CERN-TH.5383/89

THE SPECTRUM OF SPINNING SUPERPARTICLES

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ABSTRACT

We construct the action and transformation rules of a massless spinning superparticle in D dimensions which has N worldline as well as M target space supersymmetries. The spectrum of physical states for different values of N , M and D is presented. We also discuss the action, symmetries and spectrum of massive spinning superparticles.

CERN-TH.5383/89

April 1989

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It is well known that the one-dimensional action for a massless spin-0 particle in D dimensions can be supersymmetrized in two different ways. One way is to introduce a local one-dimensional worldline supersymmetry [1,2]. The thus obtained supersymmetric model is called the "spinning particle" and describes, upon quantization, a massless spin- $\frac{1}{2}$ particle in D dimensions. Supersymmetric actions with N (N arbitrary) local worldline supersymmetries have also been considered [3,4,5]. The authors of [4] called the corresponding model the N -extended spinning particle. Upon quantization, the model describes a massless spin- $\frac{N}{2}$ particle in D dimensions. In order to obtain this result it is necessary to gauge the $O(N)$ symmetry that rotates the different supersymmetries into each other, i.e. to consider gauged N -extended worldline supersymmetry.

The other way to supersymmetrize the action for a spin-0 particle is to introduce a rigid D -dimensional target space supersymmetry. In this way one obtains the "superparticle" action [6] which, upon quantization, describes a massless supermultiplet in D dimensions. It turns out that these superparticle actions are invariant under an additional local fermionic so-called "kappa-symmetry" [7]. Actions with M (M arbitrary) rigid target space supersymmetries have also been considered [8] and they describe supermultiplets which form representations of the M -extended super-Poincare algebra.

Instead of introducing either worldline or target space supersymmetry separately it is also possible to introduce these two supersymmetries at the same time. In [9] an action was constructed with one local worldline supersymmetry and one local 4-dimensional target space supersymmetry. The model is called the spinning superparticle and describes, upon quantization, a 4-dimensional scalar and vector multiplet. A similar action, but with 10-dimensional instead of 4-dimensional target space supersymmetry was considered in [10] and was shown to describe two 10-dimensional gravitini multiplets.

It is the purpose of this paper to give a unified treatment of the above cases considered. To be precise, we will present the action and transformation rules of a massless spinning superparticle in D dimensions which has N local worldline and M rigid target space supersymmetries. This model will be called the massless (N, M, D) spinning superparticle and we will give the spectrum of physical states for different values of N, M and D . At the end of this letter we will also discuss the action, transformation rules and spectrum of the massive (N, M, D) spinning superparticle.

The dynamics of the (N, M, D) spinning superparticle can be described in terms of two sets of fields. The first set are the coordinates of the spinning superparticle:

$\{X^\mu(\tau), \Psi_\mu^i(\tau), \theta_\alpha^m(\tau)\}$. The parameter τ is an evolution parameter. The X^μ ($\mu = 1, \dots, D-1$) are commuting variables, whereas the Ψ_μ^i ($i = 1, \dots, N$) and θ_α^m ($m = 1, \dots, M$) are anticommuting variables. The μ and α are vector and spinor indices respectively of the spacetime Lorentz group $SO(1, D-1)$. The range of the α index is given by $\alpha = 1, \dots, 4\zeta$ where ζ is the real dimension of the spinor representation of $SO(D-1, 1)^*$. The i index is a vector index of $O(N)$. The Ψ_μ^i (θ_α^m) are the N worldline (M target space) supersymmetric partners of X^μ . The second set of variables are the one-dimensional N -extended supergravity gauge fields: $\{V(\tau), \chi^i(\tau), f^{ij}(\tau) = -f^{ji}(\tau)\}$. The V is the einbein, the χ^i are the N gravitini and the f^{ij} are the $O(N)$ gauge fields. In terms of the variables introduced above the action of the massless (N, M, D) spinning superparticle is given by

$$S = \frac{1}{2} \int d\tau \{V P^\mu P^\nu \eta_{\mu\nu} - \Psi_\mu^i \mathcal{D}_0 \Psi_\nu^i \eta^{\mu\nu}\} \quad (1)$$

Here $\eta_{\mu\nu}$ is the Minkowski spacetime metric, $\eta_{\mu\nu} = \text{diag}(-, +, \dots, +)$, and P_μ and $\mathcal{D}_0$ are defined by

$$P_\mu \equiv V^{-1}(\dot{X}_\mu + i\dot{\theta}^m \gamma_\mu \theta^m - \chi^i \Psi_\mu^i) \quad (2)$$

$$\mathcal{D}_0 \Psi_\mu^i \equiv \dot{\Psi}_\mu^i - f^{ij} \Psi_\mu^j \quad (3)$$

The γ^μ are the γ -matrices of the $SO(1, D-1)$ Lorentz group. The properties of these γ -matrices and of the spacetime spinor θ depend on the value of D and are given in [11,12]

The action (1) is invariant under five types of gauge transformations: worldline diffeomorphism invariance with parameter $\xi(\tau)$, N -extended worldline supersymmetry with parameter $\epsilon^i(\tau)$, $O(N)$ invariance with parameter $b^{ij}(\tau) = -b^{ji}(\tau)$, local M -extended " κ -invariance" with parameter $\kappa_\alpha^m(\tau)$ and new bosonic gauge invariances with parameter λ^{mn} . The ξ, b and λ (ϵ and κ) are commuting (anti-commuting) parameters. Note that κ is a spacetime spinor and worldsheet scalar.

The infinitesimal worldline reparametrizations are (we omit from now on the spacetime spinor index)

$$\begin{aligned} \delta X^\mu &= \xi \dot{X}^\mu & \delta V &= (\xi V) \\ \delta \Psi_\mu^i &= \xi \dot{\Psi}_\mu^i & \delta \chi^i &= (\xi \chi^i) \\ \delta \theta^m &= \xi \dot{\theta}^m & \delta f^{ij} &= (\xi f^{ij}) \end{aligned} \quad (4)$$

* The specific value of ζ for different values of D is given in the Table

The worldline supersymmetries are:

$$\begin{aligned}\delta X^\mu &= \epsilon^i \Psi_i^\mu & \delta V &= 2\epsilon^i \chi_i \\ \delta \Psi_\mu^i &= P_\mu \epsilon^i & \delta \chi^i &= \mathcal{D}_0 \epsilon^i\end{aligned}\tag{5}$$

The $O(N)$ gauge transformations are

$$\begin{aligned}\delta \Psi_\mu^i &= b^{ij} \Psi_\mu^j & \delta \chi^i &= b^{ij} \chi_j \\ \delta f^{ij} &= \mathcal{D}_0 b^{ij}\end{aligned}\tag{6}$$

The κ -transformations are

$$\begin{aligned}\delta X^\mu &= i\bar{\theta}^m \gamma^\mu \delta \theta^m & \delta V &= -4\bar{\theta}^m \kappa^m \\ \delta \theta^m &= i\gamma_\mu \kappa^m P^\mu\end{aligned}\tag{7}$$

The bosonic λ invariances are

$$\begin{aligned}\delta X^\mu &= i\bar{\theta}^m \gamma^\mu \delta \theta^m \\ \delta \theta^m &= \lambda^{mn} \dot{\theta}^n\end{aligned}\tag{8}$$

The parameter λ^{mn} satisfies

$$C\gamma^\mu \lambda^{mn} = (C\gamma^\mu \lambda^{nm})^T\tag{9}$$

where T indicates a transposition in the spinor indices. E.g. in $D = 4$ the general solution of eq. (9) is given by $\lambda^{mn} = \lambda_1^{(mn)} + \lambda_2^{[mn]} \gamma_5$. Note that in general this λ -invariance is not an independent symmetry. For instance the $\lambda_1^{(mn)}$ transformations in $D = 4$ can be obtained as a field-dependent κ -transformation with parameter given by

$$\kappa^m = -i\lambda_1^{(mn)} \gamma_\mu \dot{\theta}^n \frac{P^\mu}{P^2}\tag{10}$$

For completeness we also list here the on-shell κ and ϵ commutator algebra:

$$\begin{aligned}[\delta(\epsilon_2), \delta(\epsilon_1)] &= \delta_{g.c.t.}(\xi) + \delta(\epsilon^i = -\xi \chi^i) \\ &\quad + \delta(b^{ij} = -\xi f^{ij}) + \delta(\lambda = -\xi) + \text{eqs. of motion} \\ [\delta(\kappa_1), \delta(\kappa_2)] &= \delta_{g.c.t.}(\tilde{\xi}) + \delta(\epsilon^i = -\tilde{\xi} \chi^i) \\ &\quad + \delta(b^{ij} = -\tilde{\xi} f^{ij}) + \delta(\lambda = -\tilde{\xi}) + \text{eqs. of motion} \\ [\delta(\kappa), \delta(\epsilon)] &= 0 + \text{eqs. of motion}\end{aligned}\tag{11}$$

where $\xi = 2V^{-1}\epsilon_2^i \epsilon_1^i$, $\tilde{\xi} = -4iV^{-1}\bar{\kappa}_2^m \gamma_\mu \kappa_1^m P^\mu$ and λ is the trace of λ^{mn} : $\lambda \equiv \frac{1}{M} \lambda^{mn} \delta_{mn}$.

Besides local gauge invariances the action (1) also has the following rigid target space symmetries: translations P^μ with parameter a^μ , Lorentz rotations M^μ_ν with parameter $\Lambda^\mu_\nu = -\Lambda^\nu_\mu$, dilatations D with parameter α and supersymmetry transformations Q^m_α with parameter ϵ^m_α ($m = 1, \dots, M$) which are spacetime spinors. The translations, Lorentz rotations and dilatations are

$$\begin{aligned}\delta X^\mu &= a^\mu + \Lambda^\mu_\nu X^\nu + \alpha X^\mu & \delta V &= 2\alpha V \\ \delta \Psi^i_\mu &= \Lambda^\nu_\mu \Psi^i_\nu & \delta \chi^i &= \alpha \chi^i \\ \delta \theta^m &= \frac{1}{4} \Lambda^{\mu\nu} \gamma_{\mu\nu} \theta^m + \frac{1}{2} \alpha \theta^m\end{aligned}\tag{12}$$

while the supersymmetries are given by

$$\begin{aligned}\delta X^\mu &= i\tilde{\epsilon}^m \gamma^\mu \theta^m \\ \delta \theta^m &= \epsilon^m\end{aligned}\tag{13}$$

The local and global transformations given so far are valid for any value of (N, M, D) . For special values of (N, M, D) the action (1) possesses additional (super) conformal invariances. First of all one can show that the N -extended spinning particle ($M = 0$) is invariant under special conformal transformations K^μ with parameter c^μ for all values of N and D . These special conformal rotations are given by [13,14,15]

$$\begin{aligned}\delta X^\mu &= 2c \cdot X X^\mu - X^2 c^\mu & \delta V &= 4c \cdot X V \\ \delta \Psi^i_\mu &= 2c \cdot \Psi^i X_\mu - 2c_\mu X \cdot \Psi^i & \delta \chi^i &= 2c \cdot X \chi^i + 2V c \cdot \Psi^i \\ \delta f^{ij} &= 4\chi^{[i} c \cdot \Psi^{j]}\end{aligned}\tag{14}$$

On the other hand, it is suggestive to think that the M -extended superparticle ($N = 0$) is superconformal invariant for those values of M and D for which there exists a corresponding M -extended superconformal algebra in D dimensions. These superconformal algebras have been classified and are given by [16]^{*}:

$$\begin{array}{ll} D = 2 & OSp(M|2) \\ D = 3 & OSp(M|4) \\ D = 4 & SU(2, 2|M) \\ D = 5, M = 1 & F_4 \\ D = 6, M = 1 & OSp(6, 2|2)\end{array}\tag{15}$$

The superconformal transformation rules of the $(0, M, 3)$ and $(0, M, 4)$ -extended superpar-

* For $M = 1, D = 2, 3, 4 \bmod 8$ more general superconformal algebras have been constructed in [17]

ticle have been given in [8]. It would be interesting to see whether the $(0, M, 2)$, $(0, 1, 5)$ and $(0, 1, 6)$ -extended superparticles are also superconformal invariant. Summarizing, we can say that the spinning particle action ($M = 0$) is conformal invariant for all values of N and D and the superparticle action ($N = 0$) is superconformal invariant for special values of M and D . We will present arguments below indicating that the spinning superparticle action $(N, M, \neq 0)$ is not superconformal invariant for any value of (N, M, D) with $D \geq 4$.

We now present the canonical quantization of the spinning superparticle. A covariant quantization for special values of (N, M, D) can be achieved (see e.g. [9] for the $(0, 1, 4)$ - and $(1, 1, 4)$ -models, [13] for the $(1, 0, 4)$ -, [2, 19] for the $(2, 0, 4)$ - and [3, 4, 5] for the general $(N, 0, 4)$ - model) but is problematic for general values of N, M and D^* . For our purposes however it is enough to discuss the canonical quantization in the lightcone gauge. We define the lightcone gauge by**

$$\begin{aligned} X_0^+ &= 0 & V &= 1 \\ \Psi_0^{+i} &= 0 & \chi^i &= 0 \\ \gamma^+ \theta^m &= 0 & f^{ij} &= 0 \end{aligned} \quad (16)$$

where $W^\pm \equiv \frac{1}{\sqrt{2}}(W^0 \pm W^{d-1})$ for any d-vector W^μ and X_0^+, Ψ_0^{+i} are the constant (τ -independent) modes of X^{+i}, Ψ^{+i} . This lightcone gauge fixes all gauge symmetries except for the τ -independent $O(N)$ -rotations with parameter b_0^{ij} . In the lightcone gauge (16) one can solve the field equations and one is left with the following set of physical variables

$$\begin{aligned} (X_0^-, p^+) & \quad \Psi_{0I}^i \\ (X_0^I, p^I) & \quad S^m \end{aligned} \quad (17)$$

Here $X_0^-, X_0^I, \Psi_{0I}^i$ ($I = 1, \dots, D-2$) are the time-independent modes of X^-, X^I, Ψ_I^i ; p^+, p^I are the time-independent modes of $\dot{X}^+, \dot{X}^I$ and S^m is defined by

$$S^m \equiv \frac{1}{2} \sqrt{p^+} \gamma^+ \gamma^- \theta^m \quad (18)$$

Using standard methods one can derive the rigid target-space transformations of the physical variables in the lightcone gauge. We only give here the translations P and the supersymmetry transformations Q . They are given by the rigid target space transformations

* For some recent progress in the covariant quantization of superparticles see however [18]

** We ignore here boundary conditions. See [20, 13, 21] for a proper treatment of the boundary terms.

(12), (13) plus compensating field-dependent gauge transformations. The specific form of the transformation rules is uniquely determined by the requirement that they leave the lightcone gauge (16) invariant. They are given by

$$\begin{aligned}\delta X_0^I &= a^I - a^+ \frac{p^I}{p^+} \\ \delta X_0^- &= a^- + \frac{1}{2} a^+ \left(\frac{p^I}{p^+} \right)^2\end{aligned}\tag{19}$$

and

$$\begin{aligned}\delta X_0^I &= \frac{2i}{\sqrt{p^+}} \bar{\alpha}^m \gamma^I S^m \\ \delta X_0^- &= -i \frac{p^I}{(\sqrt{p^+})^3} \bar{\alpha}^m \gamma^I S^m + \frac{i}{\sqrt{p^+}} \bar{\beta}^m \gamma^- S^m \\ \delta S^m &= \frac{1}{2} \frac{p^I}{\sqrt{p^+}} \gamma^I \gamma^+ \alpha^m + \sqrt{p^+} \beta^m\end{aligned}\tag{20}$$

respectively. The supersymmetry parameters α^m and β^m are defined by $\alpha^m \equiv \frac{1}{2} \gamma^- \gamma^+ \epsilon^m$ and $\beta^m \equiv \frac{1}{2} \gamma^+ \gamma^- \epsilon^m$. Using Dirac's quantization procedure one can also derive the Dirac brackets of the physical variables and the expressions for the P and Q Noether charges. They are given by

$$\begin{aligned}\{X_0^-, p^+\} &= 1 & \{S^m, \bar{S}^m\} &= \frac{i}{4} \delta^{mn} \gamma^+ \\ \{X_0^I, p^J\} &= \delta^{IJ} & \{\Psi_{0I}^i, \Psi_{0J}^j\} &= \delta^{ij} \delta_{IJ}\end{aligned}\tag{21}$$

and

$$\begin{aligned}P^+ &= p^+ & Q_+^m &= -\frac{2i}{\sqrt{p^+}} p^I \gamma_I S^m \\ P^I &= p^I & Q_-^m &= -2i \sqrt{p^+} \gamma^- S^m \\ P^- &= \frac{-1}{2p^+} (p^I)^2\end{aligned}\tag{22}$$

respectively. Here $Q_+^m \equiv \frac{1}{2} \gamma^+ \gamma^- Q^m$ and $Q_-^m \equiv \frac{1}{2} \gamma^- \gamma^+ Q^m$. Furthermore the global $O(N)$ invariance leads to the constraint

$$\Psi_{0I}^{[i} \Psi_{0I}^{j]} = 0\tag{23}$$

The P and Q Noether charges satisfy the following M -extended super-Poincare algebra in D dimensions

$$\begin{aligned}\{Q_+^m, \bar{Q}_+^n\} &= +\frac{i}{2} P^- \gamma^+ \delta^{mn} \\ \{Q_+^m, \bar{Q}_-^n\} &= +\frac{i}{4} P^I \gamma_I \gamma^+ \gamma^- \delta^{mn} \\ \{Q_-^m, \bar{Q}_-^n\} &= +\frac{i}{2} P^+ \gamma^- \delta^{mn}\end{aligned}\tag{24}$$

We are now in a position to calculate the spectrum of physical states of the (N, M, D) spinning superparticle. It is clear from the Dirac brackets (21), that the spectrum of the

spinning superparticle is given by the direct product of the spectra of the $(N, 0, D)$ spinning particle and the $(0, M, D)$ superparticle. Therefore we first discuss these two special cases. Then we can proceed to the general case.

a. $(N, 0, D)$.

For $M = 0$ there is no target-space supersymmetry and the particle states span representations of the bosonic D -dimensional Poincaré algebra or its conformal extension. If $N = 0$ as well, we obtain the simple scalar theory in D dimensions, described completely by the co-ordinates X^μ and the associated momenta, thus reducing eq. (21) to the Heisenberg subalgebra generated by (X^I, X^-, p^I, p^+) . The spectrum consists of a single spinless particle in D dimensions. If $N \geq 1$ the Heisenberg algebra is supplemented by the Clifford algebra of the Ψ^I , which can be represented by gamma-matrices γ^I of the transverse Lorentz group. For $N = 1$ this is the only modification and the groundstate of the theory is a spinor of the transverse Lorentz group. However, for $N \geq 2$ we must take into account the extra constraint of eq. (23), which leaves no solution for odd D , whilst for even D the spectrum is given by $SO(D - 2)$ representations characterized by Young tableaux consisting of $\frac{D-2}{2}$ rows and $\frac{N}{2}$ columns^{*}. According to ref.[22] these are precisely the representations of the D -dimensional light-cone conformal group $SO(D - 2, 2)^{**}$. Actually, there are always *two* such representations for any $N \geq 1$, which generalizing the $D = 4$ case correspond to multispinors with only dotted or undotted indices, respectively. For even N these representations describe two bosonic states of opposite duality, for odd N two tensor-spinors of opposite duality-chirality. As a consequence, the spectrum of the $(N, 0, D)$ spinning particle is *reducible* for any even D and $N \geq 1$, with both representations describing conformal particle states, whilst for odd D only the cases $N = (0, 1)$ exist and describe conformal representations as well.

b. $(0, M, D)$.

Next we turn to the M -extended superparticle models, for which $N = 0$. They are characterized by the reduced form of the algebra (21) obtained by ignoring the Ψ -operators and the constraint (23). We choose a fixed light-like D -momentum

^{*} A column of width one-half represents a spinor index

^{**} For $N = 2$ the spectrum contains additional representations [4], [15] but these will not be considered here.

$$p^+ = \omega, \quad p^- = p^I = 0. \quad (25)$$

In this frame $Q_-^m \sim S^m$ and $\{Q_-, Q_-\} \neq 0$, whereas Q_+ anti-commutes with itself and with Q_- . Therefore Q_+ generates only zero-norm states which can be ignored. For the creation of physical states we only need to consider the action of the $2\zeta M$ real components of Q_-^m on a Clifford vacuum $|\Omega\rangle$. These components of Q_- transform under the transverse Lorentz group $SO(D-2)$ and the automorphism group $\mathcal{A}$ of the super-Poincaré algebra, and generate a Clifford algebra $\mathcal{C}_{M,D}$ of real dimension $2^{\zeta M}$. In the Table we have listed the $SO(D-2) \times \mathcal{A}$ group for $D = 2, \dots, 11$, and for $M = 1$ the particular multiplet of real dimension 2ζ to which Q_- (or S) belongs. For general $M \geq 1$ the components of Q_-^m belong to M such multiplets. In order to obtain the spectrum of the physical states of the $(0, M, D)$ superparticle one must decompose the $2^{\zeta M}$ dimensional representation of $\mathcal{C}_{M,D}$ into $SO(D-2) \times \mathcal{A}$ representations. This can be done for general M and D (see e.g. [12]). In the table we have indicated the multiplet structure for $M = 1$.

For specific values of M and D , for which there exists a superconformal algebra^{*}, the $(0, M, D)$ spectrum consists of superconformal representations only. For instance, for $M = 1$ we see from the table, that the spectrum in $D = 2, \dots, 6$ contains only scalar multiplets, i.e. scalars and spinors, which belong to conformal representations. However, for $D = 7, \dots, 11$ the spectrum contains Yang-Mills or gravity multiplets which are not conformal representations. Summarizing, the spinning particle spectrum ($M = 0$) is conformally invariant for all values of N and D , but the superparticle spectrum ($N = 0$) is conformally invariant only for special values of M and D .

c. $(N, M, D), (N, M) \geq 1$.

Finally we turn to the spinning superparticle, the models with both $(N, M) \neq 0$. We observe that the spectrum is not conformally invariant for any $D \geq 4$. For even D this is immediately related to the fact that the spinning particle spectrum is reducible, i.e. contains one state with only "undotted" spinor indices α and one state with only "dotted" indices $\dot{\alpha}$. Therefore, even if the superparticle spectrum is conformal (made up of states with only dotted or undotted indices), the spinning superparticle will always contain one

* Cf. the discussion surrounding eq.(15)

multiplet with mixed indices $\alpha\dot{\alpha}$. For instance, from the table we see that the (1,1,4) spinning-superparticle spectrum contains a conformal 2+2 Yang-Mills multiplet, but also a non-conformal 2+2 scalar multiplet [9]. Similarly, the (1,1,6) spinning superparticle spectrum contains a conformal 4+4 tensor multiplet and a non-conformal 4+4 Yang-Mills multiplet.

For odd D we observe, that the only allowed values of N are (0,1), and that the only dimensions in which the simple superconformal representations of [14] exists are $D = 3$ for arbitrary M , and $D = 5$ for $M = 1$. The $D = 5$ case can be excluded, since the table shows the spectrum to be a non-conformal 4+4 Yang-Mills multiplet. This confirms our statement about the non-existence of conformal spinning superparticles in $D \geq 4$. We are left with only the $D = 2$ and $D = 3$ cases, for which conformal invariance is impossible if there are spin $s \geq 1$ states, thus eliminating $M > 2$ for $D = 3$ and $M > 4$ for $D = 2$. For example, from the table we deduce the existence of conformal 1+1 scalar multiplets for the (1,1,2) and (1,1,3) models.

For $D \geq 4$ an irreducible spectrum of conformal states for the spinning particle can be achieved by imposing a duality-chirality constraint [15]. It will be interesting to see what the corresponding modification of the spinning superparticle is. Probably, this modified theory will be superconformal for all values of M and D for which the superconformal algebra of [14] exists.

We now also consider the massive (N, M, D) spinning superparticle. The massive $(1, 0, D)$ spinning particle has already been considered in [2] and the general massive $(N, 0, D)$ spinning particle has been considered in [3,15]. Recently, also the spectrum of the (1,1,4) massive spinning superparticle has been calculated [23]. We here treat the general case. The action for the massive (N, M, D) spinning superparticle is given by

$$S = S_0 + S_m \tag{26}$$

where S_0 is the action for the massless (N, M, D) spinning superparticle given in eq. (1) and S_m is given by (m is a mass parameter).

$$S_m = \frac{1}{2} \int d\tau \{ V m^2 + \Psi_5^i \mathcal{D}_0 \Psi_5^i + 2m \chi^i \Psi_5^i - 2m \bar{\theta}^m \dot{\theta}^m \} \tag{27}$$

The action (27) makes only sense for those values of D for which $\bar{\theta}^m \dot{\theta}^m$ is not a total derivative, e.g. $D = 9$. If it is a total derivative, one should take $M = 2$ and replace $\bar{\theta}^m \dot{\theta}^m$ by $\bar{\theta}^m \theta^n \delta_{mn}$.

by $\bar{\theta}^m \theta^n \epsilon_{mn}$ [23]. The massive spinning superparticle action is invariant under the same symmetries as the massless spinning superparticle except for the conformal symmetries (dilatations D and special conformal boosts K^μ). However the transformation rules now involve extra terms containing the mass parameter. In the case of the supersymmetry and the " κ " transformations these extra terms are given by

$$\begin{aligned}\delta_m X^\mu &= i \bar{\theta}^m \gamma^\mu \delta_m \theta^m & \delta_m \Psi_5^i &= m \epsilon^i \\ \delta_m \theta^m &= m \kappa^m\end{aligned}\tag{28}$$

In the lightcone gauge (16) one obtains the following mass corrections to the transformations given in eqs. (19) and (20)

$$\begin{aligned}\delta_m X_0^- &= \frac{m}{(\sqrt{p^+})^3} \bar{\alpha}^m S^m + \frac{1}{2} a^+ \frac{m^2}{(p^+)^2} \\ \delta_m S^m &= -\frac{i}{2} \frac{m}{\sqrt{p^+}} \gamma^+ \alpha^m\end{aligned}\tag{29}$$

The corresponding Noether charges are given by

$$\begin{aligned}P^- &= \frac{-1}{2p^+} (p^I p^I + m^2) \\ Q_+^m &= \frac{-2}{\sqrt{p^+}} (i p^I \gamma_I - m) S^m\end{aligned}\tag{30}$$

The P and Q Noether charges now obey a supersymmetry algebra involving a central charge Z :

$$\{Q_+^m, \bar{Q}_-^n\} = \frac{1}{4} (i p^I \gamma_I - m) \gamma^+ \gamma^- \delta^{mn}\tag{31}$$

We are now able to analyze the spectrum of the massive (N, M, D) spinning superparticle. As for the massless spinning superparticle one finds that the spectrum is given by the direct product of the massive $(N, 0, D)$ spinning particle spectrum and the massive $(0, M, D)$ superparticle spectrum. The spectrum of the general massive $(N, 0, D)$ spinning particle can be found in [3,15]. The spectrum of the massive $(0, M, D)$ superparticle is given by the smallest massive supermultiplet of the M -extended D -dimensional super-Poincare algebra with a central charge given by (31). For $D = 4, \dots, 9$ these multiplets have been given in [12]. As an example let us consider the $(1, 2, 4)$ massive spinning superparticle. The spectrum is given by the direct product of the massive $(1, 0, 4)$ spinning

particle spectrum and the massive $(0, 2, 4)$ superparticle spectrum:

$$(2, 1) \otimes \{(1, 2) + (2, 1)\} = (2, 2) + (1, 1) + (3, 1) \quad (32)$$

where the first entry corresponds to the transverse $SO(3)$ group and the second entry to the $SU(2)$ automorphism group. We thus confirm the result of [23] that the massive $(1, 2, 4)$ spinning superparticle spectrum is given by a massive gauge hypermultiplet [24] which consists of a complex scalar and vector and two Dirac spinors.

One way to generalize the results of this paper is to consider the massless or massive (N, M, D) spinning superparticle in a curved background. For the $(N, 0, D)$ spinning particle this has already been done in [4]. It would be interesting to see what restrictions on the background one obtains for general values of N, M and D . It would also be of interest to investigate the covariant quantization of the spinning superparticle along the lines of [18].

ACKNOWLEDGEMENTS

One of the authors (E.B.) would like to thank the NIKHEF for financial support and its hospitality and R. Kallosh for useful discussions.

D	$SO(D-2) \times \mathcal{A}$	rep S	Spectrum $(0, 1, D)$	Spectrum $(1, 1, D)$
2	—	1	1 + 1 Scalar	1 + 1 Scalar
3	—	1	1 + 1 Scalar	1 + 1 Scalar
4	$SO(2) \times U(1)$	$(\frac{1}{2}, 1) + (-\frac{1}{2}, -1)$	2 + 2 Scalar	2 + 2 Scalar, 2 + 2 Y.M.
5	$SO(3) \times USp(2)$	(2, 2)	4 + 4 Scalar	4 + 4 Y.M.
6	$SO(4) \times USp(2)$	(2, 1; 2)	4 + 4 Scalar	4 + 4 Tensor 4 + 4 Y.M.
7	$SO(5) \times USp(2)$	(4, 2)	8 + 8 Y.M.	64 + 64 Gravitino
8	$SO(6) \times U(1)$	$(4, 1) + (\bar{4}, -1)$	8 + 8 Y.M.	64 + 64 Gravitino 64 + 64 Gravitino'
9	$SO(7)$	8	8 + 8 Y.M.	64 + 64 Gravitino
10	$SO(8)$	8 ₊	8 + 8 Y.M.	64 + 64 Gravitino 64 + 64 Gravitino'
11	$SO(9)$	16	128 + 128 Gravity	—

Table: For each spacetime dimension $D = 2, \dots, 11$ the second column gives the transverse Lorentz group $\times$ Automorphism group $SO(D-2) \otimes \mathcal{A}$. The third column gives, for $M = 1$, the particular multiplet, of real dimension 2ζ , to which $Q_- \sim S$ belongs. The fourth (fifth) column gives the multiplet structure of the spectrum of the $(0, 1, D)$ superparticle ($(1, 1, D)$ spinning superparticle). Scalar, Tensor, Yang- Mills (Y.M.) and Gravity indicates a supermultiplet whose highest dimensional $SO(D-2)$ representation is a scalar, antisymmetric tensor, vector or symmetric traceless tensor respectively. Gravitino indicates a multiplet whose highest spin state is a vector-spinor. Multiplets with spin higher than two are not considered here.

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