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COVARIANT QUANTIZATION OF TYPE II A,B GREEN-SCHWARZ SUPERSTRING

R.E.Kallosh*

Theory Division, CERN

CH-1211 Geneva 23, Switzerland

ABSTRACT

The Green-Schwarz type II superstring theory is quantized in a Lorentz covariant gauge with a manifest generalized space-time supersymmetry. The difference in the quantization procedure comparatively to heterotic string is related to the existence of bosonic λ -symmetry, which is not independent of κ -symmetry. The gauge fixed action corresponds to a free conformal theory with a correct counting of degrees of freedom and vanishing conformal anomaly.

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* Bitnet address: KALLOSHR@CERNVM

On leave of absence from: Lebedev Physical Institute, Moscow, 117924, USSR

The manifestly space-time supersymmetric formulation of superstring type II A,B theories was presented by Green and Schwarz [1]. However, the quantization of these theories was not yet proposed in a covariant gauge* as distinct from the heterotic superstring [3], where the Lorentz-covariant, world-sheet covariant quantization by means of additional variables is available [4].

Recently new Lorentz-covariant gauges were found for the superparticle [5,6] and for the heterotic superstring [7,8]. The infinite reducibility of κ -symmetry, which was revealed two years ago [9,10], in these new gauges is no longer a drawback of the theory but a property which made it possible to find variables in which the action is quadratic, corresponding to a free theory. In the heterotic superstring theory it was also found that the gauge fixed action corresponds to a conformal theory with fields with all conformal weights (p,q) , $q \geq 0, p \geq q$ [7,8].

In quantizing the heterotic string we have used the solution (in the minimal sector) [9,11] of the master equation of Batalin and Vilkovisky [12], which is necessary for the BV quantization.

It turns out, however, that for the gauge fixing function which we have chosen in [7] it is not necessary to know this solution. It is sufficient to know that 1) the classical transformation rule of the zweibein contains $\partial_z \theta$; 2) there is no non-closure function in the gauge algebra of the form η^{ij} , where both i and j are related to the metric.

In this paper we will try to find a set of minimal fields which describe the gauge independent content of the gauge algebra. Then we will find the minimal requirements on the solution of the master equation which are necessary for obtaining the quadratic gauge-fixed action in a specially chosen gauge. We will check that these requirements are fulfilled.**

The reason why the heterotic string quantization performed in [7,8] cannot be used straightforwardly for type II superstring is the presence of bosonic λ -symmetry. This symmetry is not independent from κ -symmetry. The quantization of this theory in the framework of a truncated ghost system was performed in [13,14].

The classical superstring action is

* A possible way to do the covariant quantization of the type II strings was discussed in [2], where the singlets of κ -symmetry were defined. They might play the role of corresponding harmonic variables.

** The chirality property of type II A or type II B θ -fields is never used. Therefore all quantization works equally well both for II A and II B theory.

$$S_{cl} = \int d^2z \, e(L_1 + L_2) \quad (1)$$

where

$$L_1 = \Pi^\mu_z \Pi_{\mu\bar{z}} \quad , \quad L_2 = \partial_{\bar{z}} X^\mu (i\bar{\theta}^1 \gamma_\mu \partial_{\bar{z}} \theta^1 - i\bar{\theta}^2 \gamma_\mu \partial_{\bar{z}} \theta^2 \\ - \partial_{\bar{z}} X^\mu (i\bar{\theta}^1 \gamma_\mu \partial_{\bar{z}} \theta^1 - i\bar{\theta}^2 \gamma_\mu \partial_{\bar{z}} \theta^2) + (i\bar{\theta}^1 \gamma^\mu \partial_z \theta^1) \times \\ \times (i\bar{\theta}^2 \gamma_\mu \partial_{\bar{z}} \theta^2) - (\bar{\theta}^1 \gamma^\mu \partial_{\bar{z}} \theta^1) (i\bar{\theta}^2 \gamma_\mu \partial_z \theta^2) \quad , \\ \Pi^\mu_z = \partial_z X^\mu - i\bar{\theta}^A \gamma^\mu \partial_z \theta^A \quad , \quad A=1,2 \quad .$$

We use the two-beins $g^{ab} = e^a_z e^b_{\bar{z}} + e^b_z e^a_{\bar{z}}$,

$$\varepsilon^{\alpha\beta} = e \left(e^{\alpha}_{\underline{m}} e^{\beta}_{\underline{n}} - e^{\beta}_{\underline{m}} e^{\alpha}_{\underline{n}} \right) ; \quad \partial_{\underline{m}} = e^{\alpha}_{\underline{m}} \partial_{\alpha} .$$

The action (1) is invariant under the local fermionic κ -symmetry transformations

$$\begin{aligned} \delta(\kappa) \theta^1 &= \cancel{\prod_{\underline{z}}} K_{\underline{z}}^1, & \delta(\kappa) \theta^2 &= \cancel{\prod_{\underline{z}}} K_{\underline{z}}^2, \\ \delta(\kappa) X^\mu &= i \bar{\theta}^A \gamma^\mu \delta(\kappa) \theta^A, \end{aligned} \quad (2)$$

$$\delta(\kappa)e^a_z = -4i\partial_z\theta^1\kappa^1_z e^a_{\bar{z}} \quad , \quad \delta(\kappa)e^a_{\bar{z}} = -4i\partial_{\bar{z}}\theta^2\kappa^2_{\bar{z}} e^a_z .$$

The infinitesimal parameters κ_a^A are Majorana-Weyl spinors, κ_a^1 is an anti-self-dual two-dimensional vector and κ_a^2 is a self-dual vector.

There also exists the local bosonic λ -symmetry of the action (1):

$$\begin{aligned}\delta(\lambda)\theta^1 &= \lambda^1_{\bar{z}} \partial_{\bar{z}} \theta^1, \\ \delta(\lambda)\theta^2 &= \lambda^2_z \partial_{\bar{z}} \theta^2, \\ \delta(\lambda)X^\mu &= i\theta^A \gamma^\mu \delta(\lambda)\theta^A,\end{aligned}\tag{3}$$

$$\delta(\lambda) e^a_z = \delta(\lambda) e^a_{\bar{z}} = 0,$$

where λ_a^1 is self-dual and λ_a^2 is anti-self-dual.

The gauge algebra of this theory was investigated in [13-15,2]. The commutator of two κ -symmetries looks as follows:

$$\begin{aligned} [\delta(\kappa_2), \delta(\kappa_1)] \varphi^i = & \delta_{g.c.}(\xi^a) + \delta(\kappa_3) + \delta_{conf}(\varrho) \\ & + \delta_{\text{Lor}}(\Lambda) + \delta(\lambda) + \eta^{ij} S_{,j} \quad , \end{aligned} \quad (4)$$

where $\xi^a, \kappa_3, \rho, \Lambda$ and λ are bilinear functions of $\kappa_{a1}^A, \kappa_{a2}^A$, given in [13-15,4]:

$$\begin{aligned} \kappa_{z3}^1 &= 8i \kappa_{z[1}^1 \partial_{\bar{z}} \bar{\theta}^1 \kappa_{z2]}^1 - 2i \gamma^\mu \partial_{\bar{z}} \theta^1 \bar{\kappa}_{z1}^1 \gamma_\mu \kappa_{z2}^1, \\ \kappa_{\bar{z}3}^2 &= 8i \kappa_{\bar{z}[1}^2 \partial_z \bar{\theta}^2 \kappa_{\bar{z}2]}^2 - 2i \gamma^\mu \partial_z \theta^2 \bar{\kappa}_{\bar{z}1}^2 \gamma_\mu \kappa_{\bar{z}2}^2, \\ \lambda_{\bar{z}}^1 &= -4i \bar{\kappa}_{\bar{z}1}^2 \not{\partial}_{\bar{z}} \kappa_{\bar{z}2}^2, \quad \lambda_z^2 = -4i \bar{\kappa}_{z1}^1 \not{\partial}_z \kappa_{z2}^1, \\ \xi^a &= 4i \bar{\kappa}_{z1}^1 \not{\partial}_{\bar{z}} \kappa_{z2}^1 e^a_{\bar{z}} + 4i \bar{\kappa}_{\bar{z}1}^2 \not{\partial}_z \kappa_{\bar{z}2}^2 e^a_z, \\ \varrho &= -\frac{1}{2} \nabla_a \xi^a = -\frac{1}{2} (\nabla_z \xi^z + \nabla_{\bar{z}} \xi^{\bar{z}}), \\ \Lambda &= \frac{1}{2} (\nabla_z \xi^z - \nabla_{\bar{z}} \xi^{\bar{z}}) + \Omega_{\bar{z}} \xi^{\bar{z}} - \Omega_z \xi^z - 32 \partial_z \bar{\theta}^1 \kappa_{z[1}^1 \partial_{\bar{z}} \bar{\theta}^2 \kappa_{\bar{z}2]}^2, \\ \Omega_z &= e_{z\theta} (\partial_{\bar{z}} e_z^\theta - \partial_z e_{\bar{z}}^\theta), \quad \Omega_{\bar{z}} = e_{\bar{z}\theta} (\partial_z e_{\bar{z}}^\theta - \partial_{\bar{z}} e_z^\theta). \end{aligned}$$

As was explained in [1,15], the commutator of two κ -symmetries produces all other gauge symmetries of the theory. The non-closure functions η^{ij} are given in [13,14]. The only information about η^{ij} which will be important for us is that

$$\eta^{e_z e_{\bar{z}}} = 0.$$

In the type II superstrings not only is κ -symmetry infinite reducible, but also λ -symmetry is not independent of κ -symmetry. From eqs. (2),(3) it is clear that it is impossible to solve the equation

$$(\delta(\lambda) + \delta(\kappa))\theta^1 = \lambda^1_{\bar{z}} \partial_{\bar{z}} \theta^1 + \cancel{\lambda^1_{\bar{z}}} \kappa^1_{\bar{z}} \Big|_{S,i=0} = 0 \quad (7)$$

if κ -symmetry is not truncated. Only when $\kappa^1_{\bar{z}} = -\frac{1}{2}\gamma^{\dagger}\bar{\gamma}\kappa^1_{\bar{z}}$ is it possible to solve eq. (7) and to reproduce λ -symmetry as some κ -symmetry with a field dependent parameter [13,14]. But now, when we know that it is possible to avoid κ -truncation, as it was discussed in [16] and realized in [5,6] for superparticle and in [7,8] for heterotic string, we would like to perform a self-consistent quantization for type II superstring and take into account λ -symmetry.

The simplest way to solve this problem is not to use any relation between λ and κ , but consider them as independent symmetries and solve the BV master equation in the minimal sector, which presents the gauge algebra of the theory. However, in choosing the gauge fixing condition we will effectively use the information that λ -symmetry in the truncated system is reproduced by κ -symmetry and is not necessary to close the gauge algebra.

The possibility to proceed like this is related to the fact that eq. (7), which was solved in the truncated system [12], is not a reducibility condition

$$R^i_{\alpha} Z^{\alpha}_{\alpha_1} + \gamma^{ij}_{\alpha_1} S^{\alpha\ell}_{,j} = 0,$$

which exists for all classical fields φ^i . The standard reducibility condition leads to a degeneracy of the zero level ghost action $\varphi^*_i R^i_{\alpha} C^{\alpha}$ of the form $\delta C^{\alpha} = Z^{\alpha}_{\alpha_1} C^{\alpha_1}$ and therefore requires the introduction of ghosts C^{α_1} for ghosts C^{α} . In our case eq. (7), which is a relation between generators of symmetries, can be written down in general notations as

$$R^{i'}_{\alpha_0} C^{\alpha_0} + R^{i'}_{\beta_0} C^{\beta_0} \Big|_{S,i=0} = 0,$$

where i' corresponds to some subset $\varphi^{i'}$ of all classical fields φ^i , and $C^{\alpha_0}, C^{\beta_0}$ form together the subset of original symmetries of the action. For example,

$$(\delta(\lambda) + \delta(\kappa)) g_{ab} \Big|_{S,i=0} \neq 0,$$

since

$$\delta(\lambda) g_{ab} = 0, \quad \delta(\kappa) g_{ab} \Big|_{S,i=0} \neq 0.$$

Therefore it is possible to use κ - and λ -symmetries in the minimal action as independent ones , they are both necessary to close the algebra. However , working out the non-minimal sector of the theory one should perform the gauge-fixing of κ -symmetry , which will simultaneously be the gauge-fixing of λ -symmetry, and there is no need in double gauge-fixing.

The slight technical difference between the quantization of heterotic string [7] and the quantization of type II , which will be performed here, is in treating reparametrization, conformal and Lorentz covariance. One can use as an intermediate variable the conformal invariant, $d = 2$ Lorentz covariant traceless metric γ_{ab} , as it was done in [9,11,13,14,7]. In this case one can use only 2 remaining bosonic symmetries, which are the symmetries under reparametrization. In such a case Φ_{min} would contain γ_{ab}, c^a , where c^a is the reparametrization ghost. In the present paper we will use an essentially equivalent but a more natural integration variable, $e^a_{\hat{a}} \equiv \{e^a_z, e^a_{\bar{z}}\}$, which is the two-dimensional zweibein. These 4 fields have symmetries of three types : reparametrization with two parameters ξ^a , conformal and Lorentz symmetry with parameters ρ and Λ respectively,

$$\begin{aligned}\delta e^a_z &= \xi^b \partial_b e^a_z - \partial_z \xi^a - (\rho - \Lambda) e^a_z , \\ \delta e^a_{\bar{z}} &= \xi^b \partial_b e^a_{\bar{z}} - \partial_{\bar{z}} \xi^a - (\rho + \Lambda) e^a_{\bar{z}} .\end{aligned}$$

The gauge algebra of both heterotic string and type II strings was presented in [13-15,2], and the existence of zweibeins essentially simplified all the expressions comparatively to the previous formulation with $P_{\pm}(\gamma_{ab})$ projectors on the left or right directions. The commutator of two fermionic κ -symmetries was investigated in more details in [2], where it was explained that it contains in the right hand side both conformal and Lorentz symmetry, and that conformal symmetry is present in any basis [15].

Therefore we will perform the quantization using 4-component zweibeins and all 4 bosonic symmetries, instead of the 2-component $d = 2$ traceless metric and 2 reparametrization symmetries.

Thus we use for S_{min} the following set of fields :

$$\begin{aligned}\Phi_{min} &= \{X^{\mu}, e^a_{\hat{a}}, \theta^1_{(\rho,0)}, \theta^2_{(0,\rho)}, \lambda^1_{\bar{z}}, \lambda^2_z, c^a, \rho, \Lambda\}, \\ \Phi_{min}^* &= \{X^*_{\mu}, e^*_{\hat{a}}, \bar{\theta}^*1_{(\rho,0)}, \bar{\theta}^*2_{(0,\rho)}, \lambda^{*1}_{\bar{z}}, \lambda^{*2}_z, c^*_a, \rho^*, \Lambda^*\},\end{aligned}\tag{8}$$

where we denoted $\theta^1 = \theta^1_{(0,0)}$, $\theta^2 = \theta^2_{(0,0)}$, and all ghosts for ghosts for $\kappa^{1,2}$ -symmetry by $\theta^1_{(p,0)}$ and $\theta^2_{(0,p)}$ respectively, $p \geq 1$; $\lambda^1_{\bar{z}}$, $\lambda^2_{\bar{z}}$, c_a, ρ, Λ are ghosts of λ -symmetry, reparametrization, conformal and Lorentz symmetry.

Now the statement that the solution of the master equation $(S,S) = 0$ in the minimal sector (8) exists, means that the fields (8) generate the gauge algebra. For the heterotic string it was shown [9,11] that the solution of the BV equation in the minimal sector is

$$S_{\min}(\Phi_{\min}, \Phi_{\min}^*) = S_{cl} + \Phi_{\min I}^* S^I(\Phi_{\min}) + \Phi_{\min I}^* \Phi_{\min J}^* S^{IJ}(\Phi_{\min}).$$

As explained in [7], the properties of this action which were essential for obtaining the quadratic gauge-fixed action in the gauge where the dependence on $\theta^1_{(p,q)}$, $\bar{\theta}^1_{\bar{z}}(p,q)$ was $\nabla_{\bar{z}} \theta^1_{(p,q)}$, $\nabla_{\bar{z}} \bar{\theta}^1_{\bar{z}}(p,q)$ respectively, were the following : i) κ -symmetry of e_z^a contains $\partial_z \theta$; ii) there is no non-closure function in the κ -symmetry algebra of the form η^{ij} , where i corresponds to e_z^a and j corresponds to e_z^a .

Before we can formulate the necessary properties of the corresponding $S_{\min}$ for the type II superstring, we must find the gauge fermion Ψ , which will define the most suitable surface $\Phi^* = \frac{\partial \Psi}{\partial \Phi}$ and take into account all symmetries which should be fixed. Also the non-minimal part of the action must be constructed by adding the auxiliary field action S_{aux} , which depends on antighosts, extraghosts, etc., and on the Lagrange multipliers to gauge conditions. The proliferation of auxiliary fields θ, π is explained in detail for the right-moving part in the heterotic string quantization [7].

To clarify the notations used in [7] and also to present the most useful variables, let us now define the antifield Φ_I^* to the field Φ^I as $(\Phi^I)^*$.

We choose the Lagrangian for non-minimal fields as follows :

$$L_{aux} = L_{\mu} + L_{\pi}^1 + L_{\bar{\pi}}^1 + (1 \rightarrow 2), \quad (9)$$

where

$$e^{-1} L_{\mu} = (\bar{c}^{\hat{a}}_a)^* \mu^{\hat{a}}_a,$$

$$L_{\pi}^1 = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} (\tilde{\theta}_{(p,q)}^1)^* \pi_{(p,q)}^1 = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} (\theta_{(p,q)}^1)^* \tilde{\pi}_{(p,q)}^1 ,$$

$$L_{\bar{\pi}}^1 = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\pi}_{\bar{z}}^{1(p,q)} (\tilde{\bar{\theta}}_{\bar{z}}^{1(p,q)})^* = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \tilde{\bar{\pi}}_{\bar{z}}^{1(p,q)} (\bar{\theta}_{\bar{z}}^{1(p,q)})^* .$$

The relation between the tweedled and untweedled variables is the following :

$$\tilde{\theta}_{(p,q)}^1 = \theta_{(p,q)}^1 + \theta_{(p+1,q+1)}^1 \eta^{z\bar{z}} ,$$

$$\tilde{\bar{\theta}}_{\bar{z}}^{1(p,q)} = \bar{\theta}_{\bar{z}}^{1(p,q)} + \bar{\theta}_{\bar{z}}^{1(p-1,q-1)} \eta^{z\bar{z}} , \quad \bar{\theta}_{\bar{z}}^{1(p,q)} = 0 \text{ for } q < 0 ,$$

$$\pi_{(p,q)}^1 = \tilde{\pi}_{(p,q)}^1 + \tilde{\pi}_{(p+1,q+1)}^1 \eta^{z\bar{z}} , \quad \tilde{\pi}_{(p,q)}^1 = 0 \text{ for } q \leq 0 ,$$

$$\bar{\pi}_{\bar{z}}^{1(p,q)} = \tilde{\bar{\pi}}_{\bar{z}}^{1(p,q)} + \tilde{\bar{\pi}}_{\bar{z}}^{1(p-1,q-1)} \eta^{z\bar{z}} , \quad \tilde{\bar{\pi}}_{\bar{z}}^{1(p,q)} = 0 \text{ for } q < 0 .$$

(10)

Thus, generically the tweedled $\tilde{\theta}, \tilde{\bar{\theta}}$ are presented as a combination of two untweedled variables and $\pi, \bar{\pi}$ are presented as a combination of two tweedled variables. However in special cases we have, according to eqs. (10), more simple relations,

$$\tilde{\bar{\theta}}^{(p,0)} = \bar{\theta}^{(p,0)} , \quad \pi_{(p,0)} = \tilde{\pi}_{(p+1,1)} , \quad \bar{\pi}^{(p,0)} = \tilde{\bar{\pi}}^{(p,0)} . \quad (11)$$

The gauge fermion is*

$$\Psi = \Psi_g + \Psi_f^1 + \Psi_f^2 ,$$

* The improvement of this gauge fixing for higher genus, i.e., the definition of the corresponding functional integral on Riemann surfaces, is under investigation.

where

$$\Psi_f = \bar{c}^{\hat{a}}_a (e^a_{\hat{a}} - \bar{e}^a_{\hat{a}}), \quad (12)$$

$$\Psi_f^1 = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\theta}_{\bar{z}}^{1(p,q)} \nabla_{\bar{z}} \tilde{\theta}_{(p,q)}^1 = \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \tilde{\bar{\theta}}_{\bar{z}}^{1(p,q)} \nabla_{\bar{z}} \theta_{(p,q)}^1, \quad ,$$

where $\bar{e}^a_{\hat{a}}$ is some background zweibein. Thus we have not used any of X^μ , $\lambda^1_{\bar{z}}$, λ^2_z , c_a , ρ , Λ in the gauge fixing. We have used 4 gauge fixing functions for bosonic symmetries including reparametrization, conformal and Lorentz symmetry. For both types of fermionic $\kappa^{1,2}$ -symmetry we have used a gauge fixing analogous to that of the heterotic string κ^1 -symmetry. There is no independent gauge fixing function for λ -symmetry in (9) and also no dependence on $\lambda^1_{\bar{z}}$, λ^2_z , which is a self-consistent way to treat λ -symmetry in this non-truncated theory.*

Thus the surface in the phase space which fixes the gauge is

$$\begin{aligned} X_\mu^* = c_a^* = \lambda^{*1\bar{z}} = \lambda^{*2z} = \mathcal{S}^* = \mathcal{A}^* = 0, \\ e^{*\hat{a}}_a = \bar{c}^{\hat{a}}_a + \delta \bar{c}^{\hat{a}}_a = -\theta^{\hat{a}}_a, \quad \delta \bar{c}^{\hat{a}}_a \equiv \frac{\partial \Psi_f(\theta)}{\partial e^a_{\hat{a}}}; \end{aligned} \quad (13)$$

$$\begin{aligned} (\bar{c}^{\hat{a}}_a)^* &= e^a_{\hat{a}} - \bar{e}^a_{\hat{a}}, \quad (\theta_{(p,0)}^1)^* = \nabla_{\bar{z}} \bar{\theta}_{\bar{z}}^{1(p,0)}, \\ (\tilde{\bar{\theta}}_{\bar{z}}^{1(p,q)})^* &= \nabla_{\bar{z}} \theta_{(p,q)}^1, \quad (\tilde{\theta}_{(p,q)}^1)^* = \nabla_{\bar{z}} \bar{\theta}_{\bar{z}}^{1(p,q)}, \\ (\bar{\theta}_{\bar{z}}^{1(p,q)})^* &= \nabla_{\bar{z}} \tilde{\theta}_{(p,q)}^1, \quad (\theta_{(p,q)}^1)^* = \nabla_{\bar{z}} \tilde{\bar{\theta}}_{\bar{z}}^{1(p,q)}. \end{aligned} \quad (14)$$

The definition of the left-moving fields θ, θ^*, π is always obtained from the right moving θ, θ^*, π by the replacements ($1 \leftrightarrow 2, z \leftrightarrow \bar{z}, p \leftrightarrow q$). For example, we get automatically from eq. (14)

$$(\theta_{o,p}^2)^* = \nabla_{\bar{z}} \bar{\theta}_{\bar{z}}^{2(o,p)}, \quad (\tilde{\bar{\theta}}_{\bar{z}}^{2(q,p)})^* = \nabla_{\bar{z}} \theta_{(q,p)}^2, \quad \text{etc.}$$

* Note that if at some stage of an exact solving BV equation in the minimal sector some additional fields y^i would appear (which seems unlikely), the only way to perform a consistent quantization would be to treat them exactly as λ^* -fields and λ -symmetries, so that $y_i^* = 0$.

In this gauge the gauge fixed action is constructed from $S_{min} + S_{aux}$ with Φ^* defined by eqs. (11)-(13). We get, using eqs. (8) (14),

$$\begin{aligned}
L_{g.f.} = & L_{min}(\Phi_{min}, \Phi_{min}^*) + b^z{}_a (\partial_z c^a - c^b \partial_b e^a{}_z \\
& + (\rho - \Lambda) e^a{}_z) + b^{\bar{z}}{}_a (\partial_{\bar{z}} c^a - c^b \partial_b e^a{}_{\bar{z}} + (\rho + \Lambda) e^a{}_{\bar{z}}) \\
& + (e^a{}_{\hat{a}} - \dot{e}^a{}_{\hat{a}}) M^{\hat{a}}{}_a + \sum_{q=0}^{\infty} \sum_{p=0}^{\infty} (\bar{\pi}^1{}_{\bar{z}}^{(p,q)} \nabla_z \theta^1{}_{(p,q)} \\
& + \bar{\theta}^1{}_{\bar{z}}^{(p,q)} \nabla_{\bar{z}} \pi^1{}_{(p,q)} + \left\{ \begin{matrix} 1 \leftrightarrow 2 \\ z \leftrightarrow \bar{z} \\ p \leftrightarrow q \end{matrix} \right\}) . \tag{15}
\end{aligned}$$

Integrating over $M^{\hat{a}}{}_a$, we get the action in a gauge where the zweibein is a background one, $e^a{}_{\hat{a}} = \dot{e}^a{}_{\hat{a}}$. The term with antighosts $b^{\hat{a}}{}_a$ for reparametrization, conformal and Lorentz symmetry can be rewritten as

$$\begin{aligned}
& b^{zz} \partial_z c_z + b^{\bar{z}\bar{z}} \partial_{\bar{z}} c_{\bar{z}} + b^{z\bar{z}} (\rho - \Lambda) + b^{\bar{z}z} (\rho + \Lambda) \\
& + b^{z\bar{z}} \partial_z c_{\bar{z}} + b^{\bar{z}z} \partial_{\bar{z}} c_z . \tag{16}
\end{aligned}$$

Redefining the conformal and Lorentz ghosts ρ and Λ ,

$$\begin{aligned}
\rho' &= \rho + \frac{1}{2} (\partial_z c_{\bar{z}} + \partial_{\bar{z}} c_z) \\
\Lambda' &= \Lambda - \frac{1}{2} (\partial_z c_{\bar{z}} - \partial_{\bar{z}} c_z) \tag{17}
\end{aligned}$$

we have this part of the action in the form

$$L_{b,c} = \bar{b} \partial c + b \bar{\partial} \bar{c} + b_{\rho} \rho' + b_{\Lambda} \Lambda' \tag{18}$$

Thus, in the background metric the gauge-fixed Lagrangian is

$$\begin{aligned}
L_{g.f.} = & L_{min}(\Phi_{min}, \Phi_{min}^*) + L_{b,c} \\
& + \sum_{q=0}^{\infty} \sum_{p=0}^{\infty} (\bar{\pi}^1{}_{\bar{z}}^{(p,q)} \nabla_z \theta^1{}_{(p,q)} + \bar{\theta}^1{}_{\bar{z}}^{(p,q)} \nabla_{\bar{z}} \pi^1{}_{(p,q)}) +
\end{aligned}$$

$$+ \sum_{P=0}^{\infty} \sum_{q=P}^{\infty} \left(\bar{\pi}_{\bar{z}}^{2(q,P)} \nabla_{\bar{z}} \theta_{(q,P)}^2 + \bar{\theta}_z^{2(q,P)} \nabla_{\bar{z}} \pi_{(q,P)}^2 \right). \quad (19)$$

Now we reached the point where it is reasonable to ask which properties of $S_{min}(\Phi = \Phi^*)$ are necessary to make the action free.

The first property which we use is that the classical Lagrangian can be presented in the form

$$L_{cl} = \partial X^\mu \bar{\partial} X^\mu + \delta \bar{\pi}_{\bar{z}}^{1(0,0)} \partial_z \theta^1 + \delta \bar{\pi}_{\bar{z}}^{2(0,0)} \partial_{\bar{z}} \theta^2. \quad (20)$$

The terms with $\partial_z \theta^1$, $\partial_{\bar{z}} \theta^2$ can be absorbed into a redefinition of $\bar{\pi}_{\bar{z}}^{1(0,0)}$ and $\bar{\pi}_{\bar{z}}^{2(0,0)}$. Any other terms in $S_{min}(\Phi, \Phi^*)$ are important only if they depend on non-zero antifields Φ^* . These are $e^{\hat{a}}_{\hat{a}} = -\theta^{\hat{a}}_{\hat{a}}$ and θ^{*1} , θ^{*2} . The four antighosts $\theta^{\hat{a}}_{\hat{a}}$ are the usual reparametrization antighosts θ^{zz} , $\theta^{\bar{z}\bar{z}}$ and conformal and Lorentz antighosts b_ρ and b_Λ . Any dependence on b_ρ and b_Λ can be absorbed into a redefinition of ρ and Λ ghosts in (18). The linear dependence on θ^{zz} , $\theta^{\bar{z}\bar{z}}$ follows from the transformation rule of $e^{\hat{a}}_{\hat{a}}$ under κ -symmetry transformations and contains $\partial_z \theta^1$, $\partial_{\bar{z}} \theta^2$ respectively. These terms again can be absorbed into a redefinition of $\bar{\pi}_{\bar{z}}^{1(0,0)}$ and $\bar{\pi}_{\bar{z}}^{2(0,0)}$. There is no bilinear term of the form $\theta \bar{\theta}$ since no corresponding non-closure function is present in the algebra. The only non-closure terms which include the zweibein are of the form $e^* X^*$, $e^* \theta^*$, see ref. [13, 14], or include the antifields for ghosts, which vanish at the surface $\Phi^* = \frac{\partial \Psi}{\partial \Phi}$. And again, as in the heterotic string theory, the dependence on $(\theta^1_{(P,0)})^*$, $(\theta^2_{(0,P)})^*$ can be arbitrary! It is always absorbed into a redefinition of $\pi^1_{(P,0)}$, $\pi^2_{(0,P)}$ respectively.

Thus we have established, using the properties of gauge algebra of the type II superstring [13-15,2], that the gauge-fixed Lagrangian in terms of shifted variables, which we denote again by π without primes, is

$$L_{g.f.} = \partial X^\mu \bar{\partial} X^\mu + \bar{\theta} \partial c + \theta \bar{\partial} \bar{c} + \bar{D}^1 \partial H^1 + H^2 \bar{\partial} \bar{D}^2. \quad (21)$$

We have used the following notations :

$$\begin{aligned} \overline{\mathcal{D}}_{\bar{z}}^1 &= \left\{ \overline{\pi}_{\bar{z}}^{1(p,q)}, \overline{\theta}_{\bar{z}}^{1(p,q)} \right\}, \quad \overline{\mathcal{D}}^2 = \left\{ \theta_{(q,p)}^2, \pi_{(q,p)}^2 \right\}, \\ \mathcal{H}_{\bar{z}}^1 &= \left\{ \theta_{(p,q)}^1, \pi_{(p,q)}^1 \right\}, \quad \mathcal{H}_{\bar{z}}^2 = \left\{ \overline{\pi}_{\bar{z}}^{2(q,p)}, \overline{\theta}_{\bar{z}}^{2(q,p)} \right\}. \end{aligned} \quad (22)$$

The same theory can be described also by eqs. (21),(22), in which $\overline{\pi}_{\bar{z}}^{1(p,q)}$ is replaced by $\tilde{\pi}_{\bar{z}}^{1(p,q)}$ and simultaneously $\theta_{(p,q)}^1$ is replaced by $\tilde{\theta}_{(p,q)}^1$ (with or without the corresponding changes in the left sector).

The generalized space-time supersymmetry, which exists for the right-moving fields in the heterotic string theory and which is described in detail in [7], can be easily used to write down the corresponding generalized space-time supersymmetry for the left-moving fields in the theory of the type II superstring. In order to do it one should make the following substitution $\overline{\mathcal{D}}_{\bar{z}}^1 \rightarrow \mathcal{H}_{\bar{z}}^2, \mathcal{H}_{\bar{z}}^1 \rightarrow \overline{\mathcal{D}}^2, z \rightarrow \bar{z}, p \rightarrow q$ in eqs. (14), (15) of ref. [7]. The action (21) has this generalized supersymmetry both in the left and in the right sectors. The counting of degrees of freedom as well as the calculation of the conformal anomaly in the left sector proceeds as in the right one. Since for the right sector it was already proved that there are effectively 4 physical degrees of freedom (complex spinors), and that the conformal anomaly vanishes [7,8], it is clear from the right-left symmetry of the Lagrangian (21) that the same properties are shared also by the left sector. This proves the consistency of the covariant quantization of the type II A,B Green-Schwarz superstring.

Finally, we would like to clarify some features of the regularization which we have used for the quantization of the heterotic string and of the type II string. In [7] we suggested to perform a summation over the branches, and then to make a summation of contributions of all branches. Alternatively, one can make a summation over the horizontals. We will demonstrate now that the regularized function is the same.*

The unregularized expression which was used in [7] for counting the (graded) number of degrees of freedom and for the calculation of the conformal anomaly was given in [7] in the form

$$\sum_{m=0}^{\infty} (-1)^m \sum_{n=0}^{\infty} (-1)^n f_n, \quad (23)$$

* I am very grateful to M.Green for an illuminating discussion of this point.

where $f_n = 1$ for the counting of the number of degrees of freedom and $f_n = 6n^2 + 6n + 1$ for the calculation of the anomaly. In eq. (22) the sum over n means the summation along the branches, see ref. [7], and the sum over m means the summation of contributions of all branches. The factor $(-1)^m(-1)^n$ appears due to the alternating statistics along each branch and between different branches. The regularized sum looks as follows :

$$\sum_{m=0}^{\infty} (-x)^m \sum_{n=0}^{\infty} (-y)^n f_n . \quad (24)$$

For the counting of degrees of freedom we have used

$$N_{x,y} = 16 \sum_{m=0}^{\infty} (-x)^m \sum_{n=0}^{\infty} (-y)^n = \frac{16}{(1+x)(1+y)} , \quad N_{x,x}^{\text{branch}} = \frac{16}{(1+x)^2} , \quad (25)$$

$$\lim_{x,y \rightarrow 1} N_{x,y} = 4 .$$

It is important to note that that this result of a formal summation of the infinite regularized sum in (25) exactly coincides also with the result of the computation in the truncated theory which is unambiguous and does not depend on the level of truncation [9].

For the anomaly we have used

$$A_{x,y} = -2 \cdot 16 \sum_{m=0}^{\infty} (-x)^m \sum_{n=0}^{\infty} (-y)^n (6n^2 + 6n + 1) =$$

$$= -2 \cdot 16 \cdot \frac{1}{1+x} \left(\frac{12}{(1+y)^3} - \frac{12}{(1+y)^2} + \frac{1}{1+y} \right) ; \quad (26)$$

$$A_{x,x}^{\text{branch}} = -2 \cdot 16 \left(\frac{12}{(1+x)^4} - \frac{12}{(1+x)^3} + \frac{1}{(1+x)^2} \right) ; \quad \lim_{x,y \rightarrow 1} A_{x,y} = -16 .$$

Let us show how the same results can be obtained by using the same original expressions (22),(23), but performing the summation over the horizontals. It is useful for this purpose to look on triangles. In the left triangle the distribution of anticommuting and commuting fields is given. In the right triangle the distribution of conformal weights along the branches, according to the action $\bar{B} \partial C$ of Ref. [7] (eq. (17)) is presented.

			1						0					
			-1		-1				0		1			
		1		1		1			0		1		2	
	-1		-1		-1		-1		0		1		2	3
1		1		1		1		1	0		1		2	3
														4

Using these triangles we can first write the non-regularized expression for the counting of degrees of freedom and for the anomaly as

$$\sum_{t=0}^{\infty} (-1)^t \sum_{s=0}^t f_s = \sum_{m=0}^{\infty} (-1)^m \sum_{n=0}^{\infty} (-1)^n f_n ,$$

where $f_s = 1$ for the counting of degrees of freedom, and $f_s = 6s^2 + 6s + 1$ for the anomaly. In the l.h.s. we have represented the summation over horizontals, in the r.h.s. — summation over branches. The result of the horizontal summation is

$$\sum_{t=0}^{\infty} (-1)^t (t+1) \quad (28)$$

for the counting of degrees of freedom, and

$$\sum_{t=0}^{\infty} (-1)^t (2t^3 + 6t^2 + 5t + 1) \quad (29)$$

for anomaly. Performing in (28), (29) the Euler regularization, as in [16,7], we get

$$\begin{aligned} N_x^{\text{hor.}} &= 16 \sum_{t=0}^{\infty} (-x)^t (t+1) = \frac{16}{(1+x)^2} = N_{x,x}^{\text{branch}} , \\ A_x^{\text{hor.}} &= -2 \cdot 16 \sum_{t=0}^{\infty} (-x)^t (2t^3 + 6t^2 + 5t + 1) = \quad (30) \\ &= -2 \cdot 16 \left(\frac{12}{(1+x)^4} - \frac{12}{(1+x)^3} + \frac{1}{(1+x)^2} \right) = A_{x,x}^{\text{branch}} . \end{aligned}$$

Thus, the regularized expression for the summation over the horizontals and over branches for $x = y$ coincide and, of course, give the same correct result in the limit $x, y \rightarrow 1$ both for the number of degrees of freedom and for the conformal anomaly. This property confirms the consistency of the regularization.

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