QUANTUM NULL (SUPER)STRINGS

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ABSTRACT

Null strings are the analog of massless particles, namely strings with zero tension or infinite slope. We present the supersymmetric extension of the null bosonic string constraint algebra and, in order to quantize covariantly, construct the corresponding BRST charge. We find that the null supersymmetric (bosonic) string is quantum consistent only in $D=10$ ($D=26$). The anomalies are generated only in the single Virasoro subalgebra associated to the tangential direction.

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The string tension T is usually taken to be an arbitrary parameter, fixed by phenomenological requirements to be (up to a numerical factor) the square of the Planck mass. The field theory limit of string theories is obtained by boosting T to infinity, thus decoupling all massive string modes from the massless ones. The latter presumably match the fundamental physical degrees of freedom observed in experiments at the low energies of particle accelerators. The opposite limiting situation, $T \simeq M_P^2 \rightarrow 0$, corresponds to interactions at an inconceivably high energy, so high that the Planck mass is effectively zero. The Regge slope $\alpha' \approx 1/T$ becomes infinite, and thus one expects that all the states in the Regge trajectories (leading and daughters) should collapse to zero mass.

One should expect also a much larger symmetry than that which is normally apparent for strings with non-zero tension. One might also speculate on two alternative mechanisms for generating dynamically a non-zero tension from the tensionless highly symmetric theory, either the generation of a scale by the renormalization group (as in QCD), or the spontaneous breakdown of the large unknown symmetry (as in Weinberg-Salam). The purpose of this letter is to analyze some basic features of the tensionless string theory, formulated directly with no string tension, rather than as the $T \rightarrow 0$ limit of usual strings. This is motivated by the analogy with point particles, for which the massless case can be studied on its own, with the bonus that all massive representations decompose as direct sums of massless ones.

In a posthumous paper, Schild formulated and studied the classical tensionless bosonic string [1, 2, 3, 4], a stringy analogue of the massless particle. The world-sheet is a null surface ($\det g=0$) and some of the reparametrization invariance can be used to fix the null gauge, in which every point of the string moves along a null geodesic. The null gauge is very similar to the conformal gauge, except that the canonical form of the metric is $\text{diag}(-1, 0)$ instead of $\text{diag}(-1, 1)$. In the null gauge, the classical equations of motion imply that $\ddot{X}^\mu = 0$. The only trace of the inherent stringiness is the fact that the velocity of each point of the string is perpendicular to the null string itself. Although there are no oscillators in the classical solution of null strings, it is too naive to think of their quantum states as mere plane waves.

A short time ago, two of us found the (local or world-sheet) supersymmetric extension of the null string constraint algebra [5], in close analogy to the original construction of the spinning string [6]. In this letter, we address the quantization of both the bosonic and supersymmetric null string theories, for simplicity closed and in a flat Minkowski space-time of dimension D . We consider the purely algebraic aspects of the theory, in a manifestly covariant formulation. Since null (super)strings constitute a general-relativistic system, all the physics is contained in the (first-class!) constraint algebra. The quantization of null

bosonic strings was considered by the authors of Ref. 2, with results in frank disagreement with ours.

The fundamental dynamical variables are the bosonic $X^\mu(\sigma, \tau)$ and $P^\mu(\sigma, \tau)$, and the fermionic $\Psi_a^\mu(\sigma, \tau)$, with equal-time graded Dirac brackets[†]

$$\begin{aligned} [P^\mu(\sigma), X^\nu(\sigma')] &= -i\eta^{\mu\nu}\delta(\sigma - \sigma') \\ [\Psi_a^\mu(\sigma), \Psi_b^\nu(\sigma')] &= \delta_{ab}\eta^{\mu\nu}\delta(\sigma - \sigma') \end{aligned} \quad (1)$$

where the space-time indices μ, ν run from 0 to D-1 (with mostly plus metric) and the local indices a, b denote the two possible directions on the world-sheet, transverse and longitudinal ($\perp$ and 1, respectively), which are normally rotated into $+$ and $-$ directions of the light-cone [8]. Not in this case!

Since we consider closed strings, P and X are periodic and each of the Ψ_a can in principle be periodic (Ramond sector, R) or antiperiodic (Neveu-Schwarz sector, N). Letting Φ denote any of the coordinates P^μ , X^μ and Ψ_a^μ , we define the Fourier modes as $\Phi(\sigma) = \frac{1}{\sqrt{2\pi}} \sum \phi_n e^{in\sigma}$, so that the non-vanishing graded Dirac brackets for the coordinate modes are

$$\begin{aligned} [p_m^\mu, x_n^\nu] &= -i\eta^{\mu\nu}\delta_{m+n} \\ [\psi_m^{a\mu}, \psi_n^{b\nu}] &= \delta_{ab}\eta^{\mu\nu}\delta_{m+n} \end{aligned} \quad (2)$$

where the mode indices of the $\psi_n^{a\mu}$ are integral or half-odd depending on the sector. In the Ramond sector(s), the zero-modes $\psi_0^{a\mu}$ are realized as Dirac Γ -matrices.

The first-class constraints for the null spinning string have been obtained [5] using the “square-root” method [9], they are

$$\begin{aligned} \mathcal{H}_\perp &= P^2 \\ \mathcal{H}_1 &= P \cdot X' + \frac{i}{2} \Psi_a \cdot \Psi'_a \\ \mathcal{S}_a &= P \cdot \Psi_a \end{aligned} \quad (3)$$

where repeated indices are summed over and the constraints for the bosonic null string [1, 2, 3, 4] are recuperated by setting $\Psi_a^\mu = 0$. Letting G denote any of the constraints $\mathcal{H}_a$, $\mathcal{S}_a$, we define Fourier modes by $G(\sigma) = \frac{1}{2\pi} \sum G_n e^{in\sigma}$, so that the algebra can be nicely written as

$$\begin{aligned} [\mathcal{H}_m^a, \mathcal{H}_n^1] &= (n - m)\mathcal{H}_{m+n}^a \\ [\mathcal{S}_m^a, \mathcal{H}_n^1] &= (\frac{n}{2} - m)\mathcal{S}_{m+n}^a \\ [\mathcal{S}_m^a, \mathcal{S}_n^b] &= \delta^{ab}\mathcal{H}_{m+n}^\perp \end{aligned} \quad (4)$$

[†] Note in passing that the equations of motion for the world-sheet spinor fields in Schild's null gauge are $\dot{\Psi}_a^\mu = 0$ [7].

with all other brackets vanishing. Again, the indices of the fermionic constraints S_m^a are integers or half-odd numbers depending on the sectors.

Note that it is impossible to define mutually orthogonal linear combinations of $\mathcal{H}_\perp$ and $\mathcal{H}_1$ so that we cannot think of the null (super)string as consisting of both left- and right-movers. We have, nevertheless, found a variation of Schild's null gauge in which all the constraints and variables are of a given (gauge-dependent but fixed) chirality [7]. The only difference in the formalism is the definition of Fourier modes, in terms of $e^{in(\sigma+\tau)}$ instead of $e^{in\sigma}$. The algebraic manipulations below are identical for both gauges, and the critical dimensions we find are of course gauge-independent.

An anomaly is only possible in the first line of (4) with $a = 1$, since the only ordering problems arise in $\mathcal{H}_0^1$, as can be seen from the explicit expressions for the constraint modes:

$$\begin{aligned}\mathcal{H}_n^\perp &= p_{n-k} \cdot p_k \\ \mathcal{H}_n^1 &= i k p_{n-k} \cdot x_k - \frac{k}{2} \psi_{n-k}^a \cdot \psi_k^a \\ S_n^a &= p_{n-k} \cdot \psi_k^a\end{aligned}\tag{5}$$

Following the standard Dirac-Batalin-Fradkin-Vilkovisky procedure for quantizing constrained systems [10], we introduce anticommuting ghosts $\eta^a(\sigma) = \sum \eta_n^a e^{in\sigma}$ associated to the bosonic constraints $\mathcal{H}_a$ and their conjugate momenta $\mathcal{P}_a(\sigma) = \sum \mathcal{P}_n^a e^{in\sigma}$, and similarly introduce ghosts $\tilde{\eta}^a(\sigma)$ and ghost momenta $\tilde{\mathcal{P}}_a(\sigma)$ for the fermionic constraints S_a . The bosonic ghosts η_n^a , $\mathcal{P}_n^a$ anticommute and are integrally moded, whereas the fermionic $\tilde{\eta}_n^a$, $\tilde{\mathcal{P}}_n^a$ commute and are half-odd or integrally moded depending, again, on the sector. The fundamental non-vanishing (graded) brackets are

$$\begin{aligned}[\mathcal{P}_n^a, \eta_m^b] &= -\delta^{ab} \delta_{m+n} \\ [\tilde{\mathcal{P}}_n^a, \tilde{\eta}_m^b] &= -\delta^{ab} \delta_{m+n}\end{aligned}\tag{6}$$

Note that $\tilde{\mathcal{P}}_n^a$ are antihermitian, all other variables (as well as the constraints, of course) hermitian.

One can immediately write down the BRST charge (compare with the usual string [11]):

$$\begin{aligned}\Omega &= \eta_k^a \mathcal{H}_{-k}^a + \tilde{\eta}_k^a S_{-k}^a + (m-n) \eta_m^\perp \eta_n^1 \mathcal{P}_{-m-n}^\perp + \frac{1}{2} (m-n) \eta_m^1 \eta_n^1 \mathcal{P}_{-m-n}^1 \\ &\quad + (m - \frac{n}{2}) \tilde{\eta}_m^a \eta_n^1 \tilde{\mathcal{P}}_{-m-n}^a - \frac{1}{2} \tilde{\eta}_m^a \tilde{\eta}_n^a \mathcal{P}_{-m-n}^\perp + \alpha \eta_0^1\end{aligned}\tag{7}$$

where all repeated indices are summed over, all products are assumed normal-ordered (positive indices to the right) and α is an unknown constant. We have already used the fact that only $\mathcal{H}_0^1$ presents ordering problems.

The single Virasoro subalgebra for the full system is generated by operators which are a sum of orthogonal terms:

$$L_m = [\Omega, \mathcal{P}_m^1] = L_m^{\text{matter}} + L_m^\perp + L_m^1 + L_m^{\tilde{1}} + L_m^{\tilde{1}} - \alpha \delta_m \quad (8)$$

where

$$\begin{aligned} L_m^{\text{matter}} &= -\mathcal{H}_m^1 = L_m^x + L_m^{\psi_\perp} + L_m^{\psi_1} \\ L_m^x &= -ikp_{m-k} \cdot x_k \\ L_m^{\psi_a} &= \frac{k}{2} \psi_{m-k}^a \cdot \psi_k^a \\ L_m^a &= (k+m) \eta_k^a \mathcal{P}_{m-k}^a \\ L_m^{\tilde{a}} &= (k+\frac{m}{2}) \tilde{\eta}_k^a \tilde{\mathcal{P}}_{m-k}^a \end{aligned} \quad (9)$$

with no sum over a in the last three expressions. Each set of the above operators satisfy an anomalous Virasoro algebra,

$$[L_m^{\dots}, L_n^{\dots}] = (m-n) L_{m+n}^{\dots} + A_m^{\dots} \delta_{m+n} \quad (10)$$

where $\dots$ stands for x, ψ_a, a and $\tilde{a}$ ($a = \perp, 1$).

Define the vacuum $|0\rangle$ to be annihilated by all positive-indexed dynamical modes, *i.e.*,

$$x_m^\mu |0\rangle = p_m^\mu |0\rangle = \psi_m^{a\mu} |0\rangle = \eta_m^a |0\rangle = \mathcal{P}_m^a |0\rangle = \tilde{\eta}_m^a |0\rangle = \tilde{\mathcal{P}}_m^a |0\rangle = 0 \quad (m > 0) \quad (11)$$

This suffices for a consistent quantum implementation of the constraints; imposing all of them yields a trivial theory or, at least, one with no critical dimensions [2]. Defining the operators

$$\begin{aligned} M_m &= [\Omega, \mathcal{P}_m^\perp] = -\mathcal{H}_m^\perp + (m+k) \eta_k^1 \mathcal{P}_{m-k}^\perp \\ F_m^a &= [\Omega, \tilde{\mathcal{P}}_m^a] = S_m^a - (m+\frac{k}{2}) \eta_k^1 \tilde{\mathcal{P}}_{m-k}^a - \tilde{\eta}_k^a \mathcal{P}_{m-k}^\perp \end{aligned} \quad (12)$$

the vacuum state is such that

$$L_0 |0\rangle = -\alpha |0\rangle$$

and

$$L_m |0\rangle = M_m |0\rangle = F_m^a |0\rangle = 0 \quad m > 0 \quad (13)$$

and physical states are constructed by acting with negative-indexed L_m , M_m and F_m^a on the vacuum. More succinctly, physical states are the cohomology classes of Ω , *i.e.*, $\Omega|\text{phys}\rangle = 0$, $|\text{phys}\rangle \approx |\text{phys}\rangle + \Omega|\text{anything}\rangle$. For completeness, note that

$$\begin{aligned} [M_m, M_n] &= [M_m, F_m^a] = 0 \\ [M_m, L_n] &= (m-n) M_{m+n} \\ [F_m^a, F_n^b] &= \delta^{ab} M_{m+n} \\ [L_m, F_n^a] &= (\frac{m}{2} - n) F_{m+n}^a \\ [L_m, L_n] &= (m-n) L_{m+n} + A_m \delta_{m+n} \end{aligned} \quad (14)$$

To compute the anomaly $A_m = \sum A_m^{\dots}$, we use the well-known trick of computing $\langle 0|[L_m, L_{-m}]|0 \rangle$ in two different ways to obtain

$$\langle 0|[L_m, L_{-m}]|0 \rangle = A_m - 2m\alpha \quad (15)$$

It is actually enough to evaluate this for $m = 1, 2$, since the Jacobi identities require that the anomaly have the form $A_m = c_3 m^3 + c_1 m$.

We find that

$$\begin{aligned} A_m^x &= \frac{D}{6}(m^3 - m) \\ A_m^\perp &= A_m^1 = \frac{1}{6}(-13m^3 + m) \end{aligned} \quad (16)$$

In the null bosonic string theory, these are all the contributions to the anomaly. Consistency requires then the unique solution $D = 26$, $\alpha = -1$. So the null bosonic string does have a critical dimension, contrary to previous claims [2]. It is remarkable that our system possesses **two** spin-2 (b-c) systems, as in the usual string from left- and right-movers, and yet the matter contribution is **different** in the null case (twice the usual), thus yielding $D = 26$ and not $D = 52$ as might have appeared from mere counting of the ghost contributions.

For our supersymmetric case, it is of course necessary to distinguish between the R and N sectors:

$$\begin{aligned} A_m^{\tilde{1}R} &= A_m^{\tilde{1}R} = \frac{1}{12}(11m^3 - 2m) \\ A_m^{\tilde{1}N} &= A_m^{\tilde{1}N} = \frac{1}{12}(11m^3 + m) \\ A_m^{\psi_\perp R} &= A_m^{\psi_\perp R} = \frac{D}{24}(m^3 + 2m) \\ A_m^{\psi_\perp N} &= A_m^{\psi_\perp N} = \frac{D}{24}(m^3 - m) \end{aligned} \quad (17)$$

Solving now Eq. (15) for the different sectors (note that the bosonic contribution is always the same), we find that the unique solutions to the vanishing of the anomaly (or, equivalently, nilpotency of the BRST charge Ω), are

$$\begin{aligned} \text{RR} : \quad & D = 10, \quad \alpha = 0 \\ \text{NN} : \quad & D = 10, \quad \alpha = -1 \\ \left. \begin{array}{l} \text{RN} \\ \text{NR} \end{array} \right\} : \quad & D = 10, \quad \alpha = -1/2 \end{aligned} \quad (18)$$

Clearly, a stringy miracle is at work, just as for the bosonic case. In this case, there are twice as many fermions and twice as many ghosts, all of the same "chirality". Note that the $(\text{mass})^2$ operator is **not** L_0 , so the ground state in the NN, RN and NR sectors (or, for that matter, in the bosonic case) may not be interpreted off-hand as a tachyon. Nevertheless, some analogue of the GSO projection may be imposed to halve the spectrum

and thus agree more closely with the $T \rightarrow 0$ limit of the usual string [7]. These matters can presumably be settled by a careful comparison of the null spinning string presented here and the null space-time supersymmetric string [12].

In the convenient Schild's null gauge, the null (super)string has only **one** Virasoro algebra, associated to the longitudinal constraint. This is consistent with the limit $T \rightarrow 0$ of the usual string, when the two (left and right) Virasoro algebras generated by the Fourier modes of $\mathcal{H}_\perp \pm 2T\mathcal{H}_1 = (P^\mu \pm TX'^\mu)^2$ (or their supersymmetric extensions) coincide ($L_m - \bar{L}_m = 0$ in this limit, $\forall m$). This limit is highly singular, however, and it is not clear at all to us that it coincides exactly with our tensionless string. In particular, note that the null spinning string has no anomalies in the fermionic (anti)commutators, in Schild's null gauge.

Naively extrapolating our construction of the fermionic generators to extended supersymmetries, understood as the addition of more fermionic constraints, it is amusing to note that the critical dimension corresponding to $N = 2$ (S_a^i , $a = \perp, 1$, $i = 1, 2$) is $D=2$, again in surprising agreement with the usual string. Our results seem to point to a new possibility for constructing physically interesting conformal models which are generated by only **one** chiral algebra ($\approx \mathcal{H}_1$), which nevertheless mixes inextricably with the $\mathcal{H}_\perp$, associated with the masses, and, in the spinning case, with the fermionic S_a . The analogy with point (super)particles is appealing, in that one would like to conjecture that all "tensionfull" (super)string theories can be understood as superpositions of null (super)strings, in strict analogy to the massive and massless representations of the Poincaré group. It remains to be seen whether null (super)strings provide more convenient building blocks for phenomenologically acceptable string theories.

These and other issues, such as the Lagrangian formulation of the null supersymmetric string, its quantization in the light-cone gauge, its physical spectrum, and its interpretation as a chiral conformal field theory, as well as the open string case, will be addressed in a longer publication [7].

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References

- [1] A. Schild, *Phys. Rev.* **D16** (1977) 1722.
- [2] F. Lizzi, B. Rai, G. Sparano and A. Srivastava, *Phys. Lett.* **182B** (1986) 326.
- [3] A. Karlhede and U. Lindström, *Class. Quantum Grav.* **3** (1986) L73.
- [4] R. Amorim and J. Barcelos-Neto, *Z. Phys.* **C38** (1988) 643.
- [5] J. Gamboa and M. Ruiz-Altaba, *La Cuerda de Schild Supersimétrica*, in *Proceedings, Sexto Simposio Chileno de Física*, ed. M. Massman *et al.*, Socialismo o Barbarie (1988) Santiago de Chile, p. 258.
- [6] P. Ramond, *Phys. Rev.* **D3** (1971) 2415; A. Neveu and J.H. Schwarz, *Nucl. Phys.* **B31** (1971) 86; *Phys. Rev.* **D4** (1971) 1109.
- [7] J. Gamboa, C. Ramírez and M. Ruiz-Altaba, *Null Spinning Strings*, CERN preprint TH-5349/89 (March 1989).
- [8] For an in-depth review, see M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press (1987) New York.
- [9] J. Gamboa and J. Zanelli, *Class. Quantum Grav.* **5** (1988) 141; *Ann. Phys. (NY)*, in press.
- [10] M. Henneaux, *Phys. Reports* **126** (1985) 1. It includes very complete references.
- [11] M. Kato and O.K. Ogawa, *Nucl. Phys.* **B212** (1983) 443; S. Hwang, *Phys. Rev.* **D28** (1983) 2614.
- [12] J. Barcelos-Neto and M. Ruiz-Altaba, *Superstrings with Zero Tension*, preprint CERN-TH.5294/89 (February 1989).