AD

H 789-2

P. N. LEBEDEV PHYSICAL INSTITUTE C1
USSR ACADEMY OF SCIENCES

CERN
BIBLIOTHEQUE

**ФИЗИЧЕСКИЙ
ИНСТИТУТ**
*имени
П.Н. Лебедева*

F I A N

REPRINT

9

E.J. FERRER, V. De la INCERA

**INHOMOGENEOUS W-CONDENSATE
IN FERMIONIC DENSE
ELECTROWEAK MATTER AT HIGH
TEMPERATURE**

CERN LIBRARIES, GENEVA

CM-P00066959

Moscow - 1989

Препринты Физического института имени П.Н. Лебедева АН СССР являются самостоятельными научными публикациями и издаются по следующим направлениям исследований Института:

- физика высоких энергий и космических лучей
- оптика и спектроскопия
- квантовая радиоп физика
- физика твердого тела
- физика космоса
- физика плазмы

В библиографических ссылках на препринты Физического института имени П.Н. Лебедева мы рекомендуем указывать: инициалы и фамилию автора, номер препринта, место издания, сокращенное наименование Института-издателя, год издания.

Пример библиографической ссылки:

И.И. Иванов. Препринт 125, Москва, ФИАН, 1986.

Preprints of the P.N. Lebedev Physical Institute of the Academy of Sciences of the USSR are its independent publications and are issued in the Institute's following fields of research:

- high energy and cosmic ray Physics
- optics and spectroscopy
- quantum Radiophysics
- solid state Physics
- cosmophysics
- plasma Physics

In bibliographical references to the P.N. Lebedev Physical Institute's preprints we recommend to indicate: the author's initials and name, preprint number, place of the publication, abbreviation of the Institute-publisher, year of the publication:

Example of a bibliographical reference:

I.I. Ivanov. Preprint 125, Moscow, FIAN, 1986.

© Физический институт им. П.Н. Лебедева АН СССР, 1988.

High Energy and Cosmic Rays Physics

Theoretical Department

Preprint N°9

E.J.Ferrer and V. de la Incera

INHOMOGENEOUS W-CONDENSATE IN FERMIONIC DENSE
ELECTROWEAK MATTER AT HIGH TEMPERATURE

Moscow 1989

INHOMOGENEOUS W-CONDENSATE IN FERMIONIC DENSE ELECTROWEAK
MATTER AT HIGH TEMPERATURE

E.J.Ferrer* and V. de la Incera*

Lebedev Physical Institute, Moscow, USSR

Abstract

We consider high-temperature electroweak matter at nonzero leptonic charge density. It is shown that the temperature growth leads to the weakening of the inhomogeneity in the W-condensate. The relation between the parameter characterizing the inhomogeneity at high and zero temperature is found. Possible implications of this weakening to the predicted observational effects in the background gravitational radiation are outlined.

* On leave of absence from the Institute of Mathematics,
Cibernetic and Physics, Ac. of Sciences of Cuba, Havana, Cuba.

1. The study of the phase transitions in the Weinberg-Salam (WS) model at finite temperature and (or) density has shown the existence of phase with charged condensates of W-mesons and Higgs fields. At zero temperature a homogeneous W-condensate arises at sufficiently high leptonic charge density $J_L > J_L^c = \frac{M_W^3}{6\pi^2}$ [1,2]. More recently, the finite temperature phase transitions of this system have been investigated under different physical conditions [3,4]. Considering the overall neutral system, that is equivalent to take the conserved gauge charges equal to zero [5], it has been proved that the W-condensate present at $T=0$ if $J_L > J_L^c$, evaporates before the scalar one when the temperature increases [4].

Systems at high fermionic densities in chirally asymmetric gauge theories have other unusual properties [6], mostly due to the possibility of fermion number non-conservation related to the anomaly in the fermionic current. The anomaly effects are usually very small, nevertheless, they are responsible for the appearance of a inhomogeneity in the W-condensate of the WS model at zero temperature and high fermionic charge density [7]. This fact appears due to the contribution to the effective action of the fermionic loops with spatial components of vector fields taken for external legs [7]. These diagrams contain a part proportional to the Chern-Simons term, the proportionality factor being an effective chemical potential. At the same time, the Chern-Simons term determines the anomaly in the fermionic charges (leptonic, baryonic) of the model. For each family of leptons (baryons) one has

$$\partial^\mu J_\mu^{L(B)} = \frac{1}{32\pi^2} \partial_\mu K^\mu = \frac{1}{32\pi^2} \partial_\mu \epsilon^{\mu\nu\sigma\tau} \left(F_{\nu\rho}^a W_\sigma^a - \frac{2}{3} g \epsilon^{abc} W_\nu^a W_\sigma^b W_\tau^c \right) \quad (1)$$

here $j_{\mu}^{L(B)}$ is the lepton (baryon) current and K_{μ} is the Chern-Simons current. At high temperature the abovementioned fermionic loops have been studied within a $SU(2)$ gauge model that contains left-handed fermion doublets with nonzero chemical potential /8/, with the purpose of looking for a topological mass term, but without considering the condensation of vector fields.

In the context of the phases transitions at finite temperature in the WS model the anomaly effects are non-essential for the determination of the transition curves /4/ (they are $O(g^2)$ corrections to the values of the chemical potentials and condensates). However, they determine the behaviour of the W-condensate inhomogeneity as a function of the temperature and leptonic charge density. To our present knowledge this behaviour remains unknown. Finding it, one can get information not only about the characteristics of the condensate at high densities and temperatures, but also about the effect of the anomalous fermion number non-conservation in this regime. Both problems are of actual interest for cosmology because they might be connected to observational phenomena related to some stages of the early Universe (see /9/ and the discussion at the end of the present paper), as well as to possible baryogenesis in different periods in the evolution of our Universe /10/. Our main goal here is to obtain the abovementioned behaviour.

2. Let us consider the WS model with one generation of massless leptons. The inclusion of baryons, as well as another generation of fermions is not essential for our purpose, thus for the sake of simplicity we do not consider them. However, the results can be extended to more general cases.

For a quantitative description of the dense fermionic electroweak matter, the usual WS Lagrangian must be complemented by the term $\mu_3 J_L$, where J_L is the leptonic charge density

$$J_L = \bar{e} \gamma_0 e + \frac{1}{2} \bar{\nu}_e \gamma_0 (1 + \gamma_5) \nu_e \quad (2)$$

and μ_3 is the leptonic chemical potential. As it has been underlined in references /1,4,5/, the leptonic charge density is the only conserved charge that can take a nonzero value without violating the equilibrium of the system. Therefore it characterizes the system.

The high temperature one-loop effective potential of this system with massless leptons and neglecting the anomalies has been calculated in a previous paper /4/. Its expression reads as follows

$$\begin{aligned} V\beta = & -\mu_1^2 C^2 + \left(\frac{e^2 C^2}{4 \sin^2 \theta_W} - \frac{(\mu_2 - \mu_4)^2}{8} + \frac{\lambda \xi^2}{4} - \frac{m^2}{2} \right) \xi^2 \\ & - \frac{11}{24\beta^2} \mu_1^2 - \frac{5}{24\beta^2} \mu_2^2 - \frac{1}{4\beta^2} \mu_3^2 + \frac{1}{3\beta^2} \mu_2 \mu_3 \\ & + \frac{1}{24\beta^2} \xi^2 \left[\frac{3e^2}{\sin^2 2\theta_W} (1 + 2 \cos^2 \theta_W) + 6\lambda \right] \\ & + \text{terms independent of the average fields} \end{aligned} \quad (3)$$

where $\beta = T^{-1}$ is the inverse temperature, C is the modulus of the homogeneous charged vector condensate

$$\langle W_i^1 \rangle = \sqrt{2} C_i \quad (4)$$

ξ denotes the Higgs condensate

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \xi/\sqrt{2} \end{pmatrix} \quad (5)$$

and the chemical potentials μ_1 , μ_2 are proportional to the average of the fourth components of the euclidean gauge fields

$$\langle W_4^3 \rangle = -\frac{i}{g} \mu_1 \quad (6a)$$

$$\langle B_4 \rangle = -\frac{i}{g'} \mu_2 \quad (6b)$$

The other parameters are the standard ones for the WS model, $e = g \sin \theta_w = g' \cos \theta_w$.

The phase transition curves obtained from (3) (see ref./4/) are represented in the plane (n_L^2, T^2) in Fig.1, where n_L denotes the average value of the leptonic charge density $n_L = \langle J_L \rangle$. In the W-condensate region, near to the transition curve (Shaded region in Fig.1), the value of C is very small but nonzero. This is the most convenient zone in order to study the spatial oscillation of the condensate at high temperature. Henceforth our discussion will concern only this region.

Let us consider the contribution of the static field configurations

$$\langle W_i^1 \rangle = \sqrt{2} C (\vec{e}_1 \otimes \vec{k} \cdot \vec{x} - \vec{e}_2 \sin \vec{k} \cdot \vec{x}) \quad (7)$$

(reported in /11/ as unstable solutions to classical Yang-Mills equations and used in /7/ at $T=0$), to the one-loop effective action at finite temperature and leptonic charge density. Here $\vec{e}_1$ and $\vec{e}_2$ are ortonormal vectors perpendicular to $\vec{k}$. The direction of the latter is fixed arbitrarily.

As it is known /11,12/, the leading contribution to the one-loop effective action at high temperature comes from the one-loop polarization operators whose external legs coincide with any of the nonzero average fields $\langle \phi \rangle$, $\langle W_i^1 \rangle$, μ_i ($i = 1, 2, 3$); μ_3 being treated as an external Abelian gauge field.

If we would consider a homogeneous condensed solution (4) for $\langle \vec{W} \rangle$, instead of (7), the effective action Γ^β would be directly given by the effective potential (3) with a volume factor, because in this case all the diagrams should be evaluated in the zero momentum limit $K_4=0$, $\mathcal{K}=|\vec{K}| \rightarrow 0$. However, as soon as an inhomogeneous condensed solution is taken into account, two extra terms appear, one obtained at tree level and the other coming from the one-loop polarization operator with spatial components of the vector field $\vec{W}^1$ in the external legs.

Let us concentrate on calculation of this polarization operator. Suppose $\mathcal{K} \ll \mu_i$, $i=1,2,3$, in analogy with the $T=0$ case. Later we will check that the results agree with this approximation.

The polarization operator for the SU(2) gauge fields of the WS model can be written in the following form

$$\begin{aligned} \Pi_{\mu\nu}^{ab} = & \Pi_1^{ab} \left(\delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2} \right) + \Pi_2^{ab} \left(\frac{K_\mu K_\nu}{K^2} + \frac{\mu_\mu \mu_\nu}{(\mu K)^2} K^2 \right. \\ & \left. - \frac{\mu_\mu K_\nu + \mu_\nu K_\mu}{\mu K} \right) + \Pi_3^{ab} \epsilon_{\mu\nu\rho\lambda} K_\rho \mu_\lambda \end{aligned} \quad (8)$$

The 4-velocity μ_μ is related to the breaking of the Lorentz invariance in statistics. The structural term with the fully antisymmetrical unit tensor $\epsilon_{\mu\nu\rho\lambda}$ comes from the fermionic loops due to the γ_5 matrix. The coefficients Π_i^{ab} ($i=1,2,3$) depend on T , k and the chemical potentials μ_i ($i=1,2,3$).

The following remarks are in order:

- 1) We have written a k -transversal Lorentz-structure. It can be justified remembering that in the case of a SU(2) theory with only one chemical potential (the leptonic one), the one-

loop polarization operator is indeed transversal/13/. In the present case one has certainly other effective chemical potentials, one of them, μ_1 , introduces the Pauli matrix τ^3 in the propagators of some particles. Nevertheless, with respect to the Lorentz-space, μ_1 does not introduce any new independent Lorentz-vector, consequently it is unable to modify the Lorentz structure and so it is not expected to violate the transversality. It is worth pointing out that even if the tensor $\pi_{\mu\nu}^{ab}$ be not transversal the analysis that we shall do below would remain valid.

2) The coefficients π_i^{ab} have not been written in the usual form $\pi_i^{ab} = \pi_i \delta^{ab}$. The reason is, once again, the presence of μ_1 , or equivalently of τ^3 in the propagators. This matrix explicitly breaks the symmetry in the internal space. Thus, besides the tensor δ^{ab} , it can appear $\delta^{a3}\delta^{b3}$ and ϵ^{ab3} with different scalar coefficients each.

In spite of these general considerations the structure ϵ^{ab3} is not present in π_3^{ab} after evaluating in $k_4=0$. To see this, note that the one-loop diagrams that contribute to $\pi_{\mu\nu}^{12}$ have the following expressions

$$\text{Diagram (9a)} = -\frac{ig^2}{4} \int d^4p \delta_\mu \frac{1+\gamma_5}{2} \frac{i}{\not{p}} \gamma_\nu \frac{1+\gamma_5}{2} \frac{1}{(\not{p}-K)} \delta(K+K') \quad (9a)$$

$$\text{Diagram (9b)} = \frac{ig^2}{4} \int d^4p \delta_\mu \frac{1+\gamma_5}{2} \frac{i}{\not{p}^*} \gamma_\nu \frac{1+\gamma_5}{2} \frac{1}{(\not{p}-K)} \delta(K+K') \quad (9b)$$

Here

$$\int dp = \beta^{-1} \sum_{p_4 = (2n+1)\pi/\beta} \int \frac{d^3 p}{(2\pi)^3}, \quad \tilde{p}_\mu^* = \begin{cases} p_i^* = p_i \\ p_4^* = p_4 - i\mu_L \end{cases}, \quad \bar{p}_\mu = \begin{cases} \bar{p}_i = p_i \\ \bar{p}_4 = p_4 - i\mu_{e_L} \end{cases} \quad (10)$$

$\mu_{e_L} = \frac{1}{2} (\mu_1 + \mu_2) - \mu_3$, $\mu_L = \frac{1}{2} (\mu_2 - \mu_1) - \mu_3$ are the left electron and neutrino chemical potentials.

The diagrams (9) fulfill (9a) ($\mu_L \leftrightarrow \mu_{e_L}$) = -(9b). After multiplying $\Pi_{\mu\nu}^{12}$ by $\varepsilon_{\mu\nu\delta\lambda} k_\delta u_\lambda$, summing in p_4 and substituting $k_4=0$, these two diagrams cancel each other, thanks to this property. Therefore at $k_4=0$, they do not contribute to Π_3^{ab} . It can be similarly shown that $\Pi_{\mu\nu}^{21}$ does not contribute to Π_3^{ab} at $k_4=0$. We conclude that the structure ε^{ab3} is absent from Π_3^{ab} at least in $k_4=0$.

3) The high-temperature leading parts of the three diagonal coefficient components Π_3^{aa} ($a = 1, 2, 3$) coincide in the $k_4=0$ small $\mathcal{U}$ limit. To understand this remark note that the tensor components $\Pi_{\mu\nu}^{aa}$ ($a = 1, 2, 3$) are all proportional to a same kind of integral

$$\frac{g^2}{2} [\delta_{\mu\delta} \delta_{\nu\lambda} + \delta_{\mu\lambda} \delta_{\nu\delta} - \delta_{\mu\nu} \delta_{\delta\lambda} + \varepsilon_{\mu\nu\delta\lambda}] \int dp \frac{\tilde{p}_\delta (p'_\lambda - k_\lambda)}{\tilde{p}^2 (p-k)^2} \quad (11)$$

$$\tilde{p}_\mu = \begin{cases} \tilde{p}_i = p_i \\ \tilde{p}_4 = p_4 - i\mu \end{cases}, \quad p'_\mu = \begin{cases} p'_i = p_i \\ p'_4 = p_4 - i\mu' \end{cases} \quad (12)$$

the effective chemical potentials being defined in each case.

For $\Pi_{\mu\nu}^{11}$, $\mu' = \mu_{e_L}$, $\mu = \mu_L$, for $\Pi_{\mu\nu}^{22}$, $\mu' = \mu_L$, $\mu = \mu_{e_L}$, for $\Pi_{\mu\nu}^{33}$, $\mu = \mu' = \mu_L$ in one of the diagrams, whilst $\mu = \mu' = \mu_{e_L}$ in the other one. The proportionality factor is equal 1 for $\Pi_{\mu\nu}^{aa}$ $a = 1, 2$, and equal 1/2 for $\Pi_{\mu\nu}^{33}$.

Multiplying (11) by $\varepsilon_{\mu\nu\sigma\lambda} k_\sigma u_\lambda$ and taking the limit of interest (high T, $k_4=0$, small $\mathcal{U}$), one finds, as we will explicitly show below, that the leading contribution is proportional to $(\mu + \mu')$. Therefore at least for the leading part, we can consider $\Pi_3^{ab} = \Pi_3 \delta^{ab}$.

Even more, the lack of knowledge of the explicit internal structure of the other coefficients Π_i^{ab} ($i=1,2$) causes no problem to us, since thanks to the lacking of severe infrared divergences, /14/ their leading k-dependent contribution to Π_{ij}^{ab} must be $\sim g^2 k^2$, and consequently can be neglected as compared to the contribution originated from Π_3^{ab} , that is linear in k as we will immediately prove. On the other hand, we must take into account also the possible contribution of the leading high temperature terms (independent on k) obtained after evaluating in $k_4=0$, $\mathcal{U}=0$, which usually gives rise to terms proportional to T^4 , T^2 . Therefore we shall search the expansion of Π_3^{ab} around $\mathcal{U}=0$ after we take $k_4=0$. If the first term in such expansion is not zero, it will give rise to a linear in $\mathcal{U}$ contribution to Π_{ij}^{ab} . In the rest frame one can write

$$\Pi_3 = -i \frac{\varepsilon_{\mu\nu\sigma\lambda} k_\sigma u_\lambda \Pi_{\mu\nu}^{ab} \delta^{ab}}{3 \mathcal{U}^2} \quad (13)$$

Taking into account the contribution of the left-handed fermionic doublet to $\Pi_{\mu\nu}^{11}$ we can write

$$\Pi_{\mu\nu}^{11} \varepsilon_{\mu\nu\sigma\lambda} k_\sigma u_\lambda = \frac{g^2}{\beta} \sum_{\substack{p_4=(2n+1)\pi/\beta}} \int \frac{d^3 p}{(2\pi)^3} \varepsilon_{\mu\nu\sigma\lambda} \varepsilon_{\mu\alpha\nu\gamma} \frac{k_\sigma u_\lambda p_\alpha^* (\bar{p}_\gamma - k_\gamma)}{p^2 (\bar{p} - k)^2} \quad (14)$$

After summing over $p_4=(2n+1)\frac{\pi}{\beta}$, changing to spherical variables and evaluating in $K_4=0$ it is obtained that

$$\begin{aligned} \Pi_{\mu\nu}^{11} \varepsilon_{\mu\nu\rho\lambda} K_\rho u_\lambda = i \frac{g^2}{8\pi^2 \beta^2} \int_0^\pi d\theta \sin\theta \int_0^\infty dx \cdot x^2 \\ \cdot \left\{ \frac{\beta x^2 - 2x \cos\theta - \beta(\mu_{e_L} - \mu_{\nu_L})^2}{D} \left[x^2 N^- - (\mu_{e_L} - \mu_{\nu_L}) x \cos\theta N^+ \right] \right. \\ \left. - \frac{2x(\mu_{e_L} - \mu_{\nu_L})}{D} \left[x^2 N^+ - (\mu_{e_L} - \mu_{\nu_L}) x \cos\theta N^- \right] \right\} \end{aligned} \quad (15)$$

$$\begin{aligned} D &= [\beta x^2 - 2x \cos\theta - \beta(\mu_{e_L} - \mu_{\nu_L})^2]^2 - 4x^2(\mu_{e_L} - \mu_{\nu_L})^2 \\ N^+ &= [n(x + \mu_{\nu_L}) + n(x - \mu_{\nu_L})] - [n(x + \mu_{e_L}) + n(x - \mu_{e_L})] \\ N^- &= [n(x + \mu_{\nu_L}) - n(x - \mu_{\nu_L})] + [n(x + \mu_{e_L}) - n(x - \mu_{e_L})] \end{aligned}$$

Here

$$n(x \pm \mu) = [1 + \exp(x \pm \beta\mu)]^{-1} \quad (16)$$

are the fermion distribution functions.

In the high temperature region the integrand in (15) (henceforth denoted by I), can be expanded in powers of β . The first nonzero term in this expansion comes from the second derivative in β , its expression being written as follows

$$\frac{\partial^2 I}{\partial \beta^2} \Big|_{\beta=0} = \frac{\mu_{e_L} + \mu_{\nu_L}}{x^2} x^2 A(x) \quad (17)$$

where

$$A(x) = \frac{2e^x}{b(1+e^x)^2} \left\{ \frac{a^2}{b} \cos^2 \theta - \frac{a}{2} - \frac{1-e^x}{1+e^x} \frac{x}{2} \sin^2 \theta (\mu_L - \mu_V)^2 \right\} \quad (18)$$

$$a = x^2 - (\mu_L - \mu_V)^2$$

$$b = x^2 \cos^2 \theta - (\mu_L - \mu_V)^2$$

Expanding (18) in powers of x and performing the integrals in x and θ , one obtains that the leading contribution to (14) has the form

$$\prod_{\mu\nu}^{11} \varepsilon_{\mu\nu\lambda} K_{\lambda} u_{\lambda} = -\frac{ig^2}{16\pi^2} (\mu_L + \mu_V) x^2 = \frac{ig^2}{16\pi^2} (2\mu_3 - \mu_2) x^2 \quad (19)$$

The calculation of $\prod_{\mu\nu}^{22}$ and $\prod_{\mu\nu}^{33}$ is completely analogous. Hence using eq.(19) and the correspondent expressions for $\prod_{\mu\nu}^{22}$, $\prod_{\mu\nu}^{33}$, (remember that in the limit of interest $\prod_{\mu\nu}^{11} = \prod_{\mu\nu}^{22} = \prod_{\mu\nu}^{33}$), as well as eq.(13), one arrives to the following result

$$\prod_3 = \frac{g^2}{16\pi^2} (2\mu_3 - \mu_2) \quad (20)$$

Now, the one-loop effective action Γ^β is straightforwardly obtained. Taking into account (3) and (20), it has the form

$$\Gamma^\beta = - \int d^3x \left[\frac{1}{4} (F_{ij}^a)^2 - \frac{g^2}{32\pi^2} (2\mu_3 - \mu_2) \varepsilon_{ijl} \partial_i W_j^a W_l^a + V^\beta \right] \quad (21)$$

The term $(F_{ij}^a)^2$ arises at the tree level due to the k -depending gauge fields. Note that (21) is similar in form to the zero-temperature effective action found in [7]. Here, however, the Chern-Simons term does not contain any nonlinearity with respect to the chemical potentials, the high temperature simplifying the

structure of this term.

The gauge noninvariance under nontrivial gauge transformation of (21) does not cause any trouble (see the discussion of this point in /6/).

Using the configuration (7) explicitly, we can write (21) in the form

$$\frac{\Gamma^\beta}{V} = - \left[\kappa^2 C^2 - \frac{g^2}{16\pi^2} (2\mu_3 - \mu_2) \kappa C^2 + V^\beta \right] \quad (22)$$

V being the volume of the system.

Taking the minimum of Γ^β with respect to κ , it is obtained

$$\kappa = \frac{g^2}{32\pi^2} (2\mu_3 - \mu_2) \quad (23)$$

It corroborates our supposition $\kappa \ll \mu_i$. In order to know the behaviour of the inhomogeneity at finite temperature we must find the explicit dependence of κ on T . It can be obtained by solving the equilibrium equations of the system for the chemical potentials that appear in (23), in the region near the W-condensate phase transition curve, where the inequality $C \ll T$ is fulfilled.

Using the effective action (22), the system of equilibrium equations has the following form

$$-\frac{1}{V} \frac{\partial \Gamma^\beta}{\partial \xi^2} = \frac{e^2 C^2}{4 \sin^2 \theta_w} - \frac{(\mu_2 - \mu_1)^2}{8} + \frac{\lambda \xi^2}{2} - \frac{m^2}{2} + \left[\frac{3e^2}{4 \sin^2 2\theta_w} (1 + 2 \cos^2 \theta_w) + 6 \right] \frac{1}{24\beta^2} = 0 \quad (24a)$$

$$-\frac{1}{V} \frac{\partial \Gamma^\beta}{\partial C^2} = -\mu_1^2 + \frac{e^2}{4 \sin^2 \theta_w} \xi^2 + \kappa^2 - \frac{g^2}{16\pi^2} (2\mu_3 - \mu_2) \kappa = 0 \quad (24b)$$

$$-\frac{1}{V} \frac{\partial \Gamma^\beta}{\partial \mu_E} = -(2C^2 + \frac{11}{12\beta^2}) \mu_1 - \frac{5}{12\beta^2} \mu_2 + \frac{1}{3\beta^2} \mu_3 + \frac{g^2}{16\pi^2} \kappa C^2 = 0 \quad (24c)$$

$$\begin{aligned}
 -\frac{1}{v} \frac{\partial \Gamma^\beta}{\partial \mu_w} &= \cos^2 \theta_w \left(2C^2 + \frac{11}{12\beta^2} \right) \mu_1 + \sin^2 \theta_w \left(\frac{\mu_3}{3\beta^2} - \frac{5\mu_2}{12\beta^2} \right) \\
 &\quad - \frac{\mu_w}{4} \xi^2 + \frac{g^2}{16\pi^2} \alpha C^2 \sin^2 \theta_w = 0
 \end{aligned} \tag{24d}$$

$$-\frac{1}{v} \frac{\partial \Gamma^\beta}{\partial \mu_3} = -\frac{\mu_3}{2\beta^2} + \frac{\mu_2}{3\beta^2} - \frac{g^2}{8\pi^2} \alpha C^2 = -n_L \tag{24e}$$

For convenience we have introduced the electric $\mu_E = \mu_1 \sin^2 \theta_w + \mu_2 \cos^2 \theta_w$ and weak-neutral $\mu_w = \mu_2 - \mu_1$ chemical potentials. Thus the equations (24c) and (24d) have a direct physical meaning, they guarantee the overall electrical and weak-neutral neutrality of the system respectively.

The α depending terms introduce a negligible modification ($O(g^2)$) to the equations obtained in case of homogeneous W-condensate /4/, therefore they will not play any crucial role in the behaviour of $\mu = \mu(n_L, T)$. After some algebra, where we considered that the temperatures of interest are not very far from the critical value $T_c^6 = 4n_L^2 / 7.7g^2$ (i.e. are near the curve "a" in Fig.1) we get

$$2\mu_3 - \mu_2 \simeq 4.4 \frac{n_L}{T^2} \tag{25}$$

Finally, from eqs.(23) and (25), it is found that

$$\alpha \simeq g^2 n_L / 8\pi^2 T^2 \tag{26}$$

It follows from eq.(26) that at each fixed leptonic charge density, the increasing of temperature weakens the effect of inhomogeneity in the W-condensate.

In order to know the degree of weakening of the inhomogeneity as compared to the effect at zero temperature let us

write eq.(26) in terms of the value of $\mathcal{L}$ at $T=0$, $\mathcal{L}^{(0)}$. For the sake of simplicity, consider a very high density situation $n_L \gg n_L^{(0)}$, here $n_L^{(0)} = M_W^3 / 6\pi^2$ is the zero temperature critical average value of leptonic charge density /1/ necessary for the condensate to arise.

At $T=0$ and very high leptonic charge density one has $\mu_1, \mu_2 \ll \mu_3$ and the densities of electrons and neutrinos are almost equal to one another /1/. Then, taking into account that $n_L = n_\nu + n_{e_L} + n_{e_R}$ where $n_\nu = \frac{1}{6\pi^2} [\mu_3 - \frac{1}{2}(\mu_2 - \mu_1)]^3$, $n_{e_L} = \frac{1}{6\pi^2} [\mu_3 - \frac{1}{2}(\mu_1 + \mu_2)]^3$ and $n_{e_R} = \frac{1}{6\pi^2} [\mu_3 - \frac{1}{2}\mu_3]^3$ one can easily find

$$\mu_3 \simeq (2\pi^2 n_L)^{1/3} \quad (27)$$

Substituting eq.(27) in the equation $\mathcal{L} = (g^2/16\pi^2)\mu_3$ that gives the value of the inhomogeneity at zero temperature (see ref. /7/), we get

$$\mathcal{L}^{(0)} = \frac{g^2}{16\pi^2} (2\pi^2 n_L)^{1/3} \quad (28)$$

Using eqs.(26) and (28) we roughly obtain

$$\mathcal{L} \sim \left(\frac{4}{\pi^2}\right)^{1/3} \frac{n_L^{2/3}}{T^2} \mathcal{L}^{(0)} \sim g^{2/3} \mathcal{L}^{(0)} \quad (29)$$

We conclude that for any value of leptonic charge density the temperature tends to eliminate the inhomogeneity of the W-condensate that was present at $T=0$. On the other hand, if the temperature is fixed and the leptonic charge density is increased, the parameter of the inhomogeneity also increases. The conjugation of this two opposite effects, characterized by the quotient $L = n_L/n_\gamma$ ($n_\gamma \sim T^3$ is the photon density in the system) determines the degree of inhomogeneity of the W-con-

densate in this state.

3. An interesting physical implication of the existence of an inhomogeneous and anisotropic W-condensate in the dense electroweak matter has been discussed in /9/. In this appear it is argued the possibility to have at present a background of gravitational waves created due to the inhomogeneous W-condensate whose amplitudes and frequencies have ranges of magnitudes accesible to the planned detectors of gravitational waves.

This argument, however, is based on the assumption that the Universe passed through an intermediate cold stage.

We would like to argue here that in the case of a hot dense Universe the magnitude of the dimensionless parameter Δ , characterizing the background of gravitational waves, decreases in several orders.

A hot Universe with large lepton asymmetry can be compatible with the present experimental data. Scenarios of this kind are presented in /14/ and in the last paper of ref. /10/.

The existance of a condensate of the form (7) gives rise to the appearance of a anisotropic part in the energy-momentum tensor /9/. The transverse traceless component of this tensor has the form

$$\theta_{\mu 0}^{TT} = 0, \quad \theta_{ij}^{TT} = -\left(\frac{1}{3} \delta_{ij} - n_i n_j\right) \theta^{TT}, \quad \theta^{TT} = \frac{\vec{H}^2}{2} \quad (30)$$

where the unit vector $\vec{n}$ is directed along $\vec{H}$, the magnetic field corresponding to the condensate (7), $|\vec{H}| = \kappa c$. This transverse traceless component determines the magnitude of the anisotropy, characterized by

$$\Delta = \frac{\theta^{TT}}{E} = \frac{\vec{H}^2}{2E} \quad (31)$$

where E is the total energy. At $T=0$ one has that $E \sim \frac{\mu^4}{12\pi^2}$ and $\Delta \sim (1-3) \cdot 10^{-4} / 9$. At high T , but zero fermion density there is no reliable procedure to calculate Δ .

In our case we have simultaneously high temperature and density and a reliable treatment is perfectly possible. In this case $E \approx \frac{10\pi^2}{3} T^4$. Working again in the region where $C \ll T$ and using eqs.(24), one arrives to the following equation

$$C^2 = \left[0.4 \left(\frac{7E^2}{\alpha} \right) - \frac{2\alpha}{7E^2} + \frac{m^2}{2T^2} - 0.8 \right] T^2 \quad (32)$$

where $E = 1 - \sqrt{1 - \alpha}$, $\alpha = 7\lambda T^6 / n_L^2$.

We considered $\lambda \sim 8^{3/4}$. At $T \gg T_0$ ($T_0^2 = \frac{m^2}{46\lambda}$) we can cost away the third term in r.h.s. of eq.(32). After this we immediatly find that in the high temperature, high density region near the transition curve for the W-condensate, it must be $\frac{7E^2}{\alpha} \gtrsim 3.45$ in order to have $C^2 \gtrsim 0$.

Let us consider for convenience the following range of typical values of $7E^2/\alpha$ near the transition curve

$$3.48 < \frac{7E^2}{\alpha} < 3.6 \quad (33)$$

From eq.(32) and (33) it is easily established that in this case the amplitude of the W-condensate lies in the range

$$0.018 < C^2 < 0.14 \quad (34)$$

Bearing in mind eq.(26), that is valid in the present region, as well as the expression for the magnitude of the anisotropy (31), one obtains

$$\Delta = \frac{3g^2}{160\pi^4} \frac{n_L^2}{T^6} C^2 \quad (35)$$

For the ranges (33), (34), one can roughly obtain

$$10^{-7} \leq \Delta \leq 10^{-6} \quad (36)$$

This result clearly show that in the case of a hot dense Universe the magnitude of the anisotropy is several orders (2-3) lower than the value obtained from a cold dense Universe. Consequently, the present amplitudes of gravitational waves originated by initial anisotropic states at high temperatures and densities would be considerably smaller than in the case that they are generated by anisotropic cold dense states.

References

1. A.D.Linde, Phys. Lett. B 86 (1979) 39
2. I.V.Krive, Yadern. Fiz. 31 (1980) 1259
3. O.K.Kalashnikov and H.Perez-Rojas, Kratk. Soob. po Fiz. 2 (1986), 23 / English translation: Sov. Phys. Lebedev Inst. Reports, Allerton Press/; Nucl. Phys. B 293 (1987) 241
4. E.J.Ferrer, V. de la Incera and A.E.Shabad, Phys. Lett. B 185 (1987) 407; Proceedings of the XI International School of Theoretical Physics "Testing of the Standard Model" held in Szczyrk-Bila, Poland, Sept. 1987 (World Scientific Publishing Company, Singapore, Jun. 1988)
5. E.J.Ferrer, V. de la Incera and A.E.Shabad, Il. Nuovo Cim. A 98 (1987) 245
6. V.A.Rubakov, Progr. Theor. Phys. 75 (1986) 366;
V.A.Matveev, V.A.Rubakov, A.N.Tavkhelidze and V.F.Tokarev, Nucl. Phys. B 282 (1987) 700
7. D.I.Deriagin, D.Yu.Grigoriev and V.A.Rubakov, Phys. Lett. B 178 (1986) 385.
8. A.N.Redlich and L.C.R.Wijewardhana, Phys. Rev. Lett. 54 (1985) 970;
K.Tsokos, Phys. Lett. B 157 (1985) 413

9. D.V.Deryagin, D.Yu.Grigoriev, V.A.Rubakov and M.V.Sazhin,
Mod. Phys. Lett. A 1 (1986) 593
10. V.A.Kuzmin, V.A.Rubakov and M.E.Shaposhnikov, Phys. Lett.B
155 (1985) 36;
M.E.Shaposhnikov, Nucl. Phys. B 287 (1987) 757;
P.Arnold and L.Mc. Lerran, Phys. Rev. D 34 (1987) 581;
E.J.Ferrer and V. de la Incera. Preprint N° 223, Lebedev
Phys. Inst., Moscow, 1988
11. A.D.Linde, Rep. Prog. Phys. 42 (1979) 389
12. L.Dolan and R.Jackiw, Phys. Rev. D 9 (1974) 3320;
S.Weinberg, Phys. Rev. D 9 (1974) 3357
13. O.K.Kalashnikov and V.V.Klimov, Yad. Fiz. 31 (1980) 1357
14. A.D.Linde, Phys. Lett. B 96 (1980) 289;
O.K.Kalashnikov and V.V.Klimov, Yad. Fiz. 33 (1981) 848;
R.Jackiw and Templeton, Phys. Rev. D 33 (1981) 2291.
15. Langacker P., Segre G. and Soni S. Phys. Rev. D 26 (1982)
3425

The realization of this work was suggested to us by E.S.Frad-
kin and A.D.Linde; we feel indebted to them by useful advices
and encouragement. We also grateful to A.E.Shabad and M.G.Sha-
poshnikov by discussions.

Fig.1. Phase transitions curves in the plane (n_L^2, T^2) of the WS model at finite temperature and leptonic charge density. The shaded region in the W-condensed phase indicates the zone when the W-condensate is very small compared to the temperature ($C \ll T$).

Препринты Физического института имени П.Н. Лебедева АН СССР рассылаются научным организациям на основе взаимного обмена.

Наш адрес: 117924, Москва В-333, Ленинский проспект. 53

Preprints of the P.N. Lebedev Physical Institute of the Academy of Sciences of the USSR are distributed by scientific organizations on the basis of mutual exchange.

Our address is: USSR, 117924, Moscow В-333, Leninsky prospect, 53

Распоряжение Президиума АН СССР № 57-199 от 30. 01. 1973 г.
Заказ № 9. Тираж 100 экз. П.л. 1,2.

Отпечатано в Отделе научно-технической информации ФИАН СССР
Москва, В-333, Ленинский проспект, 53