the assumptions (5.2). m_1 is zero at our level of sophistication.

Since there is no lepton number in the present model lepton flavour changing processes are allowed. The branching ratios of the decays $\mu \rightarrow e\gamma$ and $\mu \rightarrow e^+e^-e$ have the most stringent experimental bounds [2]. Using the couplings of Sect. 4 it turns out that with (5.2) the charged Higgs loop dominates over the W loop in $\mu \rightarrow e\gamma$ [11,12] leading to a very mild lower bound on the charged Higgs mass in the GeV range. The decay $\mu \rightarrow e^+e^-e$ through neutral physical Higgs scalars at the tree level is far below the experimental bound with the assumption for Γ'_2 (5.2).

In the present model only the lightest neutrino N_1 is stable. The other light ones can decay like $N_i \rightarrow N_j + \gamma$ ($i > j$) [13]. However, despite of this the cosmological bound for stable neutrinos can hardly be evaded. Only if m_3 is close to its experimental upper bound of 35 MeV [2] is there a slight possibility for N_3 to decay fast enough via a charged Higgs loop (see [10] for a summary on cosmological bounds for unstable neutrinos). Note that there is no tree level decay $N_i \rightarrow N_j + N_k + N_\ell$ ($i = 2, 3$) through Z^0 or Φ_b^0 exchange as can be seen from (4.12) and (4.14).

Neutrino oscillations of the three light degrees of freedom are governed by the matrix U_L^ℓ . The orders of magnitude can be read off from (5.3). The largest mixing can occur between the μ and the τ neutrino. Because of the smallness of m_1, m_2 there are no constraints from reactor and accelerator experiments [12]. With the mixing angle Θ between the first and second neutrino generation being of order m_e/m_μ and with $m_2 \sim 10^{-2}$ eV the model lies just at the edge in the $\Delta m^2 - \sin^2(2\Theta)$ plane for having a sizable MSW effect [14] for solar neutrinos [15].

6 Conclusions

In this paper we have considered a model which combines the seesaw mechanism and radiative mass production to generate small Majorana neutrino masses. Starting from the SM with n_L left-handed lepton doublets and n_H Higgs doublets we have added n_R neutrino gauge singlets with a Majorana mass term. These singlets also give rise to a Yukawa term. The point in our discussion is to assume $n_L > n_R$. Then, with a heavy scale for the Majorana masses there are n_R heavy neutral leptons, n_R neutrinos which are light because of the seesaw mechanism and $n_L - n_R$ massless neutrinos at the tree level [5]. We have shown in this paper that at the one-loop level $n_L - n_R - n_0$ with $n_0 = \max(0, n_L - n_H n_R)$ of these massless states become massive with naturally small and calculable masses which are generated by the exchange of neutral physical Higgs scalars. The remaining n_0 massless neutrinos become massive at the two-loop level [6,7]. For the one-loop mechanism to work we need at least $n_H = 2$ Higgs doublets. However, we want to stress at this point that the interesting and attractive feature of this model is the absence of exotic fermions or bosons or contrived horizontal symmetries. It is well conceivable that the right-handed singlets occur in some larger multiplets of a grand unified theory and that the above-mentioned Majorana mass term arises quite naturally with a large scale in such a theory. Thus we expect our model to show rather general features.

The minimal case of the model is represented by $n_L = 3, n_R = 1, n_H = 2$. We have discussed this case in detail. As a basis for a phenomenological discussion we have extended the hierarchy of the charged lepton masses to the Yukawa couplings in a plausible way and we have assumed the light neutrino masses to obey the cosmological bound for stable neutrinos. We have seen in this spirit that the minimal case is in accord with experimental constraints and that the right-handed scale is of order $M_R \gtrsim 10^4$ TeV. Then the third generation neutrino has a mass of order $m_3 \lesssim 100$ eV through the seesaw mechanism, $m_2 \lesssim 0.1$ eV in the second generation is produced at the one-loop level and

$m_1 = 0$ at this level of accuracy. However, neutrino mixing is rather small with the above phenomenological assumptions and hard to detect. Also the MSW mechanism for solar neutrinos [14] is hard to realize [15]. The largest mixing occurs between the second and third generation with a mixing angle of order m_μ/m_τ .

As a concluding remark we want to point out the simplicity of the basic assumptions in our model and we stress once more that no exotic particles are needed in this scheme to generate naturally light neutrino masses. It is rather the interplay between two mechanisms, the seesaw mechanism and radiative mass production, which leads to this result.

Acknowledgement

It's a pleasure for us to thank S.T. Petcov for discussions. Furthermore we are grateful to G. Ecker for a careful reading of the manuscript.

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