SUPER-POINCARÉ COVARIANT QUANTIZED GS HETEROTIC STRING

AS A FREE CONFORMAL THEORY

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A B S T R A C T

The Green-Schwarz action for the heterotic string is quantized in a super-Poincaré covariant gauge. The gauge-fixed action is completely quadratic and corresponds to a free conformal theory. The theory has a correct counting of degrees of freedom. The conformal anomaly vanishes.

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In this paper we will consider the quantization of the heterotic string in the Green-Schwarz formulation

$$S_{cl} = \int d^2z \left\{ e \Pi_{\bar{z}}^{\mu} \Pi_{\bar{z}}^{\mu} + i \partial_{\bar{z}} X^{\mu} \bar{\theta} \Gamma_{\mu} \partial_{\bar{z}} \theta - i \partial_{\bar{z}} X_{\mu} \bar{\theta} \Gamma^{\mu} \partial_{\bar{z}} \theta - \frac{i}{2} \bar{\psi}^I \partial_{\bar{z}} \psi^I \right\}. \quad (1)$$

This action has a fermionic κ symmetry, which is infinite-reducible [1-3], since

$$\delta \theta^{\alpha} = \not{\Pi}_{\bar{z}}^{\alpha\beta} K_{z\beta}; \quad \delta K_{z\beta} = \not{\Pi}_{\bar{z}\beta\gamma} K^{\gamma}_{zz}; \dots; \delta K_{\underbrace{z\dots z}_P} = \not{\Pi}_{\bar{z}} K_{\underbrace{z\dots z}_{P+1}}, \quad (2)$$

and $\Pi_{\bar{z}}^{\mu} \Pi_{\bar{z}}^{\mu} = 0$ is an equation of motion.

In [1] we proposed to quantize this theory using the Batalin-Vilkovisky method [4]. The gauge-independent content of the theory was presented by the solution of the BV master equation in the minimal sector. This minimal sector includes $\Phi_{min}^A = \{X^{\mu}, \sqrt{g} g^{ab} = \gamma^{ab}, \theta^{\alpha} \equiv \theta_{0,0}^{\alpha}, C_{\alpha z} \equiv \theta_{1,0}^{\alpha}, C_{zz}^{\alpha} \equiv \theta_{2,0}^{\alpha}, \dots, C_a\}$, where $C_{\underbrace{z\dots z}_p} \equiv \theta_{p,0}$ are κ -symmetry ghosts for ghosts, and C_a are the reparametrization ghosts.

The solution of the master equation was given in [1,5] in the form

$$S_{min}(\Phi_{min}, \Phi_{min}^*) = S_{cl} + \Phi_A^* S^A(\Phi) + \Phi_A^* \Phi_B^* S^{AB}(\Phi). \quad (3)$$

This solution is quite complicated, since it takes into account both on-shell infinite reducibility and the non-closure of the algebra,

$$S_{min} = S_{cl} + \Phi_A^* \delta_{g.c.} \Phi_A - 8 \gamma^{*zz} \partial_{\bar{z}} \bar{\theta} \theta_{1,0} + \sum_{P=0}^{\infty} \bar{\theta}^{*P,0} W_{P,0} + X_{\mu}^* W^{\mu} + C^{*z} W_z. \quad (4)$$

We have used the compact notation, in which $\delta_{g.c.} \Phi^A$ is the general covariant transformation of all fields Φ^A with C_a as a parameter. The functions $W_{\kappa,0}, W^{\mu}, W_z$ depend on γ^{*zz} and on Φ_{min}^A . In particular,

$$W_{P,0} = \gamma^{*zz} \theta_{P+2,0} + V_{P,0}(\Phi_{min}^A). \quad (5)$$

Since the proof of consistency of BV quantization existed only for gauge systems with finite reducibility [4], we proposed in [1] a special procedure of truncation of an infinite ghost system. In this way it was possible to satisfy the completeness condition for BV quantization, to obtain the correct counting of

degrees of freedom and to prove that the conformal anomaly vanishes in this theory [6].

It was shown also that the quantized theory is independent of the level of truncation as well as of the type of truncation. The proof was given by integrating off the last level of ghosts and Lagrange multipliers. A simple illustration of this theorem was given by using the truncated triangles, see Figs. 1 and 2 in Ref. [7]. For practical applications we have used the first level truncation [6].

Recently the new Lorentz covariant gauges with infinite ghost system were proposed and used for quantization of the superparticle [8-10]. The main idea was to use the gauge condition $\dot{\theta}_\alpha = 0$. Since equations of motion are $\Pi_z \dot{\theta} = 0$, this gauge fixing on-shell is equivalent only to eight conditions, which is the number of independent κ -symmetries. The superparticle theory in these gauges was found to be free [8-10]. It was conjectured [9-11] that one can find similar gauges in the superstring theory, in which the quantized GS string will be quadratic in all fields. However, it was not clear if this conjecture is actually true and, if true, in which particular variables the gauge-fixed action is quadratic.

In this paper we will perform an explicit quantization of the GS heterotic superstring in super-Poincaré covariant gauges and prove that the gauge-fixed action is free, i.e., only quadratic in all fields. Moreover, it will be shown that the quantized GS theory in proper variables is a conformal theory. This quantization will provide a limit $n \rightarrow \infty$ of the truncated ghost system [1].

In what follows we will investigate the theory with the action (3), (4), but the only essential property of this action which will be necessary for the derivation of the final result of this paper is the following. The term linear in γ^{*zz} contains $\partial_z \theta$ and the terms with $\gamma^{*z\bar{z}}$ and $\gamma^{*zz} \gamma^{*z\bar{z}}$ are absent. Higher powers of $\gamma, \bar{\gamma}$ are absent because they are anticommuting fields. The dependence on X_μ^*, C^{*a} and $\theta^{*p,0}$ can be arbitrary! The essential information about the theory is that the transformation rules of the metric are

$$\delta \gamma_{zz} \sim \partial_z \bar{\theta} \kappa_z, \quad \delta \gamma_{\bar{z}\bar{z}} = 0,$$

and that there is no non-closure function in the algebra, corresponding to $\gamma^{*zz} \gamma^{*z\bar{z}}$. Any other details about the solution of master equation, e.g., the fact that there is no dependence on Φ^* higher than quadratic, are relevant to BRST rules but irrelevant for the derivation of the main result of this paper.

We use the solution of BV equation in the minimal sector (4) and enlarge the minimal sector by introducing antighosts, extra ghosts and Lagrange multiplier ghosts

$$S(\Phi, \Phi^*) = S_{\min} + \bar{b}_{ab}^* P_{cd}^{ab} \lambda^{cd} + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\theta}^{*p,q} \pi_{p,q} + \bar{\pi}_{\bar{z}}^{p,q} \theta_{p,q}^* z, \quad (6)$$

where $q \geq 0, p \geq q$,

$$\Phi = \{X, \gamma, b, c, \lambda; \theta_{p,q}, \pi_{p,q}, \bar{\theta}_{\bar{z}}^{p,q}, \bar{\pi}_{\bar{z}}^{p,q}\}.$$

The gauge fermion Ψ is defined as follows:

$$\Psi = \bar{b}^{ab} P_{ab}^{cd} \gamma_{cd} + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\theta}_{\bar{z}}^{p,q} \nabla_{\bar{z}} \tilde{\theta}_{p,q}. \quad (7)$$

We use the following notations in Eqs. (6) and (7)

$$\theta_{p,q} = \theta_{\underbrace{z \dots z}_p \underbrace{\bar{z} \dots \bar{z}}_q}, \quad \bar{\theta}^{p,q} = \bar{\theta}_{\underbrace{\bar{z} \dots \bar{z}}_p \underbrace{z \dots z}_q},$$

$$\theta_{p,q}^* z = \theta_{p+1,q}^*, \quad \bar{\theta}^{p+1,q} = \bar{\theta}_{\bar{z}}^{p,q}. \quad \tilde{\theta}_{p,q} = \theta_{p,q} + \theta_{p+1,q+1} \gamma^{z\bar{z}} \quad (8)$$

An analogous definition can be given for $\pi_{p,q}, \bar{\pi}^{p,q}$. The terms with γ, γ^* related to reparametrization symmetry are explained in Ref. [1]. We choose here the conformal gauge; however, any other one may be chosen as well. In particular we may use the harmonic gauge to fix reparametrization.

The proliferation of auxiliary fields is shown in Figs. 1 and 2. The only difference between triangles is that in Fig. 1 at the top of the triangle θ_{00} is an anticommuting spinor field and π_{00} at the top of the triangle in Fig. 2 is a commuting spinor field. The statistics of the fields alternates from one horizontal line to another one. The conformal weights of fields in curved space are $(p-q)$ for both $\theta_{p,q}$ and $\pi_{p,q}$, and $(-p+q)$ for both $\bar{\theta}^{p,q}$ and $\bar{\pi}^{p,q}$ *). In the Figures, they are given at the top of each field. The conformal weights of all fields coincides exactly with the ghost numbers, as defined in [4]. Also the rule for conformal weights coincide with that for ghost numbers: the sum of conformal weights of any two fields connected by an arrow equals -1.

*) This corresponds to a standard definition of conformal dimensions in $d = 2$ conformal theory.

The bold line in the right-hand side of the triangle on Fig. 1 is the "algebraic branch", which contains the classical field θ and ghosts of the minimal sector, generating the gauge algebra. The conformal weights are increasing along this branch. The next branch, starting with $\frac{-1}{\theta} 1,0$, gives antighosts. The absolute values of conformal weights are also increasing along this branch. The next branches contain extraghosts, antighosts for extraghosts, etc.

The surface $\Phi^* = (\partial\Psi/\partial\Phi)$ is given by

$$\gamma^* = b, \quad X_\mu^* = c_a^* = 0,$$

$$\theta_{p,q,z}^* = \nabla_z \theta_{p,q}, \quad \bar{\theta}^{*p,q} = \nabla_{\bar{z}} \bar{\theta}^{p,q}. \quad (9)$$

The gauge-fixed Lagrangian is obtained from $S(\Phi, \Phi^*)$ in Eq. (6) at the surface $\Phi^* = \delta\Psi/\delta\Phi$:

$$\begin{aligned} L_{g.f.} = & L_{cl} + b \partial c + \bar{b} \bar{\partial} \bar{c} + \lambda^{zz} \gamma_{zz} + \lambda^{\bar{z}\bar{z}} \gamma_{\bar{z}\bar{z}} \\ & + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \left\{ \bar{\pi}_{\bar{z}}^{p,q} \nabla_z \theta_{p,q} + \bar{\theta}_{\bar{z}}^{p,q} \nabla_z \pi_{p,q} \right\} \\ & - 8 b^{zz} \partial_z \bar{\theta} \theta_z - \sum_{p=0}^{\infty} \bar{\theta}_{\bar{z}}^{p,0} \nabla_z W_{p,0}(\gamma^*=b, \dots). \end{aligned} \quad (10)$$

The classical Lagrangian can be presented as

$$L_{cl} = \partial_z X^\mu \partial_{\bar{z}} X^\mu + \delta \bar{\pi}_{\bar{z}}^{0,0} \nabla_z \theta - \frac{i}{2} \bar{\Psi} \partial_z \Psi.$$

We perform the shift of variables in Eq. (10)*)

$$\begin{aligned} \bar{\pi}_{\bar{z}}'^{0,0} &= \bar{\pi}_{\bar{z}}^{0,0} + \delta \bar{\pi}_{\bar{z}}^{0,0} - 8 b_{\bar{z}}^{zz} \theta_z, \\ \pi_{p,0}' &= \pi_{p,0} + b^{zz} \theta_{p+2,0} + V_{p,0}, \end{aligned} \quad (11)$$

and integrate over the Lagrange multipliers λ_{zz} , $\lambda_{\bar{z}\bar{z}}$ to get the conformal gauge. The function $V_{p,0}$ is defined in Eqs. (4) and (5).

*) Our quantization is different from the generalization of the superparticle quantization [8]. The difference is in the distribution of conformal weights (or ghost numbers) along the horizontal lines. Therefore we never use the redefinition $\tilde{\pi}_{p,q} = \pi_{p,q} + \pi_{p-2,q-2}$, as in [8].

Any dependence of $S(\phi_{\min}, \phi_{\min}^*)$ on X_μ^* and C^{*a} is irrelevant since $X_\mu^* = C^{*a} = 0$ in the gauge fixed theory. Any dependence on $\bar{\theta}^{*p,0}$ on the surface (9) becomes a dependence on $(\partial\bar{\theta})^{p,0}$, and these terms are excluded by a shift of $\pi_{p,0}$. Any dependence on $\partial_z \theta$ (like that from the classical action and from $b^{zz} \partial_z \theta \theta_z$) can also be shifted into $\pi^{1,0}$. Thus the absence of terms $\bar{b}^{zz} S_{zz}$ and $\bar{b}^{*zz} b^{*zz} S_{zz,zz}$, was the only essential property of $S(\Phi_{\min}, \Phi_{\min}^*)$ with $\Phi^* = \partial\psi/\partial\Phi$ for the gauge fermion ψ in Eq. (7).

The resulting gauge fixed Lagrangian describes a free, manifestly super-Poincaré covariant conformal system (we omit primes in $\pi, \bar{\pi}$),

$$\begin{aligned} L_{g.f.} = & \partial_z X^\mu \partial_{\bar{z}} X^\mu + b \partial c + \bar{b} \bar{\partial} \bar{c} - \frac{i}{2} \bar{\psi}^I \partial_{\bar{z}} \psi^I \\ & + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \left\{ \bar{\pi}_{\bar{z}}^{p,q} \partial_z \theta_{p,q} + \bar{\theta}_{\bar{z}}^{p,q} \partial_z \pi_{p,q} \right\}. \end{aligned} \quad (12)$$

Using the notations $\mathcal{D}_{\bar{z}}(\mathcal{H})$ for the complete triangles in Figs. 1 and 2 we can rewrite the Lagrangian in the form

$$\begin{aligned} L_{g.f.} = & \partial_z X^\mu \partial_{\bar{z}} X^\mu + b \partial c + \bar{b} \bar{\partial} \bar{c} - \frac{i}{2} \bar{\psi}^I \partial_{\bar{z}} \psi^I \\ & + \bar{\mathcal{D}}_{\bar{z}} \partial_z (\mathcal{H}), \end{aligned}$$

$$\mathcal{H} = \{ \theta_{p,q}, \pi_{p,q} \}, \quad \bar{\mathcal{D}}_{\bar{z}} = \{ \bar{\pi}_{\bar{z}}^{p,q}, \bar{\theta}_{\bar{z}}^{p,q} \}. \quad (13)$$

Thus we have shown that the gauge-fixed GS action for the heterotic string describes a free conformal field theory, since every term in (12) is of the form $b\partial c$, with the conformal weight of c equal to $\lambda = p-q$, that of c equal to $1-\lambda = 1-p+q$. We keep the notations with two numbers for the purpose of labelling fields.

The generalized space-time supersymmetry^{*)} is realized as follows:

$$\delta \mathcal{H} = \mathcal{E}, \quad \delta \bar{\mathcal{D}}_{\bar{z}} = \bar{\Omega}_{\bar{z}}, \quad \partial_z \mathcal{E} = \partial_z \bar{\Omega}_{\bar{z}} = 0,$$

$$\mathcal{E} = \{ \varepsilon_{p,q}, \omega_{p,q} \}, \quad \bar{\Omega}_{\bar{z}} = \{ \bar{\omega}_{\bar{z}}, \bar{\varepsilon}_{\bar{z}} \}. \quad (14)$$

These symmetries can be supplemented by transformations of X , $\mathcal{H}$ and $\bar{\mathcal{D}}_{\bar{z}}$. In particular we have

*) This kind of symmetry was discussed in [9].

$$\delta X^\mu = \bar{\mathcal{E}} \Gamma^\mu \Theta + \bar{\mathcal{D}}_{\bar{z}} \Gamma^\mu \Omega_z, \quad \partial \bar{\mathcal{E}} = \partial \Omega_z = 0, \quad (15)$$

$$\delta \bar{\mathcal{D}}_{\bar{z}} = -2 \bar{\mathcal{E}} \Gamma_\mu \partial_{\bar{z}} X^\mu, \quad \delta \Theta = 2 \Gamma_\mu \Omega_z \partial_{\bar{z}} X^\mu,$$

$$\bar{\mathcal{E}} = \{ \bar{\varepsilon}^{p,q}, \bar{\omega}^{p,q} \}, \quad \Omega_z = \{ \omega_{p,q,z}, \varepsilon_{p,q,z} \}.$$

The usual space-time supersymmetry works only on the top of the triangle,

$$\delta \theta_{0,0} = \varepsilon_{0,0}, \quad \delta X^\mu = \bar{\varepsilon}^{0,0} \Gamma^\mu \theta_{0,0}, \quad \delta \bar{\pi}_{\bar{z}}^{0,0} = -2 \bar{\varepsilon}^{0,0} \Gamma_\mu \partial_{\bar{z}} X^\mu. \quad (16)$$

Let us show that despite the gauge system is infinite, it has the proper completeness condition [4], correct counting of physical degrees of freedom and the conformal anomaly vanishes. For this purpose, it is useful to present the π, θ part of the Lagrangian as a sum over branches,

$$L_{\mathcal{D}, \Theta} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{1-n}^{(m)} \partial C_n^{(m)},$$

$$C_{n=p-q}^{(0)} = \{ \theta_{0,0}, \theta_{1,0}, \theta_{2,0}, \dots \}, \quad C_{n=p-q}^{(1)} = \{ \pi_{0,0}, \pi_{1,0}, \pi_{2,0}, \dots \},$$

$$B_{1-n}^{(0)} = \{ \bar{\pi}_{\bar{z}}^{0,0}, \bar{\pi}_{\bar{z}}^{1,0}, \dots \}, \quad B_n^{(1)} = \{ \bar{\theta}_{\bar{z}}^{0,0}, \bar{\theta}_{\bar{z}}^{1,0}, \dots \}, \text{ etc.} \quad (17)$$

The completeness condition of BV requires that the (graded) number of degrees of freedom along the zero branch should be equal to $\text{rank} |\delta^2 S_{cl} / \delta \theta^\alpha \delta \theta^\beta|_{\text{on-shell}}$, which is equal to 8.

The fact that the completeness condition [4]

$$\text{rank} \left| \frac{\delta^2 S_{cl}}{\delta \theta^\alpha \delta \theta^\beta} \right|_{\text{on-shell}} = n_{\theta_{0,0}} - (m_{\theta_{1,0}} - (m_{\theta_{2,0}} - (\dots))) = 8$$

can be satisfied without truncation of the ghost sector was explained in [12]. We used the old rule, as given by Euler [13]: the sum $\sum_{n=0}^{\infty} a_n$ is defined as $\lim_{x \rightarrow 1-0} (\sum_{n=0}^{\infty} a_n x^n)$ when this limit exists. In this way we get

$$16 - 16 + 16 - \dots \Rightarrow \lim_{x \rightarrow 1-0} \sum_{n=0}^{\infty} 16 (-x)^n = \lim_{x \rightarrow 1-0} \frac{16}{1+x} = 8. \quad (18)$$

The zero branch starts with anticommuting spinor $\theta_{0,0}$ of conformal weight zero. The conformal weight increases along the branch by +1 with every vertex, and the statistics alternates along the branch. The first branch starts from commuting spinor $\pi_{0,0}$ of conformal weight 0, and again the conformal weights grow along the branch and statistics alternates. The next pair of branches has the same structure, etc.

The counting of a number of physical degrees of freedom and of conformal anomaly according to action (17) before regularization looks like

$$\sum_{m=0}^{\infty} (-1)^m \sum_{n=0}^{\infty} (-1)^n f_n , \quad (19)$$

where $f_n = 1$ for counting degrees of freedom and $f_n = 6n^2 + 6n + 1$ for counting anomaly. We have used the fact that the statistics alternates along the branch and between branches.

We use the same regularization as before and get

$$\sum_{m=0}^{\infty} (-x)^m \sum_{n=0}^{\infty} (-y)^n f_n . \quad (20)$$

For the number of degrees of freedom this gives

$$16 \lim_{x,y \rightarrow 1} \frac{1}{1+x} \cdot \frac{1}{1+y} = 4 . \quad (21)$$

For the conformal anomaly we use the equations^{*)}

$$\sum_{m=0}^{\infty} (-x)^m \sum_{n=0}^{\infty} (-y)^n (6n^2 + 6n + 1) = \frac{1}{1+x} \left(\frac{12}{(1+y)^3} - \frac{12}{(1+y)^2} + \frac{1}{1+y} \right) \equiv \frac{f(y)}{1+x} ,$$

$$\lim_{y \rightarrow 1} f(y) = -1 . \quad (22)$$

This shows that the contribution to anomaly from one branch is

$$2 \cdot 16 \lim_{y \rightarrow 1} f(y) = -32 , \quad (23)$$

and the contribution from all branches is

$$2 \cdot 16 \lim_{x,y \rightarrow 1} \frac{f(y)}{1+x} = -16 . \quad (24)$$

This is necessary to cancel conformal anomaly from x, b, c system, which is equal to -10+26.

We will discuss in the next publication [14] the relation between the fields of the action (12), (13) and the representations of orthosymplectic algebras [15,9,10].

*) This differs from the expression for the conformal anomaly, proposed in [10]. since we do not have $n+1$ conformal fields of the weight n .

In conclusion, we have found that the GS heterotic string gauge-fixed action is a free conformal field theory (12), (13), with manifest space-time Lorentz covariance and generalized supersymmetry. It contains besides bosonic string fields X^μ and b, c a double set of all conformal fields $\theta_{p,q}, \pi_{p,q}$ with conformal weights (p, q) and $\bar{\theta}^{p,q}, \bar{\pi}^{p,q}$ with conformal weights $(-p, -q)$ for $p \leq q, 0 \leq q < \infty$.

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FIGURE CAPTIONS

Fig. 1 Diagram for $\theta_{p,q}$, $\bar{\theta}^{p,q}$.

Fig. 2 Diagram for $\pi_{p,q}$, $\bar{\pi}^{p,q}$.

- Figure 1 -

- Figure 2 -