EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/89-34
February 23rd, 1989

CLIC NOTE 89

ATTAINING SUPERHIGH ENERGIES WITH e^+e^- COLLISIONS

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Lectures given at the Advanced Study Institute
on Techniques and Concepts of High Energy Physics,
St.Crox, Virgin Islands, July 14-25, 1988.

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1. INTRODUCTION

The purpose of the lectures given at the St. Croix Summer School was to introduce newcomers to the fastly developing field of high energy linear colliders. The main emphasis was on the *limitations* posed to the choice of the parameters by the physics of bunch-bunch collisions. Unfortunately very little time was left to discuss the different components of linear colliders, which today are the subject of R&D programs in many laboratories. The interest in this novel accelerator activity is justified by the fact that, in the last twenty years, electron-positron collisions have been producing a continuous stream of first rate physics results. To continue on the same trail it is now necessary to push the centre-of-mass energy to values larger than the $W \approx 200$ GeV achievable with LEP 200, which should be running in 1993-1994. In 1975 it was shown by B. Richter¹ that, due to synchrotron radiation, the optimal cost of e^+e^- storage rings increases as W^2 and it was quite natural to speculate² on the advantages of firing, one against the other, electron and positron bunches accelerated to hundreds of GeV by opposite linacs, since in this case there is no radiation and the cost is obviously proportional to W . A few months after the proposal, which prompted a lot of speculations, I was informed that ten years before M. Tigner had suggested³ a similar approach to produce electron-electron collisions at few GeV; this proposal had been forgotten, since at few GeV's synchrotron radiation was not yet a problem and storage rings work beautifully. During these ten years, some suggestions had been advanced by G.I. Budker and collaborators in Novosibirsk but not published and were unknown in the West, so that in preparing a review talk for the 1979 Lepton-Photon Conference, I went through the literature and collected all possible precedents. The interested reader can find them described in Ref. 4.

The crossing over point between the cost of circular and linear e^+e^- colliders depends on many assumptions, but everybody agrees that it is around $W \approx 300$ GeV⁴. Higher energy can be achieved only by linear colliders. At present the designs of 1-2 TeV linear colliders are under study in Europe, Japan, USA and USSR. To transform them into 'projects' will take at least three years, since many problems are still open. Most of them are consequences of the fact that the required luminosities are at least 10 times greater than the largest luminosities obtained at the present electron-positron storage rings: $L \approx 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$.

2. LUMINOSITY REQUIREMENTS

At $W \approx 1$ TeV we shall need very large luminosities because the point-like cross-section decreases as $1/W^2$. Fig. 1 is taken from the summary talk I gave in 1987 at the '*La Thuile Workshop on physics at future accelerators*', where the physics potentials of LHC (a 16 TeV proton-proton collider) and CLIC (a 2 TeV electron-positron collider) were compared⁵.

Fig. 1 Electron-positron cross section as a function of the centre-of-mass energy. The dashed line is the point-like cross section. The continuous line is measured up to about 45 GeV and computed above. The peak due to a 'superstring inspired' Z' is very high. Compositeness with a form-factor scale $\Lambda \approx 0.5$ TeV would have striking consequences on the total cross section. Sparticles ($\tilde{e}\tilde{e}$, $\tilde{W}\tilde{W}$ and $\tilde{\mu}\tilde{\mu}$) are expected to be produced with cross sections which are of the order of the point-like cross sections (dashed areas). The thick dotted line represents the cross section for the production of neutral higgsos having a mass of 0.5 TeV. (Figure adapted from Ref.5.)

The continuous curve represents what is known today of the hadronic 'annihilation' cross section, i.e. the sum of the cross sections of all the processes in which the positron and the electron disappear after having annihilated. The figure is based on experimental data up to $W \approx 45$ GeV and on the predictions of the Standard Model above 45 GeV. Since the overall trend is roughly parallel to the dashed curve, which represents the 'point-like' cross section applicable to $\mu^+\mu^-$ and $\tau^+\tau^-$ production, the figure proves that, in the explored energy range, quarks (and leptons) are point-like particles. The four sets of peaks appearing below about 10 GeV are due to the pair-creation of quark-antiquark pairs, which together have the same quantum numbers as the electron-positron pairs and, immediately after production, whirl thousands of times one around the other in a resonance state. The four sets are due to the production of unstable bound states of the quark pairs $u\bar{u}$ and $d\bar{d}$ (ρ and ω), $s\bar{s}$ (ϕ), $c\bar{c}$ (J/ψ) and $b\bar{b}$ (Υ). We would like to complete the figure by plotting the set of resonances due to $t\bar{t}$ bound states, but at present we only know that they must lie above $W \approx 80$ GeV and below $W \approx 400$ GeV. There are also indications that the lower limit could be as high as ~ 100 GeV.

The Z peak is not only taller than the others but, according to the Standard Model, it is also of a very different nature, because the Z boson is a *single* elementary particle having the quantum numbers of the initial electron-positron pair and not a composite system. After climbing on the Z^0 peak in 1989, LEP 200 will open the possibility of exploring the much smaller shoulder due to the production of pairs of charged bosons: $e^+ + e^- \rightarrow W^+ + W^-$.

The point-like cross section decreases as W^{-2} , so that at $W = 2$ TeV (CLIC) $\sigma_{\mu\mu} \approx 2.2 \times 10^{-38}$ cm²: by running one third of a calendar year at a luminosity $L = 10^{33}$ cm⁻²s⁻¹, only ~ 200 μ -pairs would be collected. Figure 1 shows that at CLIC the rate of production of W pairs in a 4π detector is 50 times larger than for μ -pairs. It is difficult to say whether such a large production rate will be 'useful' for physics or should be considered only as a source of background. Let us now consider the effect of 'new' physics.

In electron-positron annihilations the superstring-inspired Z' appears as an enormous peak, which is drawn in Fig. 1 for $m_{Z'} = 1$ TeV. Clearly at a 1 TeV collider it would not only be seen, but could be studied in detail. The cross section for a higgson of mass $m_H = 0.5$ TeV is large, since it corresponds to $\geq 10^3$ events per year at $W = 2$ TeV and $L = 10^{33}$ cm⁻²s⁻¹. Compositeness would be signalled by a flattening electron-positron total cross section, as indicated by the dash-dotted line of Fig. 1. For a value of the parameter Λ of the order of 0.5 TeV (i.e. for distances $d \approx 4 \times 10^{-17}$ cm) the effect shown in the figure is striking, so much so that we are reminded of the surprise of the physicists working in 1969 at Adone when, thanks to the pair-production of point-like quarks, they found a hadron production rate which greatly exceeded the expectations.

In Fig. 1 the shaded areas indicate the cross section ranges of three sparticle channels produced in the annihilation of an electron-positron pair. No unique value can be given because various parameters enter the calculations. In general one can state that selectron pairs are produced about 10 times more abundantly than wino pairs, whose cross section is of the same order as the point-like one (~ 200 events per year for $L = 10^{33}$ cm⁻²s⁻¹). Smuons are expected to be somewhat rarer, but are very interesting because it was shown at the *La Thuile Workshop* that, if found in the reachable mass range, their bosonic nature (spin $J = 0$) can be *proven* by measuring the angular distribution of the decay muons.

Fig. 1 shows that, around $W \approx 1$ TeV, the cross sections of new 'expected' phenomena fall in two broad categories: (i) more or less standard Z' production (with $m_{Z'} \approx W$) and compositeness (with a scale $\Lambda \approx W$) have $10^{-36} \leq \sigma \leq 10^{-34}$ cm² and (ii) SUSY particle production (with $m_{\text{SUSY}} \approx W/2$) and neutral higgson (with $m_H \approx W/2$) have $10^{-38} \leq \sigma \leq 10^{-36}$ cm². In the first case a luminosity $L = (W/\text{TeV})^2 10^{31}$ cm⁻²s⁻¹ gives between 10^2 and 10^4 events/year, where a running year is taken to be equivalent to 10^7 seconds. In the second case, one needs a luminosity $L = (W/\text{TeV})^2 10^{33}$ cm⁻²s⁻¹ to obtain 10^2 - 10^4 events/year. In short, these are the arguments which at the *La Thuile Workshop* brought physicists to require the 'large' luminosity

$$L \geq (W/\text{TeV})^2 10^{33} \text{ cm}^{-2} \text{ s}^{-1} \quad (1)$$

when entering in the new energy regime opened by TeV linear colliders⁵. Note that this luminosity is needed for *each interaction region*. Since at least two detectors have to be foreseen and, with a linear collider, the two interaction regions can only be served by sharing the available bunch-bunch collision rate, Eq. (1) implies that the collider has to provide a luminosity *twice* larger.

3. DISRUPTION

In the Introduction the opening remarks concerned the poor scaling laws of storage rings when increasing the center-of-mass energy. By now it is well understood that also the scaling laws of linear colliders are *not* very favourable: when in a 'Gedanken' experiment we increase the energy and pass from center-of-mass energies around $W = 100$ GeV (as today achieved, as far as energy is concerned, at the SLAC Linear Collider = SLC) to one TeV and, even more, to many TeV's, one is obliged to change the bunch-bunch 'regime'. At low energies the electrons and positrons radiate classically, when

bent by the megagauss magnetic fields produced by the opposite bunch. At many TeV's, to obtain the needed large luminosities ($L > 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$) the dimensions and densities of the bunches have to be modified and the average critical energy of the radiated photons $\bar{E}_c$ becomes larger than the beam energy E_0 ; this is the so-called *quantum regime*, which entails bunches which must be few microns long and have transverse dimensions of some tens of Angstroms. As for storage rings, synchrotron radiation is the cause of the poor scaling laws of linear electron-positron colliders which I want to discuss. More detailed arguments along the same lines can be found in the presentation which I gave at the CERN-USA Accelerator School⁶ in 1986.

We call *disruption* the focusing (or defocusing) produced on the particles of one bunch by the electric and magnetic fields of the opposite bunch. Its intensity is described by the '*disruption parameter*' D .

An electron (positron) deflected by the opposite bunch — which is supposed to have a 'round' cross-section of r.m.s. radius σ_x and length σ_z and to contain N particles — radiates electromagnetic energy. A typical particle at a distance σ_x from the axis of the bunch sees a magnetic field proportional to the incoming current ($\propto N/\sigma_z$) and inversely proportional to σ_x : $B \propto N/(\sigma_x \sigma_z)$. The radius of curvature ρ in this field, supposed to be uniform, is proportional to γ/B and thus

$$\rho = \gamma \sigma_x \sigma_z / (N r_e), \quad (2)$$

where $r_e = 2.82 \cdot 10^{-13} \text{ cm}$ is the classical electron radius. The deflection angle is $\theta = \sigma_z/\rho$ and, for small deflections, the focal distance is $F = \sigma_x/\theta = \sigma_x \rho/\sigma_z$. The disruption parameter is defined as $D = \sigma_z/F$, so that it is *small* when the focal distance is *large* with respect to the length of the bunch. By combining $D = \sigma_z^2/\sigma_x \rho$ with Eq. (2) we obtain, for round bunches of r.m.s. radius σ_x and r.m.s. length σ_z ,

$$D = \frac{r_e N \sigma_z}{\gamma \sigma_x^2}, \quad (3)$$

where $\gamma = E_0/mc^2$. When $D \leq 0.5$ the focusing is very little and one can indeed consider the bunch as a lens of focal length F ; for $D \geq 0.5$ there is a 'pinch' effect and the effective transverse dimensions of the bunches during the collisions are *smaller* than what one would compute from the properties of the final focus system.

The *natural* transverse area of the bunch is such that

$$\sigma_x^2 = \epsilon_n \beta^* / \gamma, \quad (4)$$

where ϵ_n is the '*invariant transverse emittance*' of the bunches (which does *not* change in a linear accelerator without wake field and space charge effects) and β^* the β -value of the final focus system at the interaction point. It can be shown⁷ that, for a fixed magnetic field at the poletip of the final focusing quadrupoles, the value of β^* increases with the energy of the focused particles as $\gamma^{1/3}$.

The pinch effect reduces the transverse dimensions with respect to σ_x , so that the effective radius can be written as $\sigma_x/\sqrt{H_D}$, where H_D is a function of D . The luminosity increases and takes the form

$$L = \frac{f_r N^2}{4\pi \sigma_x^2} H_D, \quad (5)$$

where f_r is the repetition rate of the bunch-bunch collisions and H_D is the '*enhancement factor*' due to the pinch effect.

For round beams Chen and Yokoya — by using simulation methods⁸ — have obtained simple formulae which express H_D as a function both of D and of the ratio $A = \sigma_z/\beta^*$:

$$\begin{aligned}
1 + 2D / 3\sqrt{\pi}, & \quad 0 \leq D \leq 0.5, \\
1 + 2D / 3\sqrt{\pi} + 0.43 [\ln(D/A)]^2, & \quad 0.5 \leq D \leq 2, \\
1.6 + 0.43 [\ln(D/A)]^2, & \quad 2 \leq D \leq 100.
\end{aligned} \tag{6}$$

These formulae are valid for round beams: $\sigma_x = \sigma_y$. Usually $D \simeq 30$ is considered to be a maximum value for the disruption parameter. For $D \geq 2$ the focusing is so strong the particles oscillate while traversing the opposite bunch. The average number of oscillations performed was computed to be $n \simeq (2D)^{1/2}/2\pi$, so that $n = 1$ for $D \simeq 20$. Chen and Yokoya found that the particles, coming from a narrow ring in the transverse plane, are focused on the beam axis within the oncoming bunch and give rise to the second term in the enhancement factor of Eqs. (6b) and (6c).

For $D = 5$ and $A = 1/4$ Eq. (6c) gives $H_D = 5.5$, a sizeable effect indeed. Recently Balakin and Solyak⁹ contested Eq. (6c) by claiming that above $D \simeq 10$ the numerically calculated enhancement factor *decreases* with D , as initially found by Hollebeek¹⁰. At $D = 20$ the two results differ by a factor 2. In the following we shall not consider disruption parameters larger than 4, and anyway the discrepancy does not influence our arguments.

By combining Eqs. (3) and (5) with the definition of *beam power*, i.e. the power of one beam,

$$P = N E_0 f_r \tag{7}$$

one gets the *first fundamental relation* of linear colliders

$$L/(10^{33} \text{ cm}^{-2} \text{ s}^{-1}) \simeq (D H_D/29) (\text{mm}/\sigma_z) (P/\text{MW}) \tag{8}$$

which express quantitatively the fact that the luminosity is proportional to beam power.

Eq. (8) implies $P > 1 \text{ MW}$ with $\sigma_z \simeq 0.1 \text{ mm}$ and $DH_D \simeq 1.5 \times 3 \simeq 5$, to obtain $L > 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ as needed for *physics reason* at $W > 1 \text{ TeV}$. This shows why high power beams made of short bunches are a *must* in TeV linear colliders.

4. NATURAL SCALING

Since by now the parameter list of SLC is well known, I reproduce it in Table 1. To scale these project parameters to a $(0.5 + 0.5) \text{ TeV}$, which I shall call the *Next Linear Collider* (NLC), it seems at first sight reasonable⁶ to keep the same type of accelerator, i.e. the same repetition rate f_r and the same bunch population N .

Luminosity is *proportional* to beam power; and beam power, for N and f_r constant, increases only *linearly* with γ . Since to follow Eq. (1) the luminosity has to be proportional to γ^2 , the needed extra factor γ can be introduced in Eq. (8) by keeping $D = \text{const}$ and by making $\sigma_z \propto \gamma^{-1}$. From Eq. (3) it is 'natural' to have $\sigma_x \propto \gamma^{-1}$ and scale the various quantities as summarised in Table 2.

In the Table I also assumed that β^* scales as $\gamma^{1/3}$, as shown in Ref. 7, so that $\epsilon_n \propto \gamma^{-4/3}$ to have the bunch radius of Eq. (4) proportional to γ^{-1} .

The results of the 'natural' scaling laws are collected in the last column of Table 1 which shows that for $W = 1 \text{ TeV}$ one would get $L = 6 \cdot 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$, close to the goal set by Eq. (1). The last line of Table 2 shows that the *new* parameter δ (to be defined in the next Section) has a frightening γ -dependence and, according to Table 1, it passes from $\simeq 10^{-3}$ at SLC to $\simeq 10$ at the 1 TeV Next Linear Collider we are considering. This is a very serious problem, which we have now to discuss.

Table 1
SLC parameters and scaling to a (0.5 + 0.5) TeV Collider

Parameter	SLC (Project values)	Scaling factor (from Table 2)	Scaled NLC
Beam energy, E_0 (GeV)	50	10	500
c.m. energy, W (GeV)	100	10	1000
Particles/bunch, N	7.2×10^{10}	1	7.2×10^{10}
Bunch radius, $\sigma_x = \sigma_y$ (μm)	1.65	0.1	0.165
Bunch length, σ_z (mm)	1.0	.1	0.1
Repetition rate, f_r (kHz)	0.18	0.18	0.18
Power/beam, P (MW)	0.1	10	1.0
β -value, β^* (mm)	5.0	2.1	10.5
Emittance, ϵ_n (m)	4×10^{-5}	0.046	1.9×10^{-6}
Disruption parameter, D	0.75	1	0.75
Enhancement factor, H_D	2.2	1	2.2
Luminosity, L ($\text{cm}^{-2}\text{s}^{-1}$)	6×10^{30}	100	6×10^{32}
Beamstrahlung parameter, δ	9×10^{-4}	10^4	9

Table 2
'Natural' scaling laws from SLC to the Next Linear Collider (NLC)

Quantity	γ -dependence	Quantity	γ -dependence
N	1	L	γ^2
f_r	1	P	γ
σ_z	γ^{-1}	$\beta_x^* = \beta_y^*$	$\gamma^{1/3}$
D	1	$\epsilon_{nx} = \epsilon_{ny}$	$\gamma^{-4/3}$
$\sigma_x = \sigma_y$	γ^{-1}	δ (defined by Eq. 9)	γ^4

5. BEAMSTRAHLUNG FOR ROUND BUNCHES

As anticipated, the problem is synchrotron radiation in the field of the opposite bunch. According to classical electrodynamics, the energy radiated per unit length by a particle, which moves along a trajectory of radius of curvature ρ , is $P_\ell = 2 r_e mc^2 \gamma^4 / (3\rho^2)$. Using for the radius of curvature Eq. (2), the *average fractional energy loss* in a length σ_z for a particle of energy E_0 which is only *slightly* deflected is $\delta \propto P_\ell \sigma_z / E_0$ and can be written in the form

$$\delta \simeq 2/9 (r_e^3 \gamma / \sigma_z) (N^2 / \sigma_x^2). \quad (9)$$

(The numerical factor, which equals 0.222, has a simple form useful for later simplifications. The exact form computed for gaussian bunches both in the transverse and the longitudinal directions is $8 \pi^{1/2} / 21 \simeq 0.215$.) The quantity δ has been dubbed '*beamstrahlung parameter*'.

Equation (9) shows that, as anticipated in Table 2, δ increases proportionally to γ^4 , if one decides to scale a collider with $N = \text{const}$, $f_r = \text{const}$, $P \propto \gamma$, $\sigma_x = \sigma_y \propto \gamma^{-1}$ and $\sigma_z \propto \gamma^{-1}$, so to have $L \propto \gamma^2$. At SLC $\delta \simeq 10^{-3}$ (Table 1), so that at NLC ($W = 1 \text{ TeV}$) with the 'natural' scaling of Table 2 one would have $\delta \simeq 10$, clearly unphysical since a particle cannot lose 10 times the energy it possesses. This is the reason for which, starting from SLC parameters of Table 1, the scaling laws of Table 2 *cannot* be followed above $E_0 \simeq 300 \text{ GeV}$.

In 1985 Himel and Siegrist showed¹¹ that at very large energies linear colliders have more favourable scaling laws than the ones implied by Eq. (9). The issue has been discussed and clarified in many recent papers¹² and I limit myself to the presentation of the main results.

In a uniform magnetic field the critical energy of the spectrum of the radiated photons has the form $E_c = (3 \hbar c \gamma^3) / (2\rho)$ so that, using Eq. (2), the *average fractional critical energy* $\bar{T} = \bar{E}_c/E$ has the form

$$\bar{T} \simeq 5/(12\alpha) (r_e^2 \gamma/\sigma_z) (N/\sigma_x), \quad (10)$$

where $\alpha = 1/137$ and the numerical factor comes from averaging on gaussian bunches¹². Himel and Siegrist showed that for large values of $\bar{T}$ the fractional energy loss is less than the classical estimate given by Eq. (9), but before introducing their result we have to consider flat bunches.

6. DISRUPTION AND BEAMSTRAHLUNG FOR FLAT AND PINCHING BUNCHES

'Flat' bunches have $\sigma_y < \sigma_x$. By introducing the aspect ratio

$$R = \sigma_x / \sigma_y \geq 1 \quad (11)$$

two disruption parameters determine now the focusing in the two planes.

$$D_x = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2 / (1 + R)], \quad (12)$$

$$D_y = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2 R / (1 + R)] = D = R D_x \geq D_x. \quad (13)$$

In the following the largest one, D_y , will still be called '*disruption parameter*' and indicated by the symbol D . For flat beams ($R > 1$) the enhancement factor is reduced with respect to the one given as a function of D in Eq. (6):

$$H \simeq H_D (1 + R)/2 R, \quad (14)$$

and the luminosity is

$$L = f_r N^2 H / (4\pi \sigma_x \sigma_y). \quad (15)$$

The *first fundamental relation* (8) becomes

$$L/(10^{33} \text{ cm}^{-2} \text{ s}^{-1}) \simeq [DH/29] [(1 + R)/2R] (\text{mm}/\sigma_z) (P/\text{MW}). \quad (16)$$

The fractional average critical energy $\bar{T}$ of Eq. (10) gets a non-trivial modification:

$$\bar{T} \simeq 5/(12\alpha) (r_e^2 \gamma/\sigma_z) (N/\sqrt{\sigma_x \sigma_y}) H_b^{1/2}(D, R), \quad (17)$$

where, following Chen¹³, I have introduced a new factor, which depends on D and R

$$H_b(D, R) \simeq 4R H_D [1 + R H_D^{(R-1)/2R}]^{-2}, \quad (18)$$

and which intervenes also in the fractional energy loss when $\bar{T} < 1$ and there is pinch effect:

$$\delta = 2/9 (r_e^3 \gamma/\sigma_z) (N^2/\sigma_x \sigma_y) H_b(D, R). \quad (19)$$

For round bunches ($R=1$) with pinch effect $H_b \approx H_D$ and the parameters $\bar{T}$ and δ are proportional to $H_D^{1/2}$ and H_D respectively. For very flat bunches ($R \gg 1$) Eq. (18) gives $H_b \approx 4R^{-1}$ and (with $\sigma_x \sigma_y = \text{const}$) $\bar{T} \propto R^{-1/2}$ while $\langle \delta \rangle \propto R^{-1}$.

Flat bunches can thus be used to reduce $\bar{T}$ and δ by large factors, while Eq. (16) is modified only by a factor 2.

7. COLLIDER REGIMES

If $\bar{T} \ll 1$ the collider runs in the so-called '*classical regime*'. By increasing the fractional critical energy $\bar{T}$ one enters in the '*quantum regime*', first studied by Himmel and Siegrist. In Ref. 6 I introduced an handy approximation, valid to $\sim 5\%$, which bridges the equations valid in the two regimes by writing:

$$\langle \epsilon \rangle \approx \delta F(\bar{T}), \quad (20)$$

where δ is given by Eq. (19) and

$$F(\bar{T}) \approx (1 + \bar{T}^{1/2} + 3\bar{T}/2)^{-4/3}. \quad (21)$$

The quantity $\langle \epsilon \rangle$ is the *average fractional energy loss*, which is equal to δ only when $\bar{T} \ll 1$ and can be much smaller than δ of Eq. (19) when $\bar{T} \geq 1$. It is generally assumed that NLC can accept relatively large values of the fractional loss, as $\langle \epsilon \rangle \approx 0.2-0.3$. This is due to the fact that such a value corresponds typically to center-of-mass spreads $\Delta W/W \approx 10\%$, which is not too large when running in the "continuum". The relation between the center-of-mass spread due to beamstrahlung $\Delta W/W$ and the fractional energy loss $\langle \epsilon \rangle$ is *not* linear. The interested reader can find the relevant arguments in Refs. 6 and 7. The function $F(\bar{T})$, called in SLAC H_T , is plotted in Fig. 2.

Fig. 2 The factor $F(\bar{T})$, which multiplies the beamstrahlung parameter δ and gives the fractional energy loss $\langle \epsilon \rangle$. The continuous line is the result of Noble numerical calculations⁷; the dashed curve is the simple analytic expression of Eq. (21).

For the scaled-up version of SLC appearing in Table 1, $R = 1$ and $H_b = H_D = 2.2$, so that from Eq. (17) $\bar{T} = 2.9$. Eq. (21) gives $F(\bar{T}) = 0.075$ and the beamstrahlung parameter $\delta = 9$ of Table 1 implies a fractional energy loss $\langle \epsilon \rangle \approx 0.7$. Such a loss is still *too* large with respect to what is generally considered acceptable ($\langle \epsilon \rangle \leq 0.3$): as anticipated, the scaling laws of Table 2 can be applied only up to $E_0 \approx 300$ GeV.

The fractional energy loss $\langle \epsilon \rangle$ of Eq. (20) can be written in a convenient form by expressing δ of Eq. (19) as a function of $\bar{T}$ and $(L/f_r)^{1/2}$. One obtains the *second fundamental relation* of linear colliders¹⁴:

$$\langle \epsilon \rangle = (16 \pi^{1/2} \alpha / 15) (H_b/H)^{1/2} (r_e^2 L/f_r)^{1/2} \bar{T} F(\bar{T}). \quad (22)$$

Since $\langle \epsilon \rangle$ is in practice fixed, the quantity $\bar{T} F(\bar{T})$ is very important because, together with R and D , it determines L/f_r .

Noble's numerical results¹² for $\bar{T} F(\bar{T})$ are plotted as a continuous line in Fig. 2, while the dashed line represents the approximation given by the simple form of Eq. (21). Three regimes can be clearly distinguished:

$$\begin{array}{ll} \text{classical regime:} & \bar{T} < 0.2 \\ \text{transition regime:} & 0.2 < \bar{T} < 10^2 \\ \text{quantum regime:} & 10^2 < \bar{T}. \end{array} \quad (23)$$

In the first regime $\langle \epsilon \rangle \propto \bar{T}$; in the second $\langle \epsilon \rangle \approx \text{constant}$ within a factor 2 and $\bar{T} F(\bar{T}) = 0.15 \pm 0.05$; and in the third $\langle \epsilon \rangle \propto \bar{T}^{-1/3}$. We shall conclude that the Next Linear Collider will run in the intermediate regime, where $\bar{T} F(\bar{T})$ is about constant. (Note that, since $\bar{T} = 2.9$, also the scaled version of SLC would have this property.)

Fig. 3 The function $\bar{T} F(\bar{T})$, which enters in the second fundamental relation of linear colliders [Eq. (22)], is plotted versus $\bar{T}$. The continuous curve is due to Noble¹² and the dashed curve represents the simple approximation⁶ of Eq. (21): $\bar{T} (1 + \bar{T}^{1/2} + 3\bar{T}/2)^{-4/3}$.

8. RELATIONS CONNECTING COLLISION PARAMETERS

We have introduced a large number of quantities to describe bunch-bunch collisions: E_0 (or γ), L , P , R , D (and $H = H(R, D)$; H_b), σ_z , f_r , N , σ_x , $\bar{T}$, $\langle \epsilon \rangle$. These *eleven* parameters are linked by the *five* equations (7), (13), (15), (17), (22), so that *six* parameters are enough to define the bunch-bunch collisions of any linear collider. To help the reader, the relevant formulae are collected in Table 3.

Table 3
Summary of collider formulae
(the symbol $\simeq$ signifies 5 – 10% accuracy)

Quantity	Formula	Eq.
Energy/beam	$E_0 = \gamma mc^2$	
r.m.s. radii	$\sigma_x = (\epsilon_{nx} \beta_x^* / \gamma)^{1/2}$ $\sigma_y = (\epsilon_{ny} \beta_y^* / \gamma)^{1/2}$ [$\sigma_x = R \sigma_y$]	(4)
Power/beam	$P = N f_r E_0$	(7)
Disruption par.	$D = (r_e N \sigma_z) (\gamma \sigma_x \sigma_y)^{-1} [2R/(1+R)]$	(13)
Pinch factor ($R = 1$)	$H_D \simeq$ Eqs. (6)	
Pinch factor ($R \neq 1$)	$H \simeq H_D (1+R)/2R$	(14)
Luminosity	$L = f_r N^2 H / (4\pi \sigma_x \sigma_y)$	(15)
Fract.critical energy	$\bar{T} \simeq (5/12\alpha) (r_e^2 \gamma / \sigma_z) (N / \sqrt{\sigma_x \sigma_y}) H_b^{1/2}(D, R)$	(17)
Beamstrahlung factor	$H_b(D, R) \simeq 4R H_D [1 + R H_D^{(R-1)/2R}]^{-2} \rightarrow 4/R$ for $R \gg 1$	(18)
Beamstrahlung par.	$\delta = 2/9 (r_e^3 \gamma / \sigma_z) (N^2 / \sigma_x \sigma_y) H_b(D, R)$	(19)
Fract.energy loss	$\langle \epsilon \rangle \simeq \delta F(\bar{T})$	(20)
Quantum factor	$F(\bar{T}) \simeq (1 + \bar{T}^{1/2} + 3\bar{T}/2)^{-4/3}$	(21)

How do we orientate ourselves in such a complicated multidimensional space? In this Section we first discuss some relations which are simple consequences of the equations of Table 3. But to make full use of them, in the next Section we have to review the scaling laws of linear accelerators.

I address, on the basis of the equations of Table 3, the questions: (i) Will NLC run with sizeable pinch effect? (ii) In which beamstrahlung regime NLC will operate? The answers will be: (i) NLC will utilize the pinch effect to increase luminosity; (ii) NLC will either run in the transition regime or very close to it. The first point is clear when looking at the first fundamental relation of Eq. (16): for a limited beam power the luminosity is proportional to DH !

To discuss the second question we have to write down some new equations. We have already met the two fundamental relations (16) and (22), which are rewritten in practical units in Table 4. The third relation in the Table is obtained by expressing $\bar{T}$ of Eq. (17) in terms of (L/f_r) . The fourth relation derives from Eqs. (16) and (24) of Table 4. It relates the parameter $\bar{T}$ to the *six* most important physical parameters (E_0 , L , P , D , R , f_r) which describe *fully* the bunch-bunch collisions. (Note that D and R appear also implicitly in the ratio H_b/H and that $H_b/H = 1$ for $R = 1$.)

Table 4
Relations derived from the equations of Table 3 (*)
(practical units are introduced for convenience of use in numerical calculations)

Relation	Eq.
$\sigma_z/\text{mm} = [\text{DH}/29] [(1+R)/(2R)] (10^{33}/L) (P/\text{MW})$	(16)
$\langle \epsilon \rangle \simeq 3.9 (H_b/H)^{1/2} (L/10^{33})^{1/2} (\text{kHz}/f_r)^{1/2} \bar{T} F(\bar{T})$	(22)
$\bar{T} \simeq 0.32 (H_b/H)^{1/2} (E_0/\text{TeV}) (L/10^{33})^{1/2} (\text{kHz}/f_r)^{1/2} (\text{mm}/\sigma_z)$	(24)
$\bar{T} \simeq 9.1 (\text{DH})^{-1} (H_b/H)^{1/2} [2R/(1+R)] (E_0/\text{TeV}) (L/10^{33})^{3/2} (\text{MW}/P) (\text{kHz}/f_r)^{1/2}$	(25)

(*) In this Table $10^{33} = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$.

When designing a collider, out of the six parameters (E_0 , L , P , D , R , f_r) the energy E_0 and the luminosity L are given by physics, and the present wisdom requires the luminosity given in Eq. (1).

P is fixed by the acceleration technology and the money available, since given a certain efficiency η_{tot} of the energy transfer from wall-plug to the accelerated beam, the power (per linac) is P/η_{tot} . (Today $\eta_{\text{tot}} < 1\%$ and in future accelerators one can expect $\eta_{\text{tot}} \leq 5\%$.)

The value of D defines the choice between *pinch effect* ($D > 0.5$) or *no pinch effect* ($D < 0.5$). As shown by Eq. (4), the aspect ratio R can be varied by using both the transverse emittances ϵ_{ny} and ϵ_{nx} and/or the ratio β_x^*/β_y^* .

Eq. (25) of Table 4 shows that, if the luminosity is scaled according to Eq. (1) and with D , R , P and f_r fixed, $\bar{T} \propto E_0^4$ and at large enough energy the collider has to run in the quantum regime ($\bar{T} > 10^2$).

For a 'standard' NLC ($W = 1 \text{ TeV}$ and $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$) with $P = 1 \text{ MW}$ (so that the *total* plug power is $\sim 100 \text{ MW}$ with $\eta_{\text{tot}} \simeq 2\%$) it follows from Eq. (25) that $\bar{T} = (C/\text{DH}) (\text{kHz}/f_r)^{1/2}$. It can be easily checked⁶ that the numerical factor is $1 < C < 10$ for all values of H and R . A standard NLC, which profits from the pinch effect ($D \geq 0.5$) and runs at the (small) SLC repetition rate ($f_r = 0.2 \text{ kHz}$), will thus have $0.2 \leq \bar{T} \leq 10$ for all reasonable values of R . However, higher rates and beam powers tend to reduce $\bar{T}$, and one can get $\bar{T} < 10^{-1}$ if the aspect ratio R is large. At higher energy (for instance at $W = 2 \text{ TeV}$, as for the CERN linear collider CLIC), the choice is again pushed to the transition regime because of the dependence $\bar{T} \propto E_0^4$ even if f_r is few kHz.

In conclusion, a low repetition rate NLC profiting from the pinch effect will have to run either close or in the *transition regime*. At higher repetition rates and for flat bunches one can choose to sit at the higher end of the classical regime. Anyway, by increasing the energy one is pushed towards the quantum regime, where the scaling laws change. This subject is not discussed here, because such a regime is not interesting for NLC; still in the last Section I shall mention some very recent results which indicate that this regime may not be accessible at all.

9. SCALING LAWS OF COPPER LINACS

It is by now generally accepted that the acceleration techniques of the next generation of linear colliders will be 'reasonable' extrapolations of the ones in use today. The most promising candidates are copper cavities running at high frequency ($10 \leq f \leq 30$ GHz). Superconducting cavities would be an *ideal solution*¹⁵, but at present the achievable gradients are too small and imply long and expensive linacs to reach the TeV energy range¹⁶.

Let us consider a normal conducting linac made of travelling wave sections of length L at frequency f . Given the *stored energy per unit length* W' , the *power dissipation per unit length* P_d' determines the decay time of the energy: W'/P_d' . The structure (unloaded) quality factor Q is defined as the ratio between this decay time and the characteristic time of the RF oscillation: $(2\pi f)^{-1} = 1/\omega$:

$$Q = \omega W' / P_d' . \quad (26)$$

The quantity τ , proportional to the decay time, fixes the time it takes to build up the field:

$$\tau = W'/(2P_d') = Q/2\omega . \quad (27)$$

(The factor $1/2$ corresponds to the usual choice made for high energy colliders¹⁷. In general $\tau = Q/\alpha$ where α is the 'attenuation constant' of the structure.)

Each one of the sections of length L is excited in a resonant mode with an electric field component in the direction of the particle motion; we suppose for simplicity that the excitation is caused by a square power pulse of frequency f , duration τ and peak power $\dot{P}_L$. The group velocity v_g , which is the velocity of the energy flow, determines the filling time so that $\tau = L/v_g$. (In the structures to be considered typically one has $v_g/c \approx 0.05-0.1$.)

The stored energy *per meter* W' (W/m) is clearly proportional to the square of the accelerating field E (V/m) and to the structure cross-section, i.e. to the square of the wavelength λ . One can thus write:

$$W' = E^2 \lambda^2 / (2\pi cZ) . \quad (28)$$

where Z is an impedance independent of λ for any given geometry of the structure. A typical value is $Z \approx 300 \Omega$, so that at $\lambda = 10$ cm and $E = 17$ MV/m (as for SLC) one needs $W' \approx 5$ J/m.

The energy is pumped into the structure with a repetition rate f_{rf} in pulses of duration τ . Typically only $\eta_1 \approx 75\%$ of the pumped energy is still there (as stored energy W') at the moment in which a bunch of N_e particles crosses the structure, is accelerated by the field E and extracts energy with efficiency

$$\eta = N_e E / W' = 2\pi cZ N_e / (E\lambda^2) . \quad (29)$$

Of course some power is spent in accelerating the electron and positron beams. When a bunch of charge N_e interacts with the structure it induces a field $E_i \propto (N_e)$ which cancels part of the accelerating field. The average particle of the bunch will thus see the field $(E - E_i/2)$ and this causes a momentum spread. If η is the *fraction* of the stored energy extracted by a bunch, when no particular attention is paid to the problem, the momentum spread is of the order of $\eta/2$. One can do better by choosing the phase of the bunch with respect to the RF wave such that, without beam loading, the particles in the tail of the bunch would see a larger accelerating field.

If b bunches of N particles each are accelerated during a *single* RF pulse ('multibunching'), and extra power is poured in the structure in between bunches to compensate for the energy extracted, the efficiency of the system can be increased without augmenting the momentum spread. Fig. 4 taken from Ref. [18], depicts how the filling of the structure is fitted to the distances between the closely spaced bunches: the 'length' of the accelerating field is longer to compensate for the decrease of the field, so that all particles are given the same amount of energy.

Fig. 4 In a multibunch scheme the structure is powered in such a way that successive bunches get the same acceleration even in presence of a decreasing field.

In the case of a multibunch collider the RF power repetition rate f_{rf} is smaller than the collider rate f_r

$$f_r = b f_{rf}, \quad (30)$$

and the fraction of stored energy which is extracted is roughly

$$\eta \simeq b N e E/W' = 2\pi c Z b N e/(E\lambda^2). \quad (31)$$

Typically for a single bunch $\eta \leq 5\%$, because of the requirement on the momentum spread, and one can hope to reach $\eta \simeq 20\%$ in the multibunch compensated scheme.

The total energy transfer efficiency from rf-power to beam is $\eta_{rf} = \eta_t \eta$ and the RF power needed for two linacs to obtain two high energy beams of power P each is

$$P_{rf} = 2 P/(\eta_t \eta) \simeq (2E_0 f_r E\lambda^2)/(2\pi c e Z \eta_t b), \quad (32)$$

where we have used Eq. (7) and the bunch population N does *not* appear. As already mentioned, in Eq. (32) typically $1/\eta_t \simeq 1.25$, so that with $\eta = 5\%$ ($\eta_{rf} = \eta_t \eta \simeq 4\%$) and $P = 1$ MW one needs $P_{rf} = 2 P/\eta_{rf} \simeq 50$ MW. Note that at the most favorable frequencies the efficiency of producing RF power from plug power is $\sim 70\%$; at the high frequencies needed for the new colliders one can hope to have $\eta_{tot}/\eta_{rf} \simeq 50\%$, so that our choice corresponds to $\simeq 100$ MW of plug power.

10. THE CHOICE OF THE RF FREQUENCY

We can now make contact with the discussion of Section 8, and in particular with the equations of Table 4. By using the quoted value $Z = 300 \Omega$, in practical units Eq. (32) becomes:

$$P_{rf}/MW \simeq (400/b) (E_0/\text{TeV}) (f_r/\text{kHz}) (E/\text{MV m}^{-1}) (\text{GHz}/f)^2, \quad (33)$$

which clearly displays the gain in RF power implied by high RF frequencies (i.e. small wavelengths) and by the use of a multibunch scheme ($b > 1$). Eq. (33) could be called the *third fundamental relation* of copper cavity linear colliders. Applied to the NLC of Table 1 with $f = 3 \text{ GHz}$, $E = 17 \text{ MV/m}$ and $f_r = 0.18 \text{ kHz}$ (as for SLC) it gives $P_{rf} \simeq 72 \text{ MW}$, a not unreasonable number. However, with such a low gradient the length of a $(0.5 + 0.5) \text{ TeV}$ collider would be much too large: $2 \times 30 \text{ km}$. If the gradient was increased by a factor 5, so that the length becomes $2 \times 6 \text{ km}$, the power would become $P_{rf} \simeq 350 \text{ MW}$. This is unreasonable because the plug power P_{ac} has typically to be twice as large.

By combining the second fundamental relation (22) with Eq. (33) one finally gets:

$$b^{1/2} f/\text{GHz} \simeq 80 (H_b/H)^{1/2} (E_0/\text{TeV})^{1/2} (L/10^{33}) (MW/P_{rf})^{1/2} (E/\text{MeV m}^{-1})^{1/2} \bar{T} F(\bar{T}) / \langle \epsilon \rangle \quad (34)$$

which does *not* contain the repetition rate. For a collider defined by the following parameters: $W = 2E_0 = 1 \text{ TeV}$, $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, $P_{rf} = 63 \text{ MW}$, $D = 2$, $H_D \simeq 3.5$, $\langle \epsilon \rangle = 0.3$, $b = 1$, $E = 100 \text{ MV m}^{-1}$, which runs in the *transition regime*, (so that $\bar{T} F(\bar{T}) = 0.15 \pm .05$), Eq. (34) implies

$$f \simeq (36 \pm 12) (H_b/H)^{1/2} \text{ GHz} \rightarrow (52 \pm 15) R^{-1/2} \text{ GHz (for } H_D = 3.5). \quad (35)$$

The first equality shows that for $R = 1$, the frequency is ten times larger than the SLC frequency. The factor $(H_b/H)^{1/2}$, which is equal to 1 for $R = 1$, decreases as $2R^{-1/2} H_D^{-1/4} \simeq 1.5 R^{-1/2}$ for $H_D \simeq 3.5$ and $R \geq 3$, and this gives the second equality in Eq. (35). Flat bunches are thus useful to decrease either the RF frequency or the plug power, since in Eq. (34) $f P_{rf}^{1/2} \propto R^{-1/2}$. For $R = 10$ and $H_D = 3.5$ from Eqs. (14) and (18) one computes $H_b/H \simeq 0.20$, so that the optimal frequency is $f \simeq 16 \text{ GHz}$.

This argument closes the general discussion of the main scaling laws. We have seen that the Next Linear Collider will have to run with some pinch effect, to increase the luminosity, and in the transition regime. Then the other parameters are practically fixed by the flatness of the bunch and the requirement of not having a too large power consumption. In particular the frequency of the structure cannot be very different from 20 GHz . Flat bunches imply a reduction of the frequency which is proportional to $R^{-1/2}$ or, for a fixed frequency, a reduction of the power $\propto R^{-1}$, as shown in Eq. (34).

11. POWER SOURCES

It is easy to write down the peak power $\hat{P}_L$ needed to produce, with a RF square pulse of frequency f and duration τ , the wanted gradient E in a copper structure of length L . As anticipated, the energy ($\hat{P}_L \tau$) of each RF pulse has to be larger by a factor $1/\eta_t \sim 1.25$ with respect to the energy ($W'L$) needed to have an accelerating field E when the bunch crosses the structure. Thus one can write

$$\hat{P}_L \tau \simeq 1.25 W'L, \quad (36)$$

and, using Eq. (27),

$$\hat{P}_L/L \simeq 2.5 \omega W'/Q. \quad (37)$$

The ratio $\hat{P}_L/L$ is nothing else than the *peak power $\hat{P}'$ per meter* of structure. W' of Eq. (28) can be written in the form $W' = c E^2 / (2\pi Z f^2)$, so that

$$\hat{P}' \simeq 2.5 c E^2 / (Z f Q). \quad (38)$$

The Q of the structure depends on its design and, for a fixed geometry, scales as $f^{-1/2}$. For a structure of the SLAC design (but having a larger opening so that the group velocity is $v_g \simeq 0.07 c$), the numerical expression

$$Q \simeq 2.2 \cdot 10^4 (\text{GHz}/f)^{1/2} \quad (39)$$

can be introduced in Eq. (38) together with the value $Z \simeq 300 \Omega$ to obtain

$$\hat{P}'/\text{MW m}^{-1} \simeq 0.11 (E/\text{MV m}^{-1})^2 (\text{GHz}/f)^{1/2}. \quad (40)$$

For a gradient $E = 100 \text{ MV m}^{-1}$ at the frequency $f = 20 \text{ GHz}$ the needed peak power is $\hat{P}' \simeq 250 \text{ MW m}^{-1}$ and, for a NLC total length of $5+5 \text{ km}$, the total peak power turns out to be 2.5 terawatts! This number gives a feeling of the difficulties to be encountered in constructing the RF power sources for NLC, especially when compared with the present total SLC peak power (which moreover runs at a much more comfortable frequency): 0.05 terawatts.

Another important factor is the duration τ of the power pulse which, combining Eqs. (27) and (39), is

$$\tau/\text{ns} \simeq 1750 (\text{GHz}/f)^{3/2}. \quad (41)$$

The length of structure which has to be powered independently is $L = v_g \tau$ i.e., for $v_g \simeq 0.07 c$,

$$L/\text{m} \simeq 37 (\text{GHz}/f)^{3/2}. \quad (42)$$

At $f = 20 \text{ GHz}$, $\tau \simeq 20 \text{ ns}$ and $L \simeq 0.4 \text{ m}$ so that the 10 km collider of our example is made of about 25,000 sections! More than 1500 power sources would be needed, even if each source is connected to 16 accelerating elements.

Power sources of the needed frequency and peak power are not available on the market. Their development is the main subject of any R & D program aiming at the construction of a future linear collider. A review of the ongoing activities would be too long. I limit myself to few short remarks.

Fig. 5 The relativistic-klystron linear accelerator considered at present by the Berkeley-Livermore-SLAC collaboration.

In USSR the Novosibirsk group is working on high frequency klystrons and gyrocons¹⁹. High frequency klystrons are also under consideration in Japan. In USA the main emphasis has been on the so-called 'relativistic klystron'²⁰ depicted in Fig. 5. Induction units accelerate an intense beam of a few MeV electrons, which excites one or more cavities which are running at the desired frequency. The same low energy bunches are used more than ones. The frequency chosen for the moment is $f = 17$ GHz.

At CERN for CLIC the concept is similar, but the electron beam would have a few GeV and would run all along the high gradient accelerating structure^{15,17}, as initially proposed by Andy Sessler for the Two Beam Accelerator²¹. The intense and highly bunched few GeV beam is accelerated with high efficiency by low frequencies superconducting cavities equal to the one which shall be used for LEP 200. The frequency chosen for the high gradient linac is $f = 30$ GHz.

The many groups involved in these activities are performing some tests. The only one which already produced concrete results has been performed at Livermore by a Berkeley-Livermore-SLAC collaboration²². Using an induction linac and a wiggler, about 2 GeV of peak power were produced for about 20 ns at $f \approx 30$ GHz. After a couple of years of uncertainty this Free Electron Laser source has been proposed again as a valid alternative to the sources described above²³.

12. PARAMETERS OF A STANDARD NLC

We can now derive a complete list of parameters for what I have chosen to call a 'standard' Next Linear Collider. The logic of the argument and the numerical values appear in Table 5. The rest of this Section is devoted to comment it.

The input parameters are dictated by physics (E_0 and L), by the desire to reduce beamstrahlung but not to have too flat bunches (R large but not larger than 10), the wish to make a reasonable use of the pinch effect ($D \approx 3.5$) and a reasonable power consumption ($P_{rf} \approx 50$ MW so that, as discussed in the last Section, $P \approx 1$ MW with $\eta = 5\%$). It is then enough to fix the fractional energy loss at the standard value $\langle \epsilon \rangle \approx 0.25$ to get all other important parameters by applying the equations mentioned in the Table. The bunch population is reasonably low ($N \approx 10^{10}$) and the transverse dimensions of the bunches are very small indeed, as in all parameter lists of linear colliders which have beam power of the order of one MW. (For superconducting colliders the power can be more than ten times larger and the cross-section of the bunches are much less demanding¹⁶.)

The Table shows that, as expected, the collider runs at the center of the transition region ($\bar{\gamma} \approx 0.4$) so that the parameter $\bar{\gamma} F(\bar{\gamma})$ equals ~ 0.15 .

Below the horizontal line appearing in the Table, I list a possible choice of the invariant emittances, which agrees with what is thought possible today with very carefully designed damping rings⁶. At SLC the damping rings give $\epsilon_{nx} \approx 3 \cdot 10^{-5}$ m, two times larger than what is needed here, but many recent studies^{24,25} have shown that the choice made in the Table ($\epsilon_{nx} = 5 \cdot 10^{-6}$ m $\epsilon_{ny} = 5 \cdot 10^{-7}$ m) is possible. Note that, to get $R = 10$, I have decided to have both $\epsilon_{ny}/\epsilon_{nx} = 10$ and $\beta_y^*/\beta_x^* = 10$; other choices are possible.

The last line in the Table shows that the parameter $\Lambda = \beta^*/\sigma$ appearing in Eq. (6) is larger than ~ 3 , as it should not to loose luminosity due to the bunch being too long with respect to the β^* value.

The main parameters of the power sources needed to run such a collider are also fixed. Table 6 collects the formulae and the results.

Table 5
Construction of a consistent set of parameters for a 'standard' NLC

Input quantities	Computed quantities and/or comment	Numerical value
1. Beam energy E_0	c.m.energy $W = 2 E_0$	$E_0 = 0.5 \text{ TeV}$ $W = 1.0 \text{ TeV}$
2. Luminosity L	physics requirement from Eq. (1)	$L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
3. Aspect ratio R	to reduce beamstrahlung	$R = 10$
4. Disruption parameter D	to have some pinch effect H_D from Eq. (6) with $A \simeq 4$ enhancement factor from Eq. (14) factor H_b from Eq. (18) $\simeq 4/R$	$D = 3.5$ $H_D \simeq 4.6$ $H \simeq 2.3$ $H_b \simeq 0.42$
5. Beam power P	to consume $P_{rf} \simeq 50 \text{ MW}$ with $\eta_{rf} = \eta_t \eta \simeq 4\%$ bunch length from Eq. (16)	$P = 1 \text{ MW}$ $\sigma_z \simeq 0.15 \text{ mm}$
6. Fractional energy loss $\langle \varepsilon \rangle$	acceptable to get $\Delta W/W \leq 10\%$ f_r from Eq. (22) with $\bar{\gamma} F(\bar{\gamma}) \simeq 0.15$ $\bar{\gamma}$ from Eq. (24) $\bar{\gamma} F(\bar{\gamma})$ from Eq. (21), it checks with above from Eq.(7): $N = P/(f_r E_0)$ from Eq. (15): $\sigma_x \sigma_y = H f_r N^2 / (4\pi L)$ $\sigma_y = \sqrt{\sigma_x \sigma_y} / R$ $\sigma_x = R \sigma_y$	$\langle \varepsilon \rangle \simeq 0.25$ $f_r \simeq 1.0 \text{ kHz}$ $\bar{\gamma} \simeq 0.45$ $\bar{\gamma} F(\bar{\gamma}) = 0.15$ $N \simeq 1.25 \cdot 10^{10}$ $\sigma_x \sigma_y \simeq 2800 \text{ nm}^2$ $\sigma_y \simeq 17 \text{ nm}$ $\sigma_x \simeq 170 \text{ nm}$
7. Invariant emittances	to get $R = 10$: $\varepsilon_{ny}/\varepsilon_{nx} = \beta_y^* / \beta_x^* = 10$ from Eq. (4): $\beta_x^* \beta_y^* = (\gamma \sigma_x \sigma_y)^2 (\varepsilon_{nx} \varepsilon_{ny})^{-1}$ $\beta_y^* = \sqrt{\beta_x^* \beta_y^*} / R$; $\beta_x^* = R \beta_y^*$ check for Eq. (6): $\Lambda = \beta_y^* / \sigma_z$	$\varepsilon_{nx} = 5 \cdot 10^{-6} \text{ m}$ $\varepsilon_{ny} = 5 \cdot 10^{-7} \text{ m}$ $\beta_x^* \beta_y^* \simeq 3.1 \text{ mm}^2$ $\beta_y \simeq 0.56 \text{ mm}$; $\beta_x \simeq 5.6 \text{ mm}$ $\Lambda \simeq 4$

Table 6
Parameters of the power sources for a 'standard' NLC

Input quantities	Comment	Numerical value
1. Beam energy		$E_0 = 0.5 \text{ TeV}$
2. Accelerating field		$E = 100 \text{ MV/m}$
	collider length = $2 \times E_0/E$	$2 \times 5 \text{ km}$
3. RF total power	to have a plug power $< 150 \text{ MW}$	$P_{\text{rf}} = 50 \text{ MW}$
4. Bunches per RF pulse	conservative choice	$b = 1$
	frequency from Eq. (34) and Table 5	$f \simeq 20 \text{ GHz}$
	wavelength of the structure	$\lambda = 15 \text{ mm}$
	stored energy/meter from Eq. (28)	$W' \simeq 4.0 \text{ J/m}$
	extraction efficiency from Eq. (31)	$\eta \simeq 5\%$
	Q-value from Eq. (39)	$Q \simeq 4.9 \cdot 10^3$
	peak power per m from Eq. (40)	$\hat{P}' \simeq 250 \text{ MW/m}$
	total peak power	2.5 TW
	duration of the RF pulse from Eq. (41)	$\tau \simeq 20 \text{ ns}$
	length of the structure (section) from Eq. (42)	$L \simeq 0.4 \text{ m}$
	total number of sections	$2 \times 12,500$

Few remarks are in order. I choose *one* bunch per RF pulse for safety reason. Without extra plug-power one could roughly double or triple the luminosity with $b=2$ or $b=3$. This may be possible if one uses structures which have slots to coupled out the higher-order modes excited by the wake field, as proposed by R. Palmer.

Once the gradient E and the RF power P_{rf} are chosen, Table 6 shows that the accelerator parameters are frozen. For the 'standard' choices ($E = 100 \text{ MV/m}$, $P_{\text{rf}} = 50 \text{ MW}$, $\eta_{\text{rf}} = 4\%$) we get $f \simeq 20 \text{ GHz}$ with all the consequences listed in the Table. In particular one needs 250 MW/m of peak power. Thus a source delivering $\sim 2.5 \text{ GW}$ for $\sim 20 \text{ ns}$, similar to the one tested at Livermore²², could power about 10 m , a not unreasonable length which corresponds to 25 sections each 0.4 m long. To power 2.5×10^4 sections one would need 1000 sources.

13. A FINAL COMPARISON

I compare in Table 7 the parameters of my 'standard' NLC with those at present considered at CERN (for CLIC, the CERN Linear Collider) at SLAC (for the TeV Linear Collider, TLC) and at Novosibirsk (for VLEPP, to be built at Serpukhov).

Table 7
Parameter lists of linear colliders

	'Standard' NLC	CLIC (CERN)	TLC (SLAC)	VLEPP (Novosibirsk)
Energy per beam, E_0 (TeV)	0.5	1.0	0.5	0.5
Gradient, E (MV/m)	100	80	185	100
Frequency, f (GHz)	20	30	17	14
Bunches per RF pulse, b	1	1	1(*)	1
Bunch repetition rate, f_r (kHz)	1.0	1.7	0.36	0.1
RF av. power, P_{rf} (MW), Eq. (33)	50	60	45	10
Particles/bunch, N	1.25×10^{10}	5×10^9	1.4×10^{10}	2×10^{11}
Power/beam, P (MW)	1.0	1.35	1.1	1.6
Disruption, D/H	3.5/2.3	3.3/2.3	5.0/2.2	7.2/2.5
Aspect ratio, R	10	5	130	150
Bunch r.m.s. radius, σ_x (nm)	170	60	390	3000
Bunch r.m.s. radius, σ_y (nm)	17	12	3	20
Bunch r.m.s. length, σ_z (μ m)	150	200	70	750
β -value, β_y^* (mm)	0.56	0.28	0.15(**)	1.3(**)
β -value, β_x^* (mm)	5.6	2.3	50	300
Inv. emittance, ϵ_{nx} (m)	5×10^{-6}	3×10^{-6}	3×10^{-6}	3×10^{-5}
Inv. emittance, ϵ_{ny} (m)	5×10^{-7}	1×10^{-6}	6×10^{-8}	3×10^{-7}
Beamstrahlung reduction, H_b	0.42	0.84	0.030	0.027
Fractional energy loss, $\langle \epsilon \rangle$	0.25	0.33	0.12	0.07
Fractional crit. energy, $\bar{Y}$	0.45	0.77	0.46	0.08
Product $\bar{Y} F(\bar{Y})$	0.15	0.18	0.15	0.05
Luminosity, L ($10^{33} \text{cm}^{-2} \text{s}^{-1}$)	1.0×10^{33}	1.1×10^{33}	1.0×10^{33}	1.3×10^{33}

(*) Multibunching (up to $b = 20$) is seriously considered to increase the luminosity and/or relax some parameters.

(**) This number, which I took $\sim 2\sigma_z$, does not appear in the parameter list available to me. Its choice constraints β_x^* and the two transverse emittances.

Before comparing the parameter lists, let me underline that at the time of writing we do not have complete parameter lists for TLC and VLEPP. In Table 7 I have used the best information available at the SLAC workshop of November 1988, which I complemented with the missing pieces. To make the comparison more meaningful the luminosity quoted in Table 7 is computed *without* multibunching ($b = 1$). However, at SLAC the consequences of choosing $b \simeq 10$ (and even $b \simeq 20$) are seriously studied; this would imply automatically either a tenfold increase of the luminosity or a relaxing of the very tight vertical dimensions of the bunches. Multibunching is considered a reserve factor for CLIC, while at VLEPP it is most probably excluded by the much greater number of particles per bunch and the consequent larger wake field effects.

The choice of the aspect ratio R distinguishes TLC and VLEPP from the 'standard' NLC and CLIC since, as discussed at length, $H_b \propto 4/R$ and $\bar{Y} \propto R^{-1/2}$. For TLC and VLEPP the chosen

value $R > 100$ implies a large suppression of beamstrahlung radiation. VLEPP even runs in the 'classical regime', since $\bar{\gamma} < 0.2$. Which are the consequences of this apparently clever choice? The difficulties have been displaced to two different areas.

By using the parameter lists of TLC and VLEPP as *typical*, we see that *two* routes are opened once $R > 100$ is chosen. Either — as for TLC — the number of particles is $N \simeq 10^{10}$ and then the bunches are very small vertically ($\sigma_y \simeq 3$ nm), or $N \simeq 10^{11}$ and the bunches transverse dimensions are ten times larger (VLEPP). In the former case the difficulty lies in providing both the very small vertical emittance at the crossing point ($\epsilon_{ny} \simeq 6 \times 10^{-8}$ m) and the β -value $\beta_y^* \simeq 0.15$ mm. In the second case the problem will be the production of such an intense bunch and the attaining of a *very large* efficiency ($\eta_{rf} \simeq 32\%$). Another problem is hidden in the preservation of the vertical emittance during acceleration of a very intense bunch.

As far as emittances are concerned let me remind that studies made in different laboratories²⁴ show that well designed and aligned damping rings can produce horizontal emittances $\epsilon_{nx} \simeq (1 \div 2) 10^{-6}$ m with $N \simeq 10^{10}$. On paper, ratios $\epsilon_{nx}/\epsilon_{ny} \simeq 10^2$ are feasible, but to maintain vertical emittances $\epsilon_{ny} \simeq 10^{-7}$ m during the acceleration in a 5 km long linac one needs²⁵ alignment accuracies of the order of 10 μ m. Note also that the elements of the final focus have to be aligned to a fraction of the final spot size, i.e. to ~ 1 nm (!) vertically for TLC and ~ 5 nm for VLEPP.

The CLIC parameter list is close, not surprisingly, to the 'standard' NLC one, with the added complications that CLIC energy is twice larger. This choice — which could be reviewed but makes the physics potentialities of CLIC and LHC equivalent *and* complementary⁴ — would require according to Eq. (1) a luminosity $L = 4 \times 10^{33}$ per interaction point (IP). The parameters considered today and listed in Table 7 miss this goal by a factor 4 (or more, if more IP's are served). The present philosophy is that multibunching will provide it, maybe in a second stage.

The 'standard' NLC and CLIC have $R \leq 10$ and vertical spot sizes similar to the ones foreseen for VLEPP. The main difference is that the repetition rate is of the order of 1 kHz and *not* 100 Hz. In this case it seems that the main difficulties are hidden in the high repetition rate, which however is not a real problem, since the relativistic klystron solution envisaged for CLIC¹⁷ allows high repetition rates because it uses CW superconducting cavities as power sources. Standard klystrons would not run above the ~ 360 Hz chosen for TLC.

The CLIC and TLC schemes need β -functions at the IP which are at the limit of what can be done. Designs exist for final focus systems which at 0.5 TeV would provide $\beta_y^* \simeq 0.3$ mm and $\beta_x^* \simeq 3$ mm with chromatic corrections for a momentum spread $\Delta p/p < 1\%$. This is more than sufficient for the 'standard' NLC and for VLEPP, but the CLIC parameters (at 1 TeV) and the TLC request (at 0.5 TeV) are at present not achievable by a factor 2 to 3.

Before closing, let me remark that in November 1988 a (until now forgotten) bunch-bunch effect appeared on the scene. This is the materialisation of the beamstrahlung photons in the megagauss magnetic field of the opposite bunch. Then the electrons and the positrons have relatively low energies and are swept by the same field, so to hit the last focusing quadrupole and even the detector. In spite of being discovered after the School, it is important to mention this phenomenon here because it will probably influence the parameter choices of future linear colliders. Pisin Chen reported results of preliminary calculations at the SLAC workshop in November 1988 and concluded that its negative effects may be limited if the fractional critical energy $\bar{\gamma}$ is chosen to be less than ~ 0.5 . Indeed the TLC parameter list of Table 7 is the one prepared to meet such a condition; previously at SLAC values $\bar{\gamma} \simeq 2$ were considered. For CLIC $\bar{\gamma} \simeq 0.8$, but little modifications are needed to have $\bar{\gamma} \simeq 0.5$; VLEPP would run in the classical regime and is not touched by the argument. Of course better calculations are needed. However, for the time being this effect looks very detrimental at energies definitely larger than $W \simeq 1 - 2$ TeV, since the arguments of Section 8 indicate that to reach them the quantum regime ($\bar{\gamma} \geq 10^2$) is a must. In conclusion one cannot exclude that this effect will represent an insurmountable difficulty on the way to many TeV linear electron-positron colliders.

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