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QUARK MASS MATRICES AND MIXINGS: IMPLICATIONS FOR THE TOP QUARK MASS*

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Abstract

We review the results on the subject of quark mass matrices that have been obtained in a series of papers written in collaboration with Jarlskog, Lindholm and Lindner. By varying the input parameters for the 3-family matrices, we obtain an upper bound of 110 GeV on the top quark mass that emerges in the hierarchical chiral symmetry-breaking framework.

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1. STRATEGY AND FITTING PROCEDURE

Since the mass and weak eigenbases are not identical, the quark eigenfields are related by two unitary transformations, U and U' , one for the up sector and one for the down sector, respectively. When the weak charged current is expressed in terms of the mass eigenbases, the nondiagonal Kobayashi - Maskawa (KM) mixing matrix¹ appears as the product $V_{KM} = UU'^{\dagger}$. On the other hand, the nondiagonal mass matrices for the Yukawa Lagrangian in the weak basis involve the unknown Yukawa couplings and vacuum expectation values. In the absence of a well-defined theory, one can attempt to invoke symmetry arguments based on the known phenomenology, as was done long ago by Fritzsch,² to postulate a set of mass matrices in the weak basis. Diagonalization of these mass matrices leads to the quark masses and unitary matrices U and U' , from which the KM matrix can be constructed. The masses and mixing matrix elements can then be compared with experiment to assess the correctness of the postulated matrices and to find a prediction for, or bound on, the unknown top quark mass.

To carry out this program and to check the accuracy of the predictions, we require knowledge of the quark masses at some common scale as well as the experimentally determined KM mixing matrix elements. Following convention, we take the quark mass determination of Gasser and Leutwyler³ and select

$$\begin{aligned} m_u &= 3.5 \text{ MeV}, & m_d &= 6.1 \text{ MeV} \\ m_c &= 1.35 \text{ GeV}, & m_s &= 120 \text{ MeV} \\ m_t &= ? & m_b &= 5.3 \text{ GeV} \end{aligned} \quad (1)$$

where m_s has been chosen at the low end of its range to obtain the best fits. For

the KM mixing matrix, we use the determination by Schubert⁴

$$\begin{aligned} &0.9754 \pm 0.0004 & 0.2206 \pm 0.0018 & 0.0000 \pm 0.0123 \\ &0.2203 \pm 0.0019 & 0.9743 \pm 0.0005 & 0.0160 \pm 0.0060 \\ &0.0101 \pm 0.0122 & 0.0449 \pm 0.0065 & 0.9989 \pm 0.0003 \\ &= |V_{ij}|; & i = u, c, t, & j = d, s, b \end{aligned} \quad (2)$$

where we note that V_{ud} , V_{us} , V_{cd} and V_{cb} are the most precisely determined experimentally, while the accuracy of the other elements has been improved by the imposition of 3-family unitarity.

The fitting procedure employed is then the following.⁵ Given a set of mass matrices M^U and M^D in the weak basis, we calculate the set of $|V_{ij}|^2$ by Jarlskog's projection operator technique⁶ which involves traces of products of two projection operators, each of which can be found simply from the mass matrices. We then fit all KM $|V_{ij}|^2$ within one standard deviation in the three-family case by varying all free parameters of the model in question. For four families, we fit just $|V_{ud}|^2$, $|V_{us}|^2$, $|V_{cd}|^2$ and $|V_{cb}|^2$ to one standard deviation accuracy. These fits lead to a physically-allowed region in the space of the free model parameters, which corresponds to an annular ring in the one phase angle *vs.* m_t plane for the models considered.

For each point in this plane, we then calculate the physically-allowed region for the $B_d - \bar{B}_d$ mixing parameter x_d determined⁷ by ARGUS and CLEO, i.e., $x_d = 0.73 \pm 0.18$. The theoretical calculation involves the evaluation of box diagrams with only W exchange in the standard Higgs model, but also charged Higgs exchange in two-doublet Higgs models. The combination of physical parameters is found to be⁸

$$m_t^2 |V_{td}^* V_{tb}|^2 R \simeq (2.0 \pm 0.5) \frac{(0.140)^2}{B_H f_H^2} \quad (3)$$

where R is sensitive to the top mass and Higgs structure, and leads to a physically-

allowed band in the same phase angle $\pi\alpha$, m_t plane.

The model is physically acceptable if overlap of the ring and band occurs for some range of m_t . For the best fitted point in the overlap region, we can then calculate other parameters of interest including the J -value for CP violation, the CP phase δ , α , for $B_s - \bar{B}_s$ mixing, the ratio $|V_{ub}|/|V_{cb}|$, the bag parameter B_K and ϵ'/ϵ .

2. THREE-FAMILY MODELS

We begin with the model proposed by Fritzsch² based on hierarchical chiral symmetry breaking with nearest-neighbor interactions for which

$$\mathbf{M}^U = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ A & B & C \end{pmatrix}, \mathbf{M}^D = \begin{pmatrix} 0 & A' & 0 \\ A'' & 0 & B' \\ A'' & B'' & C'' \end{pmatrix} \quad (4)$$

after a phase transformation. The eight model parameters can be adjusted to explain the ten independent physical parameters (6 quark masses, 3 KM angles and 1 phase), and we find it convenient to plot the $\phi_{\mu\pi}$ vs. m_t plane for a very tightly constrained ϕ_{A_s} ($\pm 2^\circ$). Figure 1(a) indicates that the Fritzsch model is not viable with standard Higgs structure, but requires the presence of at least two Higgs doublets.⁵ The physical top mass is found on the lower scale to be about 90 GeV, after running it up from the 1 GeV scale. Consideration of the renormalization effects on the mass matrices as they are evolved from a high chiral symmetry-breaking scale down to 1 GeV changes the results by 5 – 10% but does not improve the standard Higgs case.⁶

We have attempted to construct a set of mass matrices which does fit the data with just one standard Higgs doublet and have been led by symmetry and phe-

nomenological arguments to the set of matrices⁹

$$\mathbf{M}^U = \begin{pmatrix} 0 & A & Ae^{i\phi_U} \\ A & 0 & B \\ Ae^{-i\phi_U} & B & C \end{pmatrix} \quad (5)$$

$$\mathbf{M}^D = \begin{pmatrix} 0 & A' & A'e^{i\phi_D} \\ A'' & 0 & B' \\ A''e^{-i\phi_D} & B'' & C'' \end{pmatrix}$$

which, before the phase rotation, exhibit hierarchical chiral symmetry breaking and equal off-diagonal radiative corrections at the last stage of the symmetry breaking. In Fig. 1(b), the phase ϕ_D has been adjusted to give a good value for the CP-violating J parameter, and good agreement is found for $m_t^{phys} \simeq 90$ GeV with just one Higgs doublet. Renormalization effects do not spoil this good agreement.⁹

An interesting difference between the predictions of the Fritzsch mass matrices in (4) and the new set in (5) is that the $|V_{ub}|/|V_{cb}|$ ratios are quite different, being 0.05 and 0.15, respectively, with the latter value close to the present upper bound.¹⁰ For other interesting comparisons, see ref. 9.

3. FOUR-FAMILY MODELS

The effect of a heavy fourth quark family has also been investigated,¹¹ where the new quarks X and Y either form a standard doublet or exist as isosinglets. For this purpose, we have fixed their masses near the infrared fixed point and set $m_{X(2/3)} = 250$ GeV and $m_{Y(-1/3)} = 240$ GeV, which minimizes the effects of evolution from the large chiral symmetry-breaking scale down to 1 GeV and also the radiative corrections to the ρ parameter. For simplicity, we extend the Fritzsch ansatz to 4×4 matrices, so that even the t and b quarks receive their masses through a seesaw mechanism or radiative corrections. There are now eight

magnitudes which can be related to the quark masses and four phases, two of which are trivially unimportant.

We can then plot the one-standard deviation physical regions for the four well-determined KM matrix elements and for $B_d - \bar{B}_d$ mixing. We find that in both cases $m_t^{phys} \simeq 70$ GeV, with two Higgs doublets required for the four standard quark doublets but just standard Higgs structure for two heavy isosinglet quarks. This suggests that a top mass well below the 90 GeV value predicted for the 3-family matrices is a good signal for physics beyond the standard model.

4. SENSITIVITY TO INPUT PARAMETERS

As reported by H. Schroeder¹² at this meeting, the best value for the $B_d - \bar{B}_d$ mixing parameter from the ARGUS and CLEO collaborations is now $x_d = 0.73 \pm 0.10$. On the other hand, the estimates for $f_B B_R^{1/2}$ have increased in new calculations, so that Altarelli¹³ has suggested one use 160 ± 40 MeV in place of the previous 140 ± 40 MeV estimate. Hence we should replace the central value of 2.0 ± 0.5 by 1.5 ± 0.2 , and the $B_d - \bar{B}_d$ mixing bands in Figs. 1 and 2 should be moved to lower values accordingly. This has the consequence that the overlaps of the rings and bands are improved, and the top mass prediction decreases slightly.

Our studies⁹ of 3-family models have led us to conclude that the quark mass matrices which satisfactorily fit the data must be of the Fritzsch-type with possibly small variations thereof at the third stage of chiral symmetry breaking, as illustrated in (5). Since the top mass is restricted, to a good approximation, already at the second stage of chiral symmetry breaking, we can obtain a reliable upper limit on the top mass by considering the matrices in (4) and (5) and varying the input parameters of (1). As m_t has already been taken at its one-standard-deviation

lower bound, we adjust only m_c and m_b . By increasing them to 1.4 GeV and 5.7 GeV, respectively,¹⁴ we find the upper bound on the top mass, as given by the KM annular ring alone, rises from 96 GeV to 110 GeV. This m_t^{phys} upper bound is to be contrasted with the upper bound of 180 - 200 GeV obtained from considerations of radiative corrections¹³ to $\sin^2 \theta_W$.

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Figure Caption

Figure 1: Physically-allowed KM mixing rings and $B_d - \bar{B}_d$ mixing bands, single-hatched for the standard Higgs model and double-hatched for the two-doublet Higgs model, for (a) the Fritzsch matrices in (4) and (b) the new matrices in (5). The charged Higgs is taken to be 50 GeV and the ratio of the VEV's equal to unity.