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BRST QUANTIZATION, $IOSp(D,2|2)$ INVARIANCE AND THE PCT THEOREM: II

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ABSTRACT

In this paper we investigate the $IOSp(D,2|2)$ extension of the Poincaré group in the *BRST*-quantization of the *spinning particle*. We construct the corresponding field theory and prove its dimensional reduction à la Parisi-Sourlas. Like for the scalar particle, we show that the PCT transformation is induced by a certain element of the $SO(D,2)$ subgroup of the $IOSp(D,2|2)$. This gives us the tools for a simple interpretation of the PCT theorem for Dirac fields. We then continue by clarifying the role of the "modular" transformation (i.e. world-line orientation reversing diffeomorphism) and indicating how the PCT-transformation could be regarded as a modular transformation in a " $SO(D,2)$ -rotated" frame. In this work we also consider *BRST*-field theories leading to D-dimensional chiral theories. In this case, of the full $IOSp(D,2|2)$ symmetry, only the subgroup $IO(D-1,1) \otimes IOSp(1,1|2)$ survives together with the (isolated) $SO(D,2)$ -transformation that implements PCT.

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1. INTRODUCTION

In a previous paper [1], henceforth referred to as (I), we presented a detailed analysis of the $IOSp(D,2|2)$ symmetry found by Neveu and West [2, 3] in the $BRST$ -field theory of the relativistic point particle [4 – 13] and the bosonic string [14 – 18]. We showed that the $IOSp(D,2|2)$ symmetry automatically leads to PCT-invariance, microcausality and the causal propagator in the physical subspace of the D -dimensional field theory. The projection on this subspace is implemented through the Parisi-Sourlas [19] mechanism. The Lagrange multiplier of the reparametrization constraint, the proper time and the associated Fradkin-Batalin-Vilkovisky [20, 21] ghosts, enter the $BRST$ -field theory in such a way that the Poincaré group $IO(D-1,1)$ gets extended to the supergroup $IOSp(D,2|2)$. The Parisi-Sourlas mechanism guarantees that the effect of the extra two bosonic coordinates is compensated for by the two fermionic variables. In (I) an element of the $SO(D,2)$ -subgroup of $IOSp(D,2|2)$ was identified which realizes the PCT transformation as a continuous rotation in the extended bosonic space.

The reader of (I) might have been puzzled why a proper treatment of the $BRST$ quantization led to PCT, microcausality and so on. The deep reason is simple: $BRST$ -invariance is there when we want to make the *Lorentz*-covariance of the quantized theory manifest (covariant quantization), and because we introduced the ghosts to exponentiate the *non-local* Faddeev-Popov determinant into a *local* action. Thanks to these ghosts the *non-local* Slavnov symmetry became the *local* $BRST$ -symmetry. So $BRST$ -invariance is a "bonus" we get when we insist on having explicit, at the quantum level, *Lorentz covariance* and *locality*.^{*} These two, as it is well-known [22] are the crucial ingredients to prove all the fundamental theorems of quantum field theory: PCT, spin & statistics, and all that. So it is not a surprise or a puzzle that the $BRST$ invariance, which somehow encodes the above-mentioned two properties, will help in proving those theorems.

In (I) we also presented two independent arguments of why the $BRST$ -field theory uniquely leads to the causal or Feynman boundary conditions for the propagator: one relies on the $SO(D,2)$ symmetry and the other on the modular invariance [23]. For the point particle a modular transformation simply consists of an orientation-reversing diffeomorphism of the world-line. It is important, however, to respect this discrete symmetry when going from first to second quantization. In (I) these points were discussed for the neutral scalar particle. It is the purpose of the present paper to give a more detailed account

^{*} The locality we get with the ghosts is at the world-sheet level, but it enforces locality also at the space-time level in second quantization.

of how one can implement modular invariance in second quantization and to generalize the results to the spinning particle.

The paper is organized as follows. In section (2) we discuss various possibilities to implement modular invariance at the second quantized level. Section (3) contains the $IOSp(D,2|2)$ -invariant $BRST$ quantization of the spinning particle with particular emphasis on the modular symmetry. The dimensional reduction of the resulting Parisi-Sourlas field theory of Dirac fermions is performed in section (4). In section (5) we show that, for QCD-type Parisi-Sourlas theories, the PCT-transformation in the physical subspace again can be obtained by a special $SO(D,2)$ -rotation. We also show that the PCT-transformation is essentially a modular transformation "seen" in an $SO(D,2)$ -rotated frame. In section (6) we demonstrate that there exists no $OSp(D,2|2)$ -invariant generalization of the usual γ_5 -matrix. Hence D-dimensional theories containing γ_5 couplings, in particular parity-violating theories, cannot be obtained as dimensionally-reduced $OSp(D,2|2)$ -invariant theories. Interaction terms containing γ_5 break $OSp(D,2|2)$ down to a certain subgroup containing the Lorentz group $O(D-1,1)$, the smaller supergroup $OSp(1,1|2)$ (which is responsible for the dimensional reduction) and the special element of $SO(D,2)$ corresponding to the PCT-transformation. In the same section we will also discuss in some detail the interplay of PCT-invariance, modular-invariance, $SO(D,2)$ -symmetry and dimensional reduction. Finally, section (7) contains some conclusions. Some technical parts are confined to the appendices.

2. IMPLEMENTATION OF MODULAR INVARIANCE

In this section we discuss various possibilities to meet the requirement of modular invariance at the level of the second quantized $BRST$ field theory. We recall that for a relativistic point particle a modular transformation simply consists of an orientation reversing diffeomorphism of the world line [22]. For the special case of a *neutral* scalar field this point has been discussed in (I) already, and we will not repeat it here again. Let us now look at a *charged* spinless particle interacting with an external electromagnetic field A_μ . Then the Hamiltonian form of the action is given by

$$S_1 = - \int_{\tau_1}^{\tau_2} d\tau \left\{ \dot{X}^\mu P_\mu - \lambda(\tau) \phi(\tau) \right\} \quad (2.1)$$

with the constraint

$$\phi(\tau) \equiv (P_\mu(\tau) - A_\mu(X(\tau)))^2 - m^2 \quad (2.2)$$

The action S_1 is invariant under the infinitesimal diffeomorphisms generated by ϕ

$$\begin{aligned}\delta F &= [F, \varepsilon(\tau)\phi(\tau)] \\ \delta \lambda(\tau) &= \dot{\varepsilon}(\tau)\end{aligned}\tag{2.3}$$

where F is any phase space function, and $\varepsilon(\tau)$ has to vanish [6] at the end-points of the trajectory $X^\mu(\tau)$, $\tau \in [\tau_1, \tau_2]$:

$$\varepsilon(\tau_1) = \varepsilon(\tau_2) = 0\tag{2.4}$$

From (2.1) we obtain the equations of motion

$$P^\mu - eA^\mu = \frac{\dot{X}^\mu}{2\lambda}\tag{2.5}$$

$$\frac{d}{d\tau}(P^\mu - eA^\mu) = eF^\mu_\nu \dot{X}^\nu\tag{2.6}$$

In the proper-time gauge^{*}, $\dot{\lambda} = 0$, they lead to

$$\ddot{X}^\mu = 2\lambda eF^\mu_\nu \dot{X}^\nu\tag{2.7}$$

Besides the diffeomorphisms (2.3), the equations of motion (2.5), (2.6), (2.7) are form-invariant under the following orientation-reversing transformations

$$\begin{aligned}X^{\mu'}(\tau) &= X^\mu(-\tau) \\ \lambda'(\tau) &= -\lambda(-\tau)\end{aligned}\tag{2.8}$$

made together with an exchange of the endpoints. (S_1 changes by a sign under this transformation). Eq. (2.8) can be addressed as a modular transformation, since it can not be obtained by iterating the transformations (2.3) starting from the identity. As was extensively discussed in ref. [23], the modular group is $\mathbf{Z}_2$, and its non-trivial element acts on the Teichmüller parameter

$$C = \int_{\tau_1}^{\tau_2} d\tau \lambda(\tau)\tag{2.9}$$

as $C \rightarrow -C$. Hence the Teichmüller space is the full real line whereas the modular space consists only of the (positive) real half line. These conclusions remain valid for $A_\mu \neq 0$.

* This is a "good" [23] gauge even for $A^\mu \neq 0$ [5].

In connection with the second order equation (2.7) two more discrete transformations are interesting. We observe that both the transformations

$$\begin{aligned} X^{\mu'}(\tau) &= X^\mu(\tau) \\ \lambda' &= -\lambda \end{aligned} \quad (2.10)$$

and

$$\begin{aligned} X^{\mu'}(\tau) &= X^\mu(-\tau) \\ \lambda' &= \lambda \end{aligned} \quad (2.11)$$

transform the equation (2.7) into

$$\ddot{X}^\mu = 2\lambda(-e)F_\nu^\mu \dot{X}^\nu \quad (2.12)$$

Obviously (2.10) or (2.11) correspond to a charge conjugation. They change the charge of the particle from $(+e)$ to $(-e)$. They are symmetries of the equation of motion only in the special case $e = 0$ or, equivalently, $A_\mu = 0$ [†]. Now let us look at the corresponding transformations in the quantized theory. The Schrödinger equation for the wave function $\Psi = \Psi(X^\mu, p_+, \tau, \tilde{x}_a)$ is obtained by requiring stationarity of the action

$$S_2 = i \int d^D x \int_{-\infty}^{\infty} dp_+ \int_{-\infty}^{\infty} d\tau \int d\tilde{x}_1 d\tilde{x}_2 \Psi^* [2ip_+ \partial_\tau + (\partial_\mu + ieA_\mu)^2 + m^2 + 2i\partial_{\tilde{x}_1} \partial_{\tilde{x}_2}] \Psi \quad (2.13)$$

For the notation and the details of the derivation we refer to (I) and [24]. Suffice it to say that S_2 is the *BRST*-invariant action resulting from the gauge fixing $\dot{\lambda} = 0$.

Since $p_+ \equiv \lambda^{-1}$ is integrated over the whole real line, the modular symmetry $\mathbf{Z}_2$ has not yet been "divided out" (cf. later.). We now turn to the quantum versions of the transformations (2.9), (2.10) and (2.11). In terms of the Schrödinger wave function Ψ the modular transformation (2.8) has the representation

$$\Psi'(X^\mu, p_+, \tau, \tilde{x}_a) = \Psi(X^\mu, -p_+, -\tau, \tilde{x}_a) \quad (2.14)$$

The above transformation leaves the action S_2 invariant. To be more precise, a true modular transformation is a product of two factors, one acting on the bosonic coordinates,

[†] The transformations (2.10) or (2.11) could be promoted to symmetries, even for $A_\mu \neq 0$, if one would include a sign change $A_\mu \rightarrow -A_\mu$ in the transformation rules. Here we consider A^μ a fixed background, however.

the other on the ghosts (see the forthcoming[25] for more details). It is easily seen that both the Batalin-Fradkin-Vilkovisky effective action of first quantization and the action S_2 of second quantization are separately invariant under both factors. Since in this paper we are mainly interested in the interplay between (p_+, τ) on one side and the space-time coordinates on the other, we shall not indicate the ghost transformations in the following; by "modular transformation" we always shall mean its bosonic part only. The charge conjugation transformations (2.10) and (2.11) are analogously represented by

$$\Psi'(X^\mu, p_+, \tau, \tilde{x}_a) = \Psi^*(X^\mu, -p_+, \tau, \tilde{x}_a) \quad (2.15)$$

and

$$\Psi'(X^\mu, p_+, \tau, \tilde{x}_a) = \Psi^*(X^\mu, p_+, -\tau, \tilde{x}_a) \quad (2.16)$$

respectively*. For $A_\mu \neq 0$ the latter transformations do not leave the action invariant. S_2 can be made invariant, however, by simultaneously changing the sign of A_μ .

To set up a (second)-quantized field theory with (2.13) as its action one has to carefully specify the functional space over which we integrate in path-integrals like

$$Z[J, J^*] = \int [d\Psi][d\Psi^*] \exp \left\{ iS_2[\Psi, \Psi^*] + i \int J^* \Psi + \Psi^* J \right\} \quad (2.17)$$

To arrive at (2.17) we already have gauge fixed the "small" diffeomorphisms (2.3) connected to the identity, but the "large" diffeomorphisms, i.e., the modular transformations (2.14) are not yet divided out.

This can be done in various ways:

(i) One restricts the functional integration to one fundamental domain of the modular group. In practice this means that we integrate over functions $\Psi(X^\mu, p_+, \tau, \tilde{x}_a)$ where $\tau \in \mathbb{R}$, but $p_+ \in [0, \infty)$ assumes only positive values. Correspondingly, the p_+ -integration in the action (2.13) is restricted to the half-line $[0, \infty)$.

(ii) One integrates over both fundamental domains, i.e., over $\Psi(X^\mu, p_+, \tau, \tilde{x}_a)$ and $\Psi(X^\mu, -p_+, -\tau, \tilde{x}_a)$ independently, but makes sure that they give rise to equal contributions to the generating functional. As compared to (i) we obtain an additional factor of 2 for each fixed value of $p_+ > 0$. These factors cancel against the corresponding factors

* An arbitrary phase factor could be included in these transformations, but for simplicity we shall put it equal to one.

coming from the normalization (which is not written explicitly in (2.17)) and they may be discarded.

(iii) one integrates over modular invariant field configurations only, i.e., over fields $\Psi(X^\mu, p_+, \tau, \tilde{x}_a)$, $p_+ \in \mathfrak{R}$, $\tau \in \mathfrak{R}$, satisfying the constraint

$$\Psi(X^\mu, p_+, \tau, \tilde{x}_a) = \Psi(X^\mu, -p_+, -\tau, \tilde{x}^a) \quad (2.18)$$

(If we "solve" the constraint by restricting p_+ to positive values, we are led back to (i).)

The different possibilities lead to the following equivalent versions of the resulting quantum field theory.

(i) The action of the theory is given by

$$S_2 = i \int d^D x \int_0^\infty dp_+ \int_{-\infty}^\infty d\tau \int d\tilde{x}_1 d\tilde{x}_2 \Psi^* \left[2ip_+ \partial_\tau + (\partial_\mu + ieA_\mu)^2 + m^2 + 2i\partial_{\tilde{x}_1} \partial_{\tilde{x}_2} \right] \Psi \quad (2.19)$$

It has no manifest $OSp(1,1|2)$ or $OSp(D,2|2)$ symmetry. It is *BRST*-invariant however, and therefore the correct projection on the physical subspace is obtained. For the physical modes one obtains the standard action of a charged scalar field in D-dimensions coupled to a background field (more details will be given in section (4)). If $A_\mu \equiv 0$ one can follow Neveu and West [2], [3], who started from (2.19) and *defined* the wave-function for negative values of p_+ by

$$\Psi(X^\mu, -p_+, \tau, \tilde{x}_a) = \Psi^*(X^\mu, p_+, \tau, \tilde{x}_a) \quad (2.20)$$

Due to this condition the Fourier-transform of Ψ w.r.t. p_+ , i.e. $\Phi(X^\mu, x_+, x_-, \tilde{x}_a)$, becomes real.[†] Re-expressing S_2 in terms of Φ one obtains a manifestly $OSp(D,2|2)$ -invariant action for the real superfield Φ . This was the formulation used in (I). It clearly fails for charged fields since, comparing (2.20) to (2.15), we see that (2.20) just expresses the invariance of Ψ under charge conjugation. Stated differently, (2.16) is a symmetry of S_2 only for $e=0$ or $A^\mu = 0$, and hence it is only in this case that (2.20) may be used to extend the p_+ -integration in (2.19).

[†] Recall from (I) that $x_+ \equiv \tau$ and x_- is conjugate to p_+ .

(ii) In the second case we can always replace the integration variables $\Psi(X^\mu, p_+, \tau \equiv x_+, \tilde{x}_a)$ by their *complex* Fourier-transforms

$$\Phi(X^\mu, x_+, x_-, \tilde{x}_a) = \int_{-\infty}^{\infty} \frac{dp_+}{2\pi} e^{ip_+ x_-} \Psi(X^\mu, p_+, x_+, \tilde{x}_a) \quad (2.21)$$

In terms of Φ the modular transformation (2.14) reads

$$\Phi'(X^\mu, x_+, x_-, \tilde{x}_a) = \Phi(X^\mu, -x_+, -x_-, \tilde{x}_a) \quad (2.22)$$

and the generating functional becomes

$$Z[J, J^*] = \int [d\Phi][d\Phi^*] \exp \frac{i}{2\pi} \int d^{D+4}x \left\{ \Phi^* [\square_{OSp} + \dots] \Phi + J^* \Phi + \Phi^* J \right\} \quad (2.23)$$

where the $\dots$ denote terms containing A_μ and $\square_{OSp}$ is the $OSp(D, 2|2)$ -invariant Laplacian (see (I) for details of the notation). Since the restriction of the functional integration to a single fundamental domain of (2.22), say to $\Phi(x_+ \in \mathbb{R}, x_- \in [0, \infty))$, would destroy the $OSp(D, 2|2)$ symmetry^b, we would like to keep $\Phi(x_+, x_-)$ and $\Phi(-x_+, -x_-)$ as independent integration variables. To make sure that the (x_+, x_-) and $(-x_+, -x_-)$ sectors contribute equally to Z , it is not sufficient to observe that the action entering (2.23) is modular invariant. As it stands, the path-integral (2.23) is purely formal and we have to give a well-defined meaning to it. As long as there is no interaction term included (which would have to be modular invariant as well, of course) the specification of (2.23) is equivalent to specifying the propagator of the theory:

$$\begin{aligned} G_{p_+} (X^{\mu'} - X^\mu, \tau' - \tau, \tilde{x}'_a - \tilde{x}_a) &= \\ &= \langle X^{\mu'}, \tau', \tilde{x}'_a | \frac{1}{2ip_+ \partial_\tau + \partial_\mu^2 + (m^2 - i\epsilon) + 2i\partial_{\tilde{x}_1} \partial_{\tilde{x}_2}} | X^\mu, \tau, \tilde{x}_a \rangle \end{aligned} \quad (2.24)$$

For simplicity we put $A_\mu \equiv 0$ and went back to the (p_+, τ) -notation. As already indicated, the only propagator consistent with modular invariance is obtained by Feynman's

^b Clearly, considering only a D-component vector field $A_\mu(x'')$ the action cannot be made fully $OSp(D, 2|2)$ invariant. However, in section (4) we shall extend $A_\mu(X'')$ to a $(D+4)$ -dimensional field $A_D(X^C)$ giving rise to a completely $OSp(D, 2|2)$ invariant generalization of (2.23).

prescription $m^2 \rightarrow (m^2 - i\epsilon)$. Explicit evaluation of (2.24)[◊] shows that

$$G_{p_+} \propto p_+^{-1} [\theta(\tau)\theta(p_+) - \theta(-\tau)\theta(-p_+)] \quad (2.25)$$

and hence

$$G_{p_+}(X^\mu, \tau, \tilde{x}_a) = G_{-p_+}(X^\mu, -\tau, \tilde{x}_a) \quad (2.26)$$

i.e. the symmetry between (p_+, τ) and $(-p_+, -\tau)$ is indeed respected by (2.26). The same conclusion holds if $A_\mu \neq 0$. On the other hand, retarded boundary conditions, for instance, lead to

$$G_{p_+} \propto p_+^{-1} \theta(\tau)$$

and hence the symmetry (2.26) is violated. To conclude, method (ii) allows for a $OSp(D, 2|2)$ -invariant form of the BRST-field theory also for complex fields provided we use the modular-invariant Feynman boundary conditions.

(iii) If one is interested in neutral fields only, there is an interesting possibility to deal with the constraint (2.18) appearing in the third method. One requires the integration variables to fulfil an auxiliary condition of the form

$$\Psi(X^\mu, p_+, \tau, \tilde{x}_a) = \Psi^*(X^\mu, p_+, -\tau, \tilde{x}_a) \quad (2.27)$$

In view of (2.16) this means that Ψ is invariant under charge conjugation, *i.e.* it describes a neutral particle. Combining (2.18) and (2.27) one obtains the constraint

$$\Psi(X^\mu, -p_+, \tau, \tilde{x}_a) = \Psi^*(X^\mu, p_+, \tau, \tilde{x}_a) \quad (2.28)$$

which makes $\Phi(X^\mu, x_+, x_-, \tilde{x}_a)$ real.^{*} Obviously (2.28) is identical to the relation (2.20) used by Neveu and West to extend the p_+ -integration in (2.19), but here (2.28) has the physical meaning of a modular invariance condition for fields fulfilling the "Majorana" condition (2.27) rather than just being a convenient definition. It is this approach to the neutral scalar particle which has been used in (I).

After minor changes the discussion given here for the scalar particle also applies to the spinning particle, to which we shall come in the following section. Since we are considering charged Dirac particles, we have to use the second method, because it is the only one allowing for complex fields and $OSp(D, 2|2)$ -invariance at the same time.

◊ See equations (5.8) and (5.9) of (I).

* The condition (2.27) has been studied by Henty et al.[26] for the spinning particle where it leads to Majorana spinors. Note also that since both (2.15) and (2.16) represent charge conjugation, in principle we could also interchange the roles played by p_+ and τ in (2.28) and (2.27).

3. $IOSp(D,2|2)$ FOR DIRAC FERMIONS

In this section we describe the origin of the $IOSp(D,2|2)$ - symmetry of the free spinning particle. We closely follow the work of Barducci et al. [10] , to which we refer for further details. We only mention the main points to the extent they are needed in the following discussion.

The massive spinning particle in D-dimensional Minkowski-space is described by the commuting position variables $X^\mu(\tau)$, $\mu = 0 \cdots D-1$ and $(D+1)$ anticommuting variables $\xi^\mu(\tau)$ and $\xi_5(\tau)$ [27] . The theory is defined by the first class constraints:

$$\phi_1 \equiv P_\mu^2 - m^2 \quad (3.1)$$

$$\phi_2 \equiv P_\mu \xi^\mu - m \xi_5 \quad (3.2)$$

Hence the Hamiltonian form of the action is

$$S_1 = \int_{\tau_1}^{\tau_2} d\tau \left[-P_\mu \dot{X}^\mu - \frac{i}{2} \{ \xi_\mu \dot{\xi}^\mu - \xi_5 \dot{\xi}_5 \} - \lambda_1 \phi_1 - \lambda_2 \phi_2 \right] \quad (3.3)$$

where $\lambda_1(\tau)$ is a commuting Lagrange multiplier (the same as the one appearing in the action of the spinless particle) and $\lambda_2(\tau)$ is an anticommuting one. The action is invariant under the gauge transformations generated by the constraints:

$$\begin{aligned} \delta F &= \left[F, \epsilon_1 \phi_1 + \epsilon_2 \phi_2 \right] \\ \delta \lambda_1 &= \dot{\epsilon}_1 + i \epsilon_2 \lambda_2 \\ \delta \lambda_2 &= \dot{\epsilon}_2 \end{aligned} \quad (3.4)$$

if $\epsilon_i(\tau_1) = \epsilon_i(\tau_2) = 0$, $i = 1, 2$. The parameter ϵ_1 gives rise to world-line reparametrizations, whereas ϵ_2 yields local supersymmetry transformations [4] , [5] . As expected, besides the "small" gauge transformation there also exist "large" ones not connected to the identity. They consist of the replacement $\tau \rightarrow -\tau$ in the functions X^μ , P_μ , ξ^μ and ξ_5 together with

$$\lambda'_i(\tau) = -\lambda_i(-\tau), \quad i = 1, 2 \quad (3.5)$$

Again, the equations of motion are invariant under this transformation. It is known [4] , [5]

,[9] that a "good" [23] gauge fixing of (3.1) is provided by the proper-time gauge where

$$\dot{\lambda}_i = 0 \quad \lambda_i \equiv \lambda_i(0) = \text{const.}$$

In a Feynman path-integral language the discrete transformation (3.5) is "divided out" in the same way as for the scalar particle, namely by reducing the integration over "Teichmüller space", $\int_{-\infty}^{\infty} d\lambda_1 \int d\lambda_2$, to an integral over "moduli space" only: $\int_0^{\infty} d\lambda_1 \int d\lambda_2$. (See Henneaux and Teitelboim [5] for a detailed discussion.)

We now follow the standard Batalin-Fradkin-Vilkovisky [20] , [21] procedure and introduce the momenta π_i canonically conjugate to the multipliers λ_i [★]

$$[\pi_i, \lambda_j] = \delta_{i,j} \quad (3.6)$$

Furthermore one introduces several pairs of canonically conjugate ghosts. With the constraints $\phi_i \approx 0$ we associate $(c^i, \bar{\rho}_i)$ and for $\pi_i \approx 0$ we use $(\rho^i, \bar{c}_i)$ [†]. Taking the Poisson bracket algebra of the constraints into account, the *BRST*-operator can be written down in a straightforward way:

$$\Omega = c^1 \phi_1 + c^2 \phi_2 - i \rho^1 \pi_1 + \rho^2 \pi_2 + \frac{i}{2} \bar{\rho}_1 (c^2)^2 \quad (3.7)$$

Note that since ϕ_2 is anticommuting, the ghosts are commuting for $i=2$. On the other hand, the fermionic ($i=1$)-ghosts, which are responsible for the world-line reparametrizations, are the same as in the quantization of the spinless particle. Our aim is to identify Ω as a special generator of *IOSp*($D, 2|2$). To this end we perform a series of canonical transformations on

★ At this point we do not yet restrict λ_1 (considered as a Schrödinger variable) to vary only over positive values. Here and in the following we use

$$[A, B] \equiv [A, B]_- \equiv AB - (-1)^{[A][B]} BA$$

and

$$[A, B]_+ \equiv AB + (-1)^{[A][B]} BA$$

† For the precise form of their graded commutation relations and their hermiticity properties, we refer to [10] .

our variables. Details can be found in [10] . Here we just state the result

$$\begin{aligned}\Omega = & \lambda_1 \frac{1}{\sqrt{2}} \left[\frac{i}{2} \left(\tilde{x}_1 + \tilde{x}_2 \right) \left(P_\mu^2 + i\epsilon_{ab} \tilde{p}^a \tilde{p}^b - m^2 \right) - i \left(\tilde{p}_1 + \tilde{p}_2 \right) \pi_1 \right. \\ & + \left. \left(\xi_1 + \xi_2 \right) \left(P^\mu \xi_\mu + \tilde{p}^a \xi_a - m \xi_5 \right) \right] \\ & + \frac{1}{\sqrt{2}} \left(\tilde{\xi}_1 - \tilde{\xi}_2 \right) \pi_2\end{aligned}\quad (3.8)$$

Here $\tilde{x}_a$ and $\tilde{p}_a$, $a=1,2$ are the new fermionic ghosts; they are defined in terms of $c^1, \bar{c}_1, \wp^1$ and $\bar{\wp}_1$ as in the scalar case (see eq.(2.6) of (I)). The new bosonic ghosts are

$$\begin{aligned}\xi_1 &= \frac{1}{\sqrt{2}}(c^2 + \bar{\wp}_2) \\ \xi_2 &= \frac{1}{\sqrt{2}}(c^2 - \bar{\wp}_2) \\ \tilde{\xi}_1 &= \frac{1}{\sqrt{2}}(\bar{c}_2 + \wp^2) \\ \tilde{\xi}_2 &= \frac{1}{\sqrt{2}}(\bar{c}_2 - \wp^2)\end{aligned}\quad (3.9)$$

The algebra of the ghosts reads now

$$\begin{aligned}[\xi_a, \xi_b] &= -i\epsilon_{ab} \\ [\tilde{\xi}_a, \tilde{\xi}_b] &= -i\epsilon_{ab} \\ [\tilde{p}^a, \tilde{x}_b] &= -i\delta_b^a\end{aligned}\quad (3.10)$$

with all the other brackets vanishing. The indices $a, b=1,2$ are raised and lowered using the spinor metric $i\epsilon_{ab}$: $\tilde{p}_a = i\epsilon_{ab}\tilde{p}^b$, etc.

The next step is the choice of a gauge-fixing function χ [20] , [21] . Following Barducci et al. [10] , we use

$$\chi = \frac{i}{2}\wp^1 = \frac{1}{\sqrt{2}}(\tilde{p}^1 + \tilde{p}^2)\quad (3.11)$$

as for the spinless particle and we obtain for the *BRST*-Hamiltonian

$$\begin{aligned}H &= [\chi, \Omega] \\ &= \frac{1}{2}\lambda_1 \left[P_\mu^2 - m^2 + i\epsilon_{ab} \tilde{p}^a \tilde{p}^b \right]\end{aligned}\quad (3.12)$$

Working in the Schrödinger picture, we can realize the commutation relations (3.6) and

(3.10) together with

$$\begin{aligned}[X_\mu, P^\nu] &= -i\delta_\nu^\mu \\ [\xi_\mu, \xi_\nu] &= -g_{\mu\nu} \\ [\xi_5, \xi_5] &= 1\end{aligned}\tag{3.13}$$

in the following way. The states are described by wave-functions Ψ depending on P_μ, p_a and λ_i . The operators P_μ, p_a and λ_i act on Ψ multiplicatively, whereas $X_\mu, \tilde{x}_a$ and π^i are represented by differential operators. The operators ξ_μ and ξ_5 can be realized in terms of γ -matrices. In addition they must contain a Klein factor

$$K = \exp[\pi(\tilde{x}_1\tilde{p}_1 + \tilde{x}_2\tilde{p}_2)]$$

which guarantees that ξ_μ and ξ_5 anticommute (rather than commute) with $\tilde{x}_a$ and $\tilde{p}_a$. In fact, defining

$$\gamma_\mu \equiv 2\xi_5\xi_\mu, \quad \gamma_5 \equiv \sqrt{2}K\xi_5, \quad \gamma_a \equiv -2\xi_5\xi_a\tag{3.14}$$

one finds the Clifford algebra

$$\begin{aligned}\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu &= 2g_{\mu\nu} \\ \gamma_\mu\gamma_a + \gamma_a\gamma_\mu &= 0 \\ \gamma_a\gamma_b - \gamma_b\gamma_a &= -2i\epsilon_{ab}\end{aligned}\tag{3.15}$$

The matrix γ_5 is defined as the product of the γ_μ in the usual way. The ghosts ξ_a and $\tilde{\xi}_a$ can be realized by a pair of bosonic creation and annihilation operators,

$$\begin{aligned}\xi_1 &= \frac{1}{\sqrt{2}}(a_1 + a_1^+) \\ \xi_2 &= \frac{i}{\sqrt{2}}(a_1 - a_1^+) \\ \tilde{\xi}_1 &= \frac{1}{\sqrt{2}}(a_2 + a_2^+) \\ \tilde{\xi}_2 &= \frac{i}{\sqrt{2}}(a_2 - a_2^+)\end{aligned}\tag{3.16}$$

where $[a_i, a_j^+] = \delta_{ij}$. Hence the wave-functions $|\Psi(P^\mu, p_a, \lambda_i, \tau)\rangle$ are $2^{[\frac{D}{2}]}$ -component Dirac spinors (in D-dimensions) and vectors in the Fock space of the oscillators (a_i, a_i^+) . Their

proper-time evolution is given by the Schrödinger equation

$$i\partial_\tau |\Psi\rangle = H |\Psi\rangle$$

which can be obtained from the following action:

$$S_2 = \int d^D P d\lambda_1 d\lambda_2 d\tau d^2 p_a \langle \Psi | (i\partial_\tau - H) i\pi_2 | \Psi \rangle \quad (3.17)$$

The scalar product $\langle \cdot | \cdot \rangle$ includes an infinite sum over the oscillator states of the Fock-space. Due to the presence of the λ_2 integration, the measure appearing in (3.17) has odd Grassmann parity. To make S_2 even, an odd metric has to be used for the scalar product. Requiring that it commutes with H and anticommutes with Ω , one is led to $i\pi_2$ as the unique possibility. Because λ_2 is nilpotent we can factor out the λ_2 -dependence of $|\Psi\rangle$ as

$$|\Psi\rangle = |\Psi_0\rangle + \lambda_2 |\Psi^+\rangle$$

and obtain an action which is independent of λ_2 :

$$S_2 = \int d^D P d\lambda_1 d\tau d^2 p_a \langle \Psi^+ | (i\partial_\tau - H) | \Psi^+ \rangle \quad (3.18)$$

Now we Fourier-transform with respect to $\tau \equiv x_+$:

$$\tilde{\Psi}^+(P_\mu, p_+, p_-, \tilde{p}_a) = \int_{-\infty}^{\infty} d\tau e^{i\tilde{p}\cdot\tau} \Psi^+(P_\mu, p_+, \tau, p_a) \quad (3.19)$$

We have also introduced the variable $p_+ = \lambda_1^{-1}$. After rescaling the wave-function by a factor of $\sqrt{p_+}$ one finds from (3.12)

$$S_2 = \int d^{D+4} p \left\langle \tilde{\Psi}^+ \left| \left\{ p_- - \frac{1}{2p_+} (P_\mu^2 - m^2 + i\epsilon_{ab} \tilde{p}^a \tilde{p}^b) \right\} \right| \tilde{\Psi}^+ \right\rangle \quad (3.20)$$

where $d^{D+4} p \equiv d^D p dp_+ dp_- d^2 p_a$. To uncover the hidden OSp -invariance of (3.20) one

defines a $2^{[\frac{D}{2}]+1}$ -component spinor by

$$|\hat{\Psi}\rangle \equiv \begin{pmatrix} |\tilde{\Psi}^+\rangle \\ |\tilde{\Psi}^-\rangle \end{pmatrix} \quad (3.21)$$

and

$$|\tilde{\Psi}^-\rangle \equiv \left(\sqrt{2}p_+\right)^{-1} \left(P^\mu \gamma_\mu + \tilde{p}^a \gamma_a - m\right) |\tilde{\Psi}^+\rangle \quad (3.22)$$

In terms of $|\hat{\Psi}\rangle$ the action (3.20) may be replaced by

$$S_{OSp} = \int d^{D+4}p \langle \hat{\Psi} | \left(p^A \Gamma_A - m \right) | \hat{\Psi} \rangle \quad (3.23)$$

where $^* p^A \Gamma_A \equiv P^\mu \Gamma_\mu + p^D \Gamma_D + p^{D+1} \Gamma_{D+1} + \tilde{p}^a \Gamma_a$. The p^D and p^{D+1} are the cartesian linear combinations of p_+ and p_- and the enlarged matrices Γ_A are defined by:

$$\Gamma_\mu = \begin{bmatrix} \gamma_\mu & 0 \\ 0 & -\gamma_\mu \end{bmatrix}, \quad \Gamma_D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \Gamma_{D+1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Gamma_a = \begin{bmatrix} \gamma_a & 0 \\ 0 & -\gamma_a \end{bmatrix} \quad (3.24)$$

To show that (3.23) is equivalent to (3.20) one has to note that the variation of S_{OSp} with respect to $|\tilde{\Psi}^-\rangle$ reproduces the constraint (3.22), which, when inserted back into (3.23) yields the original form (3.20) of the action (this requires a special choice of the spinor metric [10], however). Since the matrices (3.24) form an orthosymplectic Clifford algebra,

$$\left[\Gamma_A, \Gamma_B \right]_+ = 2\eta_{BA} \quad (3.25)$$

it is easy to check that S_{OSp} is invariant under the transformations of $IOSp(D, 2|2)$ generated by the "momenta" $P_A \equiv (P_\mu, P_D, P_{D+1}, \tilde{p}_a)$ and the "angular momenta"

$$J_{AB} = X_A P_B - (-1)^{[A][B]} X_B P_A + \frac{1}{2} \Sigma_{AB} \quad (3.26)$$

where

$$\Sigma_{AB} \equiv \frac{i}{2} \left[\Gamma_A, \Gamma_B \right] \quad (3.27)$$

Their commutation rules are given in eqs. (2.26) and (2.27) of (I). The wave-function

* We use the same convention as in (I). In particular, $X'' \equiv (X^\mu, X^D, X^{D+1})$ and $X^A \equiv (X'', \tilde{x}^a) \equiv (X^\mu, X^r)$ where $r \equiv (D, D+1, a)$ is an $OSp(1, 1|2)$ index. The $OSp(D, 2|2)$ invariant metric is defined by $\eta_{\alpha, \beta} = \text{diag}[+1, -1, -1, \dots, -1, +1]$, $\eta_{ab} = i\epsilon_{ab}$ and $\eta_{ra} = 0$. The variables X^A and P^A are canonically conjugate in the sense that $[X_A, P_B] = -i\eta_{BA}$.

transforms according to the spinor representation of $IOSp(D,2|2)$

$$\Psi'(X'^A) = S\Psi(X^A), \quad S = \exp\left\{-\frac{i}{4}\omega^{AB}\Sigma_{AB}\right\} \quad (3.28)$$

whereas X^A transforms according to the vector representation:

$$X'^A = \Lambda^A_B X^B \quad (3.29)$$

(From now on we shall write Ψ instead of $|\tilde{\Psi}\rangle$.) Note that, due to the presence of the Γ_a -matrices, the representations of the orthosymplectic Clifford algebra (3.25) are always infinite dimensional. Therefore we must be prepared to deal with possibly ill-defined sums resulting from the Fock-space scalar product. In section (4) we shall adopt the dimensional regularization scheme suggested by McClain et al. [28] to give a meaning to these sums. As was noted in ref. [10], the $BRST$ operator is nothing but a special linear combination of the $Osp(D,2|2)$ generators:

$$\Omega = -\frac{i}{\sqrt{2}}(J_{-1} + J_{-2}) = -\frac{i}{\sqrt{2}}\sum_a \left\{ p_a x_- - p_- x_a - 2i\Gamma_a \Gamma_- \right\} \quad (3.30)$$

Using (3.22) and the Schrödinger equation, one can show that (3.30) coincides with (3.8) up to a term which vanishes in the vacuum sector of the (a_2, a_2^+) -oscillator. Note that, since the action does not contain the $\tilde{\xi}_a$ -variables, we always can restrict the wavefunction to the vacuum of this oscillator.

Now we ask how the modular transformation (3.5) is represented in the space of the wave-functions Ψ and under what kind of discrete transformation S_{OSp} is invariant. First of all, since for the gauge choice used here, Ψ and thus S_{OSp} do not depend on λ_2 anymore, a modular transformation simply consists of a simultaneous sign change in τ and λ_1 (like for the scalar particle) or, equivalently, a sign change of p_+ and p_- or of P_D and P_{D+1} . Therefore we make the following ansatz for a modular transformation:

$$\Psi_{mod}(P_\mu, P_D, P_{D+1}, \tilde{p}_a) = S_{mod}\Psi(P_\mu, -P_D, -P_{D+1}, \tilde{p}_a) \quad (3.31)$$

The matrix S_{mod} has to be determined in such a way that (3.23) is invariant under (3.31). Consequently S_{mod} has to commute with Γ_μ and Γ_a and to anticommute with Γ_D and

Γ_{D+1} in order to compensate the minus sign coming from P_D and P_{D+1} . If we require that the modular transformation is an involution, i.e. $S_{mod}^2 = 1$, the only solution is

$$S_{mod} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3.32)$$

The "-1" in S_{mod} is easily understood: it changes the sign of Ψ^- in order to leave (3.22) invariant.

4. DIMENSIONAL REDUCTION

In this section we explain how the dimensional reduction of quantum field theory involving the action (3.23) comes about. The basic tool will be the Parisi-Sourlas mechanism which can be considered a formalized way of computing the *BRST*-cohomology.^{*} In this paper we do not deal with the derivation of Yang-Mills theories from first-quantization.[†] Nevertheless the higher dimensional theory that we shall study, will be given by (3.23) coupled to a Yang-Mills field $A^C(X^B)$ which transform as a vector under $OSp(D,2|2)$. The dimensional reduction of this kind of theory is interesting in its own right. It has been discussed in a perturbative context by McClain et al.[28] . Here we shall give a non-perturbative proof along the lines of Cardy[31] . This analysis is necessary because, in order to derive the PCT-theorem from the $OSp(D,2|2)$ ward-identities, we have to know how the group acts on the physical subspace. The novel aspect, compared to the scalar theories considered so far, is that the dimensional reduction of a multi-component field does not only involve a reduction of the arguments on which the field depends, i.e. $X^B \rightarrow X^\mu$, but also a reduction in the number of field components: $A^C \rightarrow A^\mu$. In this situation the standard "magic formula" of Parisi-Sourlas dimensional reduction[19] is not sufficient. In the appendix A, to which the reader might turn at this point, we obtain a more general integral formula which also can be applied in field-space.

The theory we shall study is defined by the $OSp(D,2|2)$ -invariant fermionic action (3.23) coupled to a quantized non-abelian gauge field. Its generating functional is given by

$$Z[Q, J_C] = \int [d\Psi d\bar{\Psi} dA^C d\beta d\bar{\beta}] \exp \left[i \int d^{D+4}X \{ L_F + L_B + \text{tr}(A^C J_C) \} + i S_Q \right] \quad (4.1)$$

^{*} For a detailed discussion of the cohomology of Ω , see for instance ref.[29] .

[†] See for instance, ref.[30] .

where

$$L_F = \bar{\Psi} [i\Gamma^C (\partial_C + igA_C) - m] \Psi \quad (4.2)$$

$$L_B = -\frac{1}{2} \text{tr}(F_{BC} F^{BC}) - \frac{1}{\alpha} \text{tr}(\partial_C A^C)^2 + \bar{\beta} \partial^C [\partial_C + igA_C^{ad}] \beta \quad (4.3)$$

and

$$S_Q = \int d^{D+4}X d^{D+4}Y \bar{\Psi}(X) Q(X, Y) \Psi(Y) \quad (4.4)$$

Several comments are in order now. The gauge field $A^C(X^B)$ is a vector under $OSp(D, 2|2)$. It takes values in the Lie algebra of some semi-simple, compact gauge group G with generators $T_{(i)}$, $A^C = A^{C(i)} T_{(i)}$ where $T_{(i)}$ is in the fundamental representation of G . (We also write $A_C^{ad} = A_C^{(i)} T_{(i)}^{ad}$ where $T_{(i)}^{ad}$ is in the adjoint representation. Furthermore, "tr" denotes the trace in the fundamental representation.) The usual differential-geometric construction [28] yields the following components of the associated field strength:

$$F_{BC} = \partial_B A_C - (-)^{[B][C]} \partial_B A_C - ig[A_B, A_C] \quad (4.5)$$

In (4.3) we added an $OSp(D, 2|2)$ -invariant gauge-fixing term $\frac{1}{\alpha} \text{tr}(\partial_C A^C)^2$ and the associated Faddeev-Popov ghosts β and $\bar{\beta}$ to the action. (For a more detailed discussion of gauge transformations and gauge-fixing in the model, see ref. [28].) The external source coupling of the fermions, eq.(4.4), is slightly unusual, but very convenient for our purposes. The bi-local function $Q(X, Y)$ is an infinite-dimensional matrix in the representation space of the Γ_A -matrices. If we know $Z[Q, J_C]$ for all matrices $Q(X, Y)$, we can take a functional derivative with respect to a certain matrix element to obtain an insertion of $\bar{\Psi}(X)$ together with $\Psi(Y)$, where $\bar{\Psi}$ and Ψ carry the same spinor indices as the matrix element we had chosen*. To simplify the dimensional reduction of Z it is advantageous to introduce a new representation for the Γ_A -matrices. We replace (3.24) by

$$\begin{aligned} \Gamma_\mu &= \gamma_\mu \otimes I \otimes I \\ \Gamma_D &= \gamma_5 \otimes i\sigma_1 \otimes I \\ \Gamma_{D+1} &= \gamma_5 \otimes \sigma_2 \otimes I \\ \Gamma_a &= -\gamma_5 \otimes \sigma_3 \otimes A_a \cdot K \end{aligned} \quad (4.6)$$

where A_1 and A_2 are the Fock-space operators $A_1 \equiv a_1 + a_1^+$, $A_2 \equiv i(a_1 - a_1^+)$. We have introduced a convenient tensor-product notation. Thus a spinor-index becomes a

* The Green's functions thus constructed contain equal numbers of Ψ 's and $\bar{\Psi}$'s. This is no loss of generality, however, since all other Green's functions vanish anyhow.

triple-index: (δ_1, δ_2, n) with[†] $\delta_1 = 1, \dots, 2^{\frac{D}{2}}$, $\delta_2 = 1, 2$, and $n = 0, 1, \dots$ labels a Fock-space basis. Note that (3.24) and (4.6) are related by an ordinary finite dimensional unitary transformation of the Γ_α 's since the Γ_a 's are the same in both representations. The advantage of the new representation is that now the generators $\Sigma_{r,s}$ of the $OSp(1,1|2)$ -subgroup of $OSp(D,2|2)$ do not act on the first spinor index δ_1 . The generators are explicitly given by

$$\begin{aligned}
\Sigma_{\mu,\nu} &= \sigma_{\mu\nu} \otimes I \otimes I \\
\Sigma_{\mu,r} &= \gamma_\mu \gamma_5 \otimes (\dots) \otimes (\dots) \\
\Sigma_{D,D+1} &= -iI \otimes \sigma_3 \otimes I \\
\Sigma_{D,a} &= -iK \cdot I \otimes \sigma_2 \otimes A_a \\
\Sigma_{D+1,a} &= K \cdot I \otimes \sigma_1 \otimes A_a \\
\Sigma_{a,b} &= \frac{i}{2} I \otimes I \otimes (A_a A_b + A_b A_a)
\end{aligned} \tag{4.7}$$

Here $\sigma_{\mu\nu}$ denotes the usual Lorentz generators; the precise form of the $(\dots)$ -terms will not be important in the following. Since the $\Sigma_{r,s}$ act on the first spinor index as a unity matrix, $OSp(1,1|2)$ is not "broken" by the external source provided Q acts on the δ_1 index only. In fact, Z can be dimensionally reduced if the sources are restricted in the following way:

$$Q_{(\delta_1, \delta_2, \bar{n}) (\delta_1, \delta_2, n)}^{res}(X^A, Y^A) = q_{\delta_1, \delta_1}(X^\mu, Y^\mu) \cdot \delta^{(4)}(X^r) \delta^{(4)}(Y^r) \delta_{\delta_2 \delta_2} \delta_{\bar{n}, n} \tag{4.8}$$

$$J_A^{res}(X^B) = \begin{cases} j_\mu(X^\nu) \delta^{(4)}(X^r), & \text{if } A = \mu; \\ 0, & \text{if } A = r. \end{cases} \tag{4.9}$$

We now show that

$$Z[Q^{res}, J_A^{res}] = Z_D[q, j] \tag{4.10}$$

where Z_D is the generating functional of a purely D-dimensional theory with a Lagrangian analogous to (4.2), (4.3). Generalizing Cardy's analysis of the scalar field [31] we define

[†] We assume D even in the following.

the following interpolating functional:

$$\begin{aligned}
Z_{\omega_1, \omega_2}[Q^{res}, J_A^{res}] = & \int [d\Psi d\bar{\Psi} dA^C d\beta d\bar{\beta}] \exp \left[\frac{i}{2\pi} \int d^{D+4} X \{ [\omega_1 + \right. \\
& + 2\pi(1 - \omega_1)\delta^{(4)}(X^r)] (L_F^1 + L_B^1) \\
& + L_F^2 + L_B^2 + \omega_2 A^r A_r \} \\
& \left. + i \int d^{D+4} X \text{tr}(A^C J_C^{res}) + i S_{Q^{res}} \right]
\end{aligned} \tag{4.11}$$

where

$$L_F^1 = \bar{\Psi} [i\Gamma^\mu (\partial_\mu + igA_\mu) - m] \Psi \tag{4.12}$$

$$L_B^1 = -\frac{1}{2} \text{tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{\alpha} \text{tr}(\partial_\mu A^\mu)^2 + \bar{\beta} \partial^\mu [\partial_\mu + igA_\mu^{ad}] \beta \tag{4.13}$$

$$L_F^2 = \bar{\psi} i\Gamma^r (\partial_r + igA_r) \psi \tag{4.14}$$

$$L_B^2 = -\frac{1}{2} \text{tr}(2F_{\mu r} F^{\mu r} + F_{rs} F^{rs}) - \frac{1}{\alpha} \text{tr}(\partial_r A^r)^2 + \bar{\beta} \partial^r [\partial_r + igA_r^{ad}] \beta \tag{4.15}$$

First we note that for $\omega_1 = 1$ and $\omega_2 = 0$ the new functional (4.11) coincides with the original one, eq.(4.1). Next we demonstrate that the $\omega_1 = 0, \omega_2 \rightarrow \infty$ limit yields Z_D . For $\omega_2 \rightarrow \infty$ we obtain a delta-functional $\delta[A_r]$ under the path-integral, so that the gauge field integration is restricted to $A_\mu(X^C)$. Inserting the special form of the restricted sources we can integrate out all field variables with $X_r \neq 0$ as in the scalar case. Those variables do not couple to the sources, and in an appropriate lattice regularization of the functional integral, they also decouple from the physical variables with $X^r=0$. What remains is

$$\begin{aligned}
Z_{0,\infty}[q, j] = & \int [d\Psi d\bar{\Psi} dA^\mu d\beta d\bar{\beta}] \exp \left[i \int d^4 X \{ L_F^1(X^\mu, 0, 0, \tilde{0}) + L_B^1(X^\mu, 0, 0, \tilde{0}) \right. \\
& + \text{tr}(A^\mu(X^\nu, 0, 0, \tilde{0}) j_\mu(x^\rho)) \} \\
& \left. + i \int d^D X d^D Y \bar{\Psi}_{\delta_1 \delta_2 n}(X^\mu, 0, 0, \tilde{0}) q_{\delta_1 \delta_1}(X^\mu, Y^\mu) \Psi_{\delta_1 \delta_2 n}(Y^\mu, 0, 0, \tilde{0}) \right]
\end{aligned} \tag{4.16}$$

The integration is over fields with $X_r=0$ only. This functional, however, is not yet of the usual D-dimensional form. In fact, the fermionic action reads

$$L_F^1(X^\mu, 0, 0, \tilde{0}) = \bar{\Psi}_{(\delta_1 \delta_2 n)} \left[i\Gamma^\mu (\partial_\mu + igA_\mu) - m \right]_{\delta_1 \delta_1} \Psi_{(\delta_1 \delta_2 n)} \tag{4.17}$$

because Γ^μ does not act on the second and the third index. This means that (4.16) seems to describe an infinity of ordinary Dirac fermions Ψ_{δ_1} parametrized by the index pair (δ_2, n) .

At this point we have to remember that, in order to give a well defined meaning to (4.1), we have to regularize the infinite sums over n . It has been shown by McClain et al. [28] that a consistent way of doing this is a modified version of the standard dimensional regularization technique. Recall the following identities in a space of (non-integer) dimension 2ω :

$$tr_L(\eta_{\mu\nu}) = 2\omega \quad (4.18)$$

$$tr I = 2^{\frac{1}{2}tr_L(\eta_{\mu\nu})} = 2^\omega \quad (4.19)$$

$$tr \gamma_\mu \gamma_\nu = 2^\omega \eta_{\mu\nu} \quad (4.20)$$

(Here tr_L is the trace w.r.t. the Lorentz indices and tr denotes the trace in spinor space). Similar trace identities can be derived for products of more than two γ matrices. We shall use a superspace generalization of these identities. Replacing $\eta_{\mu\nu}$ by the $OSp(1,1|2)$ -invariant metric η_{rs} of (δ_2, n) -space and tr_L by the corresponding supertrace we have, in analogy with above

$$tr I \equiv tr(I \otimes I) = 2^{\frac{1}{2}str(g_{rs})} = 1 \quad (4.21)$$

since $str(g_{rs}) = 2 - 2 = 0$. Accordingly the regularized dimension of (δ_1, δ_2, n) -space is

$$tr(I \otimes I \otimes I) = 2^{\frac{1}{2}str(g_{AB})} = 2^{\frac{D}{2}} \quad (4.22)$$

In this way one can obtain the OSp -generalization of trace-identities like (4.20) which have been used in the perturbative calculations of ref. [27]. From (4.21) we learn that the dimension of (δ_2, n) -space is effectively unity, which means that the n -point functions generated by (4.16) are in fact the same as those of the genuinely D -dimensional functional Z_D . So far we have only shown that Z_{ω_1, ω_2} reproduces, in a certain limit, both Z and Z_D . To establish (4.10) it remains to prove that Z_{ω_1, ω_2} actually does not depend on ω_1 and ω_2 . The ω_1 -independence follows in the usual way from:

$$\begin{aligned} \frac{\partial}{\partial \omega_1} \ln Z_{\omega_1 \omega_2} [Q^{res}, J_A^{res}] &= \frac{i}{2\pi} \int d^{D+4} X [1 - 2\pi \delta^{(4)}(x^r)] \cdot \\ &\cdot \left\langle (L_F^1 + L_B^1)(X^A) \right\rangle_{\omega_1, \omega_2}^{Q^{res}, J^{res}} \end{aligned} \quad (4.23)$$

The RHS of (4.23) vanishes by virtue of the "magic formula" from the appendix. It may be applied to $\langle L_F^1 + L_B^1 \rangle$ because the latter depends on X^r only through the $OSp(1,1|2)$ -

invariant quantity $x_+x_- + i\tilde{x}_1\tilde{x}_2$. For this to be true it is essential that the $OSp(1,1|2)$ -generators of (4.7) act on the restricted source Q^{res} like the identity, so that $OSp(1,1|2)$ remains unbroken. The ω_2 -independence follows in an analogous fashion:

$$\frac{\partial}{\partial\omega_2}Z_{\omega_1,\omega_2}[Q^{res}, J_A^{res}] = \int [dA_r] \left(\frac{i}{2\pi} \int d^{D+4}X A^r A_r \right) \dots \quad (4.24)$$

The dots stand for an $OSp(1,1|2)$ -invariant functional of A^r whose form can be read from (4.11). (Its invariance follows from the fact that a transformation of A^r can be absorbed by an appropriate change of the remaining integration variables.) Applying formula (A13) to (4.24), we find that the RHS of (4.24) indeed vanishes, *i.e.* that Z_{ω_1,ω_2} is independent of ω_2 . This concludes our proof of equation (4.10). It is important to note that, as we discussed for the scalar case at length in (I), dimensional reduction occurs only if eq.(A13) is applicable which, in turn, means only if Feynman boundary conditions are used for the propagator. In this case a Wick rotation to Euclidean (internal) space is possible. Rotating A^r in the same way as X^r leaves the path-integral convergent and hence equation (A13) is applicable.

Before closing this section, it is instructive to look at another limiting case of Z_{ω_1,ω_2} : $\omega_1 = 1, \omega_2 \rightarrow \infty$. Integrating out the $X^r \neq 0$ components, one has :

$$\begin{aligned} Z[Q^{res}, J_C^{res}] &= Z_{1,\infty}[Q^{res}, J_C^{res}] \\ &= \int d\mu \exp i \int d^D X L_B^1(A_\mu(\hat{x}), \beta(\hat{x}), \bar{\beta}(\hat{x})) \\ &\quad \cdot \int [d\Psi(X^A) d\bar{\Psi}(X^A)] \exp \{ i \int d^{D+4} X \bar{\Psi} [i\gamma^\mu (\partial_\mu \\ &\quad + igA_\mu(\hat{x})) + i\Gamma^r \partial_r] \Psi + \dots \} \end{aligned} \quad (4.25)$$

where $\hat{x} \equiv (X^\mu, 0, 0, \bar{0})$, $d\mu = [dA_\mu(\hat{x}) d\beta(\hat{x}) d\bar{\beta}(\hat{x})]$ and the $\dots$ denote the source terms. Let us assume for a moment that $A_\mu(x^\nu)$ is an abelian background gauge field. Then, without any major changes, we can repeat the first-quantization procedure of section (3) with the constraints

$$\tilde{\phi}_1 \equiv (P_\mu - gA_\mu)^2 - m^2 \quad (4.26)$$

$$\tilde{\phi}_2 \equiv (P_\mu - gA_\mu)\xi^\mu - m\xi_5 \quad (4.27)$$

rather than (3.2) and (3.1). This would lead to the fermionic action in the curly brackets of (4.25) instead of (3.23). It is the analogue of eq.(2.19). Obviously this action is

not $OSp(D,2|2)$ -invariant since a priori the vector-field has no A_r components and A_μ depends on X_ν only. Equation (4.25) shows, however, that if one considers A_μ dynamical, i.e. includes $[dA_\mu]$ and the corresponding gauge-fixed Yang-Mills action into the path-integral, the resulting theory can be rewritten in a perfectly $OSp(D,2|2)$ -invariant way, since, due to eq.(A13), the additional bosonic and fermionic components of A_C have a mutually compensating effect.

5. THE PCT-TRANSFORMATION

In (I) we argued that the PCT-theorem [32] [33] [34] [23] for a hermitian scalar field can be obtained from the Ward identities of the $SO(4,2)$ subgroup of $OSp(4,2|2)^*$. A special element of this group could be identified which induces the PCT-transformation in the physical subspace. We now show that also in the spinor representation of $SO(4,2)$ this group element leads to the correct PCT-transformation law. Acting on $X^\alpha \equiv (X^\mu, X^D, X^{D+1})$ the transformation is simply

$$(\Lambda_{PCT})^\alpha_\beta = \text{diag}[-1, -1, -1, -1, -1, -1] \quad (5.1)$$

It is continuously connected to the identity, since it can be obtained by three rotations of angle π in the (X^0, X^{D+1}) , (X^1, X^2) and (X^3, X^D) -planes, respectively. For this to be possible, *the existence of a second time-coordinate, X^{D+1} is crucial*. Representing the Γ -matrices as in (4.6), the corresponding spinor transformation reads

$$S_{PCT} = \exp\left[\frac{i\pi}{2}\{\Sigma^{0,D+1} + \Sigma^{1,2} + \Sigma^{3,D}\}\right] = \exp\left[-\frac{\pi}{2}M_1\right]\exp\left[-\frac{\pi}{2}M_2\right]\exp\left[-\frac{\pi}{2}M_3\right] \quad (5.2)$$

with

$$\begin{aligned} M_1 &\equiv \gamma^0 \gamma_5 \otimes \sigma_2 \otimes I \\ M_2 &\equiv \gamma^1 \gamma^2 \otimes I \otimes I \\ M_3 &\equiv \gamma_5 \gamma^3 \otimes i\sigma_1 \otimes I \end{aligned} \quad (5.3)$$

A trivial calculation yields

$$S_{PCT} = i\gamma_5 \otimes \sigma_3 \otimes I \quad (5.4)$$

Next we derive the associated Ward-identities. To be as general as possible, we consider an OSp -invariant interacting theory like (4.1) but also include a complex scalar field Φ .

* For simplicity we put $D=4$ from now on. The generalization to other (even) dimensionalities is obvious.

Then the action is $S_{OSp} = S_{OSp}[\Phi, \Phi^*, \Psi, \bar{\Psi}, A^C]$. The respective transformations are:

$$\begin{aligned}
\Phi^{PCT}(X^\alpha, \tilde{X}^a) &= \Phi(-X^\alpha, \tilde{X}^a) \\
\Phi^{*PCT}(X^\alpha, \tilde{X}^a) &= \Phi^*(-X^\alpha, \tilde{X}^a) \\
\Psi^{PCT}(X^\alpha, \tilde{X}^a) &= S_{PCT}\Psi(-X^\alpha, \tilde{X}^a) \\
\bar{\Psi}^{PCT}(X^\alpha, \tilde{X}^a) &= \bar{\Psi}(-X^\alpha, \tilde{X}^a)S_{PCT}^{-1} \\
A_C^{PCT}(X^\alpha, \tilde{X}^a) &= -(-)^{[C]}A_C(-X^\alpha, \tilde{X}^a)
\end{aligned} \tag{5.5}$$

The Green's functions can be derived from a path-integral of the form:[†]

$$\begin{aligned}
Z[J^{res*}, J^{res}, Q^{res}, J_C^{res}] &= \int [d\Phi d\Phi^* d\Psi d\bar{\Psi} dA] \exp\{iS_{OSp}[\Phi, \Phi^*, \Psi, \bar{\Psi}, A^C] + \\
&+ i \int d^4X [j^*(X^\mu)\Phi(X^\mu, 0, 0, \tilde{0}) + j(X^\mu)\Phi^*(X^\mu, 0, 0, \tilde{0}) \\
&+ j^\mu(X^\rho)A_\mu(X_\rho, 0, 0, \tilde{0})] \\
&+ i \int d^4X d^4Y \bar{\Psi}(X^\mu, 0, 0, \tilde{0})q(X^\mu, Y^\mu)\Psi(Y^\mu, 0, 0, \tilde{0})\}
\end{aligned} \tag{5.6}$$

Now we change the integration variables from Φ, Ψ etc. to $\Phi^\Lambda, \Psi^\Lambda, \dots$ where Λ is some $O\text{Sp}$ -transformation. Since the action and the measure are invariant, the only change appears in the source-terms. Equating the functional derivatives of (5.6) and those of the new functional, we obtain a set of Ward-identities in the physical X^μ -space for any Λ .^{*}

For $\Lambda = \Lambda_{PCT}$ they read:

$$\begin{aligned}
&< 0 | \hat{\phi}(v^\mu) \dots \hat{\phi}^\dagger(w^\mu) \dots \hat{\psi}_\alpha(x^\mu) \dots \hat{\bar{\psi}}_\beta(y^\mu) \dots \hat{A}_\nu(z^\mu) \dots | 0 > = \\
&< 0 | \dots (-\hat{A}_\nu)(z^\mu) \dots [-i\hat{\bar{\psi}}(-y^\mu)\gamma_5]_\beta \dots [-i\gamma_5\hat{\psi}(-x^\mu)]_\alpha \dots \hat{\phi}^\dagger(w^\mu) \dots \hat{\phi}(v^\mu) | 0 >
\end{aligned} \tag{5.7}$$

A priori the path-integral gives T-ordered products only. However, in commuting the operators to any order, the commutator terms drop on both sides of (5.7). In writing down this Ward-identity we have identified functional derivatives like $-i\frac{\delta}{\delta j^*(X^\mu)}$ with 4-dimensional field operators $\hat{\phi}(X^\mu)$ etc. Since the derivative brings down a factor of $\Phi(X^\mu, 0, 0, \tilde{0})$ in (5.6), it is clear that $\hat{\phi}(X^\mu)$ is the operatorial counterpart of $\phi(X^\mu) \equiv \Phi(X^\mu, 0, 0, \tilde{0})$ in the 4-dimensional field-theory obtained after dimensional reduction. To derive these identities

† For simplicity we consider an abelian vector field; the presence of the Faddeev-Popov ghosts would not change our conclusions.

* The derivation is analogous to the one contained in (I) to which we refer the reader.

it is essential to know how the sources have to be restricted in order to project on the physical subspace. Setting the X^r -components of all fields to zero means that the sign-change of X^D and X^{D+1} is physically irrelevant. Moreover making q diagonal in the (δ_2, n) -indices implies that the σ_3 -matrix in S_{PCT} of (5.4) has no effect since it commutes with q . (The indices $\alpha\beta\cdots$ in (5.7) are δ_1 -type spinor indices belonging to the physical subspace.) It is well known [23] that (5.7) is nothing else than the PCT-theorem of a 4-dimensional theory with interacting scalar, spinor and vector fields. Assuming PCT-invariance of the vacuum, eq.(5.7) corresponds to the following transformation laws of the field operators under $\Theta \equiv PCT$:

$$\begin{aligned}
\Theta \hat{\phi}(x) \Theta^\dagger &= \hat{\phi}^\dagger(-x) \\
\Theta \hat{\phi}^\dagger(x) \Theta^\dagger &= \hat{\phi}(-x) \\
\Theta \hat{\psi}_\alpha(x) \Theta^\dagger &= i(\gamma_5)_{\alpha,\beta} \hat{\psi}_\beta^\dagger(-x) \\
\Theta \hat{\bar{\psi}}_\alpha(x) \Theta^\dagger &= -i \hat{\bar{\psi}}_\beta(-x) (\gamma_5 \gamma_0)_{\beta\alpha} \\
\Theta \hat{A}_\mu(x) \Theta^\dagger &= -\hat{A}_\mu(-x)
\end{aligned} \tag{5.8}$$

It is important to realize the difference between these equations and the $SO(4,2)$ -transformation laws (5.5). The Λ_{PCT} -rotation transforms $\Phi(X^\mu, 0, 0, \tilde{0})$ into $\Phi(-X^\mu, 0, 0, \tilde{0})$ and $\Phi^*(X^\mu, 0, 0, \tilde{0})$ into $\Phi^*(X^\mu, 0, 0, \tilde{0})$, whereas Θ , acting on operators, maps $\hat{\phi}(x)$ onto $\hat{\phi}^\dagger(-x)$ and $\hat{\phi}^\dagger(x)$ onto $\hat{\phi}(-x)$. This difference is due to the *antilinear* nature of Θ : when it acts on a field-operator it gives the hermitian adjoint, while when it acts on a state (a one-particle wave function for instance) it does not involve a complex conjugation. The same remark applies also to ψ and $\bar{\psi}$. The path-integral variables appearing in (5.5) behave like one-particle wave-functions. It is only because these are not complex conjugated by Θ that the PCT-transformation can be represented by an $SO(4,2)$ -rotation. On the other hand, both the separate C and the separate T transformations involve a complex conjugation of the one-particle wave-functions and therefore they cannot be represented by a $SO(4,2)$ -rotation. The situation is different for the product CT which involves no complex conjugation of ϕ and ψ . In fact, a theory like (4.1) is invariant under P and CT separately. The CT-transformation is simply the π -rotation in the time-plane (X^0, X^{D+1}) which appeared in (5.2) already. On spinors it acts via

$$\begin{aligned}
S_{CT} &= \exp\left[-\frac{\pi}{2} M_1\right] = \\
&= -i\gamma^1\gamma^2\gamma^3 \otimes \sigma_2 \otimes I
\end{aligned} \tag{5.9}$$

Hence the physical projection of $\Psi^{CT}(x^A) = S_{CT}\Psi(-X^0, \vec{X}, X^D, -X^{D+1}, \tilde{x}^a)$ yields $\psi^{CT}(X^\mu) = -i\gamma^1\gamma^2\gamma^3\psi(-X^0, \vec{X})$, which is the usual rule for a Dirac wave-function (with a

special choice of phases). Deriving the Ward-identities from (5.9) as above one can read off the transformation law for the operators. Again, because of the antilinearity of CT, they involve a hermitian conjugation of $\hat{\phi}$ and $\hat{\psi}$. Also the vector field A^C transforms correctly under this rotation. Its physical part is changed from $(A^0, \vec{A})$ to $(-A^0, \vec{A})$, as it should. Furthermore, we also can infer the parity invariance of (5.2) from its OSp -invariance. The relevant transformations are the two rotations in the (X^3, X^D) and (X^1, X^2) -planes, respectively, which we encountered in (5.2) already. For spinors we have

$$\begin{aligned} S_P &= \exp\left[-\frac{\pi}{2}M_2\right]\exp\left[-\frac{\pi}{2}M_3\right] \\ &= \gamma^0 \otimes \sigma_1 \otimes I \end{aligned} \quad (5.10)$$

The physical projection of the OSp -rotation yields

$$\psi_P(X^0, \vec{X}) = \gamma^0 \psi(X^0, -\vec{X}) \quad (5.11)$$

which, again, coincides with the standard transformation law. Needless to say that also the vector field behaves in the correct way. To conclude, for any $OSp(4,2|2)$ -invariant theory containing scalars, Dirac fermions and vector fields, both P and CT can be identified with certain OSp -transformations. Therefore, not only PCT, but also P and CT separately are symmetries of the reduced theory. This means that in any theory in which P and CT are broken, also $OSp(4,2|2)$ must be broken. This can be the case in theories of Weyl fermions (or of Dirac fermions with γ_5 coupling) in 4-dimensions. In section (6) we shall come back to this problem in more detail.

For the time being, let us only look at theories where $OSp(4,2|2)$ is intact and let us investigate the relation between the PCT-transformation and modular invariance. We shall see that *the PCT-transformation is a modular transformation "seen" in a $SO(4,2)$ -rotated frame*. Consider first a scalar Φ ; from eq.(2.22) we know the form of a modular-transformation

$$\begin{aligned} \Phi_{mod}(X^\mu, X^D, X^{D+1}, \tilde{x}^a) &= \Phi(X^\mu, -X^D, -X^{D+1}, \tilde{x}^a) \\ \Phi_{mod}(X^\mu, X^D, X^{D+1}, \tilde{x}_a) &= \Phi(\Lambda_{mod}^\alpha{}_\beta X^\beta, \tilde{x}^a) \end{aligned} \quad (5.12)$$

where $\Lambda_{mod} \equiv \text{diag}[1, 1, 1, 1, -1, -1]$. Note that $\Lambda_{mod} \in SO(4,2)$, but Λ_{mod} is not continuously connected to the identity. Next observe that, by a special $SO(4,2)$ -rotation, we can interchange the two time-variables X^0 and X^{D+1} , namely by a $\frac{\pi}{2}$ -rotation in the

(X^0, X^{D+1}) -plane. Similarly we can exchange the new spatial coordinate X^D with one of the X^i , X^3 -say. Denoting this set of rotations by Λ_{ex} , its action on X^α is

$$\Lambda_{ex}(X^0, X^1, X^2, X^3, X^D, X^{D+1}) = (X^{D+1}, X^1, X^2, X^D, -X^3, -X^0) \quad (5.13)$$

If we now apply a modular transformation (5.12) on $\Lambda_{ex}^\alpha{}_\beta X^\beta$ it changes $(-X^3, -X^0)$ to $(+X^3, +X^0)$. Finally we apply the inverse exchange transformation Λ_{ex}^{-1} to bring X^3 and X^0 back to their original positions:

$$\Lambda_{ex}^{-1} \Lambda_{mod} \Lambda_{ex}(X^0, X^1, X^2, X^3, X^D, X^{D+1}) = (-X^0, X^1, X^2, -X^3, X^D, X^{D+1}) \quad (5.14)$$

Obviously this transformation is a PCT-transformation followed by a trivial π -rotation $R_{1,2}(\pi)$ in the 1 – 2-plane:

$$\Lambda_{ex}^{-1} \Lambda_{mod} \Lambda_{ex} = \tilde{\Lambda}_{PCT} R_{1,2}(\pi) \quad (5.15)$$

Here $\tilde{\Lambda}_{PCT} \in SO(4, 2)$ is a "genuine" PCT-transformations changing only the signs of X^μ but leaving X^D and X^{D+1} unchanged. Like Λ_{mod} it is not in the identity-component of $SO(4, 2)$. This simple argument shows that, in a $OSp(4, 2|2)$ -invariant theory, the PCT-transformation and the modular transformation are essentially the same. For the sequence of transformations $\Lambda_{ex}^{-1} \Lambda_{mod} \Lambda_{ex} R_{1,2}(\pi)$ we can derive Ward identities in the usual way. Since both the measure and the action are both modular-invariant and invariant under the connected component of $SO(4, 2)$, we again obtain the PCT-theorem. The argument is easily generalized for fermions. In the basis (4.6), the modular transformation matrix (3.32) entering (3.31) reads:

$$S_{mod} = I \otimes \sigma_3 \otimes I \quad (5.16)$$

The spinor representation matrix for the exchange transformation is

$$S_{ex} = \exp\left[-\frac{\pi}{4} M_1\right] \exp\left[-\frac{\pi}{4} M_3\right] \quad (5.17)$$

It can be easily verified that

$$S_{ex}^{-1} S_{mod} S_{ex} = [i\gamma_5 \otimes I \otimes I] S_{1,2}(\pi) \equiv \tilde{S}_{PCT} \cdot S_{1,2}(\pi) \quad (5.18)$$

where $S_{1,2}(\pi) \equiv \exp\left[\frac{\pi}{2} M_2\right]$ corresponds to the rotation in the (1-2)-plane and $\tilde{S}_{PCT}$ denotes a "genuine" PCT-transformation without the σ_3 -matrix appearing in (5.4). (From a 4-dimensional point of view, S_{PCT} and $\tilde{S}_{PCT}$ are equivalent, of course.) We again conclude

that a PCT-transformation is nothing but a modular transformation "seen" in a rotated frame. The $SO(4,2)$ symmetry allows us to relate "world-line physics", namely modular invariance, to "space-time" physics, *i.e.* the PCT theorem in the present case. In (I) we already encountered a similar situation when we observed that the $BRST$ -field theory is consistent only with Feynman boundary conditions. Dimensional reduction requires the additional momenta to appear as $2[p_+p_- + i\tilde{p}^1\tilde{p}^2] + i\epsilon$ in the propagator. Because of the $SO(4,2)$ -invariance, however, the complete denominator reads

$$P_\mu^2 - m^2 + 2[p_+p_- + i\tilde{p}^1\tilde{p}^2] + i\epsilon$$

i.e. the "world-line Feynman propagator" implies a "space-time Feynman propagator". This kind of world-sheet/space-time correspondence is particularly intriguing in string theory.

In (I) we showed for the scalar field that also microcausality (or to be precise, weak local commutativity [22], [34]) follows from $SO(4,2)$ -invariance. This proof can not be generalized in a simple way to higher spin fields. However, we know from Jost's [34] classical work on the PCT-theorem, that under very general conditions PCT-invariance and weak local commutativity are equivalent. Thus for spinor and vector fields we may still say that $SO(4,2)$ embodies causality in a two-fold way: it enforces causal boundary conditions for the propagator and via the PCT-theorem it guarantees weak local commutativity in the dimensionally reduced theory.

6. CHIRAL THEORIES

In the previous sections we considered a vector-like theory with the vector-current of the fermions coupled to an $OSp(4,2|2)$ -gauge field. With the same methods we also could discuss OSp -invariant Yukawa couplings like $\bar{\Psi}\Psi\Phi$. A natural question is whether there exists a higher-dimensional generalization of the 4-dimensional γ_5 -matrix.

Is it possible to find a matrix $\hat{\Gamma}$ such that $\bar{\Psi}\hat{\Gamma}\Psi\Phi$ is $OSp(4,2|2)$ -invariant and dimensionally reduces to a pseudo-scalar Yukawa term $\bar{\psi}\gamma_5\psi\phi$? Unfortunately the answer is no. This does not come unexpected: in the last section we showed that a 4-dimensional parity transformation can be obtained as the physical projection of a (proper) $SO(4,2)$ -rotation. Hence, from a 4-dimensional point of view, the Σ_{AB} -spinor representation contains both left-handed and right-handed particles. For a massive Dirac-fermion this is nothing unusual, of course, since chirality can be changed by a simple 4-dimensional boost already.

What is new, however, is that in the massless case no $OSp(4,2|2)$ -covariant separation of a Dirac-spinor into two Weyl-spinors is possible. Stated differently, any spinor matrix $\hat{\Gamma}$ commuting with all the generators Σ_{AB} of the connected component of $OSp(4,2|2)$ is a multiple of the identity, *i.e.*, there exists no analogue of γ_5 .^{*} This is most easily seen as follows. We start from the ansatz:

$$\hat{\Gamma} = G_1 \otimes G_2 \otimes G_3 \quad (6.1)$$

where G_1, G_2 are matrices while G_3 is a Fock-space operator. Requiring commutativity with $\Sigma_{\mu,\nu}$ and $\Sigma_{D,D+1}$ of eq.(4.7) immediately implies $G_1 = \alpha_1 + \alpha_2 \gamma_5$ and $G_2 = \beta_0 + \beta_3 \sigma_3$, where the α 's and the β 's are constants. Now let us look at the special $Sp(2)$ generator

$$\Sigma_{1,1} + \Sigma_{2,2} = 4iI \otimes I \otimes [a_1^\dagger a_1 + \frac{1}{2}] \quad (6.2)$$

If $\hat{\Gamma}$ is to commute with $\Sigma_{1,1} + \Sigma_{2,2}$ the operator G_3 has to commute with the particle number operator $a_1^\dagger a_1$ of our Fock-space. This is possible only if G_3 is proportional to the identity. Finally, commuting $\hat{\Gamma}$ with $\Sigma_{D,\alpha}$ and $\Sigma_{\mu,r}$ implies $\beta_3=0$ and $\alpha_2=0$, respectively. Hence $\hat{\Gamma}$ must be a multiple of the identity operator. Note that, in order to arrive at this conclusion, the presence of the "fermionic" Γ_α -matrices is essential. In even dimensions it is always possible to find one and only one matrix Γ_5 anticommuting with all Γ_α 's: it is simply the product of all the Γ_α -matrices. However, in constructing the orthosymplectic Clifford-algebra (4.6) this possibility has been exhausted already in order to make the Γ_α anticommute with the Γ_α .

We conclude that there can be no $OSp(4,2|2)$ -invariant pseudo-scalar or axial-vector couplings like $\bar{\Psi}\hat{\Gamma}\Psi\Phi$ or $\bar{\Psi}\hat{\Gamma}\Gamma^C\Psi A_C$, respectively.[†] On the other hand we should not forget that the full $OSp(4,2|2)$ -symmetry is not really necessary for a consistent *BRST*-field theory, *i.e.* we could allow $\hat{\Gamma}$ to break $OSp(4,2|2)$ down to some subgroup of transformations commuting with $\hat{\Gamma}$. The only indispensable requirements are that this subgroup contains the Lorentz transformations and the *BRST*-transformation. Furthermore, $\hat{\Gamma}$ should not spoil modular invariance. (Recall from the appendix that *BRST*-invariance alone, *i.e.* invariance under J_{-a} , is sufficient to obtain dimensional reduction; full $OSp(1,1|2)$ -symmetry

^{*} This conclusion holds for all (even) D .

[†] A one-particle action for a chiral spinning particle has been given in ref. [35]. From the above discussion and its different constraint structure, we do not expect that it has a hidden $OSp(D,2|2)$ -symmetry, but work is in progress [25] to find the full symmetry associated to this model together with its modular-invariance and to study its second-quantized version.

is not compulsory.) Breaking down $OSp(4,2|2)$ to some of its subgroups, we have no apriori guarantee that the PCT-rotation S_{PCT} is contained in this subgroup. However, it turns out that requiring $[\hat{\Gamma}, \Sigma_{\mu\nu}] = 0$ together with $[\hat{\Gamma}, S_{mod}] = 0$ is sufficient to have $[\hat{\Gamma}, S_{PCT}] = 0$. In fact, the most general $\hat{\Gamma}$ which is Lorentz and modular invariant is of the form

$$\hat{\Gamma} = (\alpha_1 + \alpha_2 \gamma_5) \otimes (\beta_0 + \beta_3 \sigma_3) \otimes G_3 \quad (6.3)$$

This matrix commutes with S_{PCT} of eq.(5.4) for all values of the constants $\alpha_1 \dots$ and any Fock-space operators G_3 . Hence the special group element of $SO(4,2)$ inducing the PCT-transformation in 4-dimension continues to be an intact symmetry for any $\hat{\Gamma}$ which fulfils the minimal requirements of Lorentz and modular invariance. Recalling eq.(3.30), the $BRST$ -invariance of (6.3) leads to the condition $[\hat{\Gamma}, \Gamma_a \Gamma_-] = 0$. It is respected by setting

$$\hat{\Gamma} = \gamma_5 \otimes I \otimes I \quad (6.4)$$

which then leaves the full $OSp(1,1|2)$ unbroken. Higher dimensional theories resulting from this choice of $\hat{\Gamma}$ are invariant under the transformations of $O(3,1) \otimes OSp(1,1|2)$ connected to the identity and under the disconnected transformation S_{mod} . Upon dimensional reduction one obtains standard pseudoscalar and pseudovector terms like $\bar{\psi} \gamma_5 \psi \phi$ and $\bar{\psi} \gamma_5 \gamma^\mu \psi A_\mu$, etc. In general the transformations of the coset

$$\frac{OSp(4,2|2)}{O(3,1) \otimes OSp(1,1|2)} \quad (6.5)$$

will no longer be symmetries of the higher-dimensional action. An exception is the isolated S_{PCT} element of $SO(4,2)$. As we have seen, a Lorentz and modular invariant $\hat{\Gamma}$ cannot break this symmetry. This observation once more stresses the close relationship between PCT and modular-invariance. Note, however, that the S_{ex} transformation relating $\tilde{S}_{PCT}$ to S_{mod} is no longer a symmetry, it does not commute with (6.4).

So far we have only shown that modular invariance is *sufficient* for PCT-invariance. Is it also *necessary*? To investigate this question we use a $\hat{\Gamma}$ which is both Lorentz and $BRST$ -invariant, but does not commute with S_{mod} :

$$\hat{\Gamma} = \gamma_5 \otimes (a + b \sigma_+) \otimes I \quad (6.6)$$

Here $\sigma_+ \equiv \sigma_1 + i\sigma_2$ and $a, b \neq 0$ are constants. As an example, let us look at a simple

Yukawa-type theory defined by the generating functional

$$Z[J, Q] = \int [d\Phi d\Psi d\bar{\Psi}] \exp[i \int d^{D+4} \{L + J\Phi\} + iS_Q] \quad (6.7)$$

with the Lagrangian

$$L = \frac{1}{2} \partial_A \Phi \partial^A \Phi - \frac{1}{2} \mu^2 \Phi^2 + \bar{\Psi} [i\Gamma^A \partial_A - m + ig\hat{\Gamma}\Phi] \Psi \quad (6.8)$$

Restricting the sources to the physical subspace and integrating out the $(X^r \neq 0)$ -variables, as in section (4), we end up with

$$\begin{aligned} Z[J^{res}, Q^{res}] = & \int [d\Phi(X^\mu, 0, 0, \bar{0})] \exp[i \int d^D X \{ (\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \\ & - \frac{1}{2} \mu^2 \Phi^2)(X^\mu, 0, 0, \bar{0}) + j(X^\mu) \Phi(X^\mu, 0, 0, \bar{0}) \}] \\ & \cdot \det[S_0^{-1}] \cdot \det[1 + igS_0 \hat{\Gamma}\Phi(X^\mu, 0, 0, \bar{0})] \end{aligned} \quad (6.9)$$

where the "propagator"

$$S_0 \equiv [i\Gamma^\mu \partial_\mu - m + q]^{-1} \quad (6.10)$$

contains the source matrix q . Expanding the second determinant of (6.9) in the usual fashion and using the $\hat{\Gamma}$ given by (6.6), one finds:

$$\det[1 + igS_0 \hat{\Gamma}\phi(X^\mu)] = - \sum_{N=1}^{\infty} \frac{(-ig)^N}{N} \text{Tr}[S_0 \gamma_5 \phi]^N \text{tr}'(a + b\sigma_+)^N \quad (6.11)$$

Here "Tr" denotes summation over the first spinor index and integration over X^μ , whereas tr' is the trace in (δ_2, n) -space. Since according to the dimensional regularization scheme $\text{tr}'\sigma_+ = 0$, we have

$$\text{tr}'(a + b\sigma_+)^N = \text{tr}'(a^N + Nb\sigma_+) = a^N$$

This means that, at least perturbatively, (6.11) and therefore also the generating functional (6.7), is actually independent of b , so that we can work equally well with the standard choice $(a = 1, b = 0)$ from the very beginning. This choice respects both modular and parity invariance. Thus it is not possible to construct a PCT-violating theory starting from (6.6). The mechanism by which this is avoided is a kind of "non-faithful" dimensional reduction: certain types of higher-dimensional interaction terms do not have 4-dimensional

counterparts. An extreme example would be the choice ($a = 0, b = 1$). In this case $\hat{\Gamma}^2 = 0$ and the interaction term disappears completely in the process of dimensional reduction. This mechanism also can take place in more general theories involving axial-vector fields, for instance. Thus we find that even spoiling modular invariance at the higher-dimensional level does not give rise to PCT-violating 4-dimensional theories. This is a reflection of Lüders[33] proof of the PCT-theorem, who showed that it is simply not possible to write down a local, Lorentz-invariant, and hermitian Lagrangian which violates PCT (with the correct spin-statistics relation assumed). Hence, rigorously speaking, in the general case when $OSp(4,2|2)$ is broken, we cannot really prove that modular invariance is also *necessary* for the PCT-invariance. It seems that the status of modular invariance is "somehow" similar to the role played by microcausality in Lüders analysis. From Jost's proof[34] it is known that the deep reason of the PCT-invariance actually is microcausality, but this is not uncovered if one just checks which types of terms may appear in a "sensible" Lagrangian. Similarly, with the present formalism, it is possible to show the equivalence of PCT-invariance and modular invariance only if $OSp(4,2|2)$ is unbroken. In the broken case modular and Lorentz invariance is sufficient for PCT-invariance, but it is not possible to prove, at the level of *Lagrangians*, that it is also necessary. (It is necessary, however, for a "faithful" dimensional reduction.) What would be needed is an analogue of Jost's axiomatic approach which does not employ any specific lagrangian. Another point of view one might adopt here is that, even using a $\hat{\Gamma}$ like (6.6), it is not possible to spoil modular invariance since the contents of the theory remain the same even putting $b = 0$ from the beginning.

7. CONCLUSIONS

In this paper we have shown that also for charged and higher-spin particles, it is possible to formulate the *BRST*-field theory in a way which allows us to combine the Poincaré generators with the *BRST*-charge in the super-group $IOSp(4,2|2)$. In this framework the PCT-transformation can be considered either as a continuous rotation in the extended bosonic space or as a modular transformation "seen" in a rotated frame, *i.e.* the modular and the PCT-transformation are essentially the same. However, we also found that in *BRST*-field theory leading to chiral 4-dimensional theories, the $OSp(4,2|2)$ is broken to $O(3,1) \otimes OSp(1,1|2)$ and so the PCT-element of $SO(4,2)$ (which nevertheless remains a symmetry) can no longer be reached from the identity. We suspect, however, that the equivalence of modular and PCT-transformation still holds. This conjecture is supported

by the fact that Lorentz and modular invariant higher dimensional Lagrangians are automatically $SPCT$ -invariant, whereas a violation of modular invariance entails a non-faithful reduction only. Clearly more work is needed [25] to settle these intriguing questions.

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APPENDIX A

An important tool in the proof of dimensional reduction is the "magic formula" [19] of Parisi-Sourlas field theory:

$$\int dx_1 dx_2 d\tilde{x}_1 d\tilde{x}_2 g(x_1^2 + x_2^2 + 2i\tilde{x}_1 \tilde{x}_2) = (2\pi i)g(0) \quad (\text{A1})$$

valid for any function vanishing at infinity. (Here $x_i(\tilde{x}_i)$ are commuting (anticommuting) variables.) In this appendix we derive a generalization of (A1) which can be used in the space of vector fields. Recall from (I) that actually it is not (A1) which appears in the dimensional reduction of the *BRST*-field theory, but rather its "Minkowski" analogue ("Minkowski" with respect to $(x_1, x_2) \equiv (X_D, X_{D+1})$ -space)

$$\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int d\tilde{x}_1 d\tilde{x}_2 g(x_1^2 - x_2^2 + 2i\tilde{x}_1 \tilde{x}_2) = \pm 2\pi g(0) \quad (\text{A2})$$

which is valid if $g(y)$, with $y \equiv x_1^2 - x_2^2 + 2i\tilde{x}_1 \tilde{x}_2$, is holomorphic in the upper (lower) y -half-plane. In (I) eq. (A2) was derived via the Fourier-transform of g . It also can be obtained by noting that, if $g(y)$ is holomorphic in the upper (lower) y -half plane, it is holomorphic for $(\text{Re} x_2)(\text{Im} x_2) < 0$ or > 0 if we keep $x_1, \tilde{x}_1$ and $\tilde{x}_2$ real. This allows for a clockwise (counter-clockwise) rotation of the x_2 -integration path, which brings (A2) back to (A1). Now let us consider a general scalar function $F(x_+, x_-, \tilde{x}_a)$ depending on the light-cone variables $x_{\pm} = (x_1 \pm x_2)/\sqrt{2}$ and the ghosts $\tilde{x}_a$, but not necessarily through their *OSp*(1,1|2)-invariant combination y . We would like to compute [36]

$$I \equiv \int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_- \int d\tilde{x}_1 d\tilde{x}_2 F(x_+, x_-, \tilde{x}_a) \quad (\text{A3})$$

Introducing the superfield expansion

$$F(x_+, x_-, \tilde{x}_a) = \psi(x_+, x_-) + \tilde{x}_a \Theta^a(x_+, x_-) + \tilde{x}_1 \tilde{x}_2 \phi(x_+, x_-) \quad (\text{A4})$$

it follows that

$$I \equiv - \int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_- \phi(x_+, x_-) \quad (\text{A5})$$

To further evaluate eq.(A5), let us assume that F is invariant under some (but not neces-

sarily all) of the $OSp(1,1|2)$ transformations generated by

$$J_{r,s} \equiv X_r P_s - (-)^{[r][s]} X_s P_r$$

where $r \equiv (1, 2, a) \equiv (\pm, a)$:

$$[J_{r,s}, F] = 0$$

There are only two interesting cases: from $[J_{-,1}, F] = 0$ as well as from $[J_{-,2}, F] = 0$ one obtains

$$i \frac{\partial \psi}{\partial x_+} = x_- \phi \quad (\text{A6})$$

(together with some irrelevant conditions on the θ 's) and both $[J_{+,1}, F] = 0$ and $[J_{+,2}, F] = 0$ imply

$$i \frac{\partial \psi}{\partial x_-} = x_+ \phi \quad (\text{A7})$$

Let us assume (A6) is valid so that we may write

$$I = -i \int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_- \frac{1}{x_-} \frac{\partial \psi}{\partial x_+} \quad (\text{A8})$$

Furthermore we make the same assumption as before, namely that F , and hence ψ is holomorphic for $(\text{Re} x_2)(\text{Im} x_2) > 0$ or for $(\text{Re} x_2)(\text{Im} x_2) < 0$ and x_1 real. This allows us to perform a Wick rotation in x_2 , so that x_+ and x_- are replaced by the complex variables $z \equiv x_1 + ix_2$ and $\bar{z} \equiv x_1 - ix_2$. Now we apply the identity

$$\int dz d\bar{z} \frac{1}{\bar{z}} \frac{\partial f(z, \bar{z})}{\partial z} = -2\pi i f(0, 0) \quad (\text{A9})$$

valid for any continuously differentiable complex function $f(z, \bar{z})$ vanishing at infinity. Thus the final result reads

$$\int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} dx_- \int d\tilde{x}_1 d\tilde{x}_2 F(x_+, x_-, \tilde{x}_a) = \pm 2\pi F(0, 0, \tilde{0}) \quad (\text{A10})$$

for any F holomorphic in one of the regions $(\text{Re} x_2)(\text{Im} x_2) > 0$ or < 0 , $x_1, \tilde{x}_a$ real, and vanishing at infinity. If we had selected (A7) instead of (A6) we would have arrived at the

same answer (A10)^{*}. Comparing (A10) to (A2) we learn that for the dimensional reduction to occur, it is by no-means necessary that the integrand depends on the invariant y only. It is completely sufficient that one of the four conditions $[J_{\pm,a}, F] = 0$ holds. In more physical terms, this means that *BRST*-invariance alone, rather than the full $OSp(1,1|2)$ -invariance, is sufficient for the dimensional reduction[†]. Equation (A10) is easily generalized to functions depending on several variables $x^{(i)} = (x_+^{(i)}, x_-^{(i)}, \tilde{x}_a^{(i)})$, $i = 1, \dots, n^*$.

$$\int d^4x^{(1)} \dots d^4x^{(n)} F(x^{(1)}, \dots, x^{(n)}) = (\pm 2\pi)^n F(0, \dots, 0) \quad (\text{A11})$$

This equation is true if the holomorphy condition, mentioned above, holds for all variables separately and if at least one of the equations $[J_{\pm,a}, F] = 0$ is valid for $J_{\pm,a} = \sum_i J_{\pm,a}^{(i)}$ where $J_{\pm,a}^{(i)}$ acts on the i -th argument. The proof of (A11) proceeds iteratively. One first shows that $F^{(1)}$ given by

$$F^{(1)}(x^{(1)}) = \int d^4x^{(2)} \dots d^4x^{(n)} F(x^{(1)}, \dots, x^{(n)}) \quad (\text{A12})$$

is $J_{\pm,a}^{(1)}$ invariant if F is $J_{\pm,a}$ invariant and then one uses (A10) to put the first argument of F to zero. The $J_{\pm,a}^{(1)}$ -invariance follows from the fact that a $J_{\pm,a}^{(1)}$ -transformation of $x^{(1)}$ can be "undone" by a transformation on $x^{(2)}, \dots, x^{(n)}$ on the RHS of (A12). This is so because F is $J^{\pm,a}$ -invariant and the Jacobian of the transformation is unity [36]. In this way one can use (A10) to successively integrate all $x^{(i)}$ to zero.

Next consider a field $A^r(x^s)$ transforming as a vector under $OSp(1,1|2)$ and a functional $F[A^r]$ which is invariant under at least one of the generators $J_{\pm,a}$. Then applying (A12) to the field variable A^r , leads to the following formal functional integration relation

$$\int \prod_{x^s} \frac{d^4 A^r(x^s)}{2\pi} F[A^r] = F[0] \quad (\text{A13})$$

To give a well-defined meaning to the LHS of (A13), we may assume a discretization of x^r -space. For (A13) to be valid one has to assume that a clockwise Wick rotation of A_2 is allowed. Equation (A13) will be the main tool in proving dimensional reduction of Yang-Mills fields.

* Note that for the special case $F = F(y)$ this holomorphy condition goes over into the more familiar requirement of y -holomorphy in the upper or lower half-plane. Recall also from (I) that it was this requirement which had the important physical consequence of selecting the *causal* propagator.

† See also R.Kirschner [37] for related observations.

* This formula is a special case of a more general integral theorem for supersymmetric invariants recently proven in ref. [36].

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