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Sea Quarks and the Hadron Spectrum

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Abstract

We present lattice Monte Carlo results demonstrating the effects of dynamical quarks on the hadron spectrum. We show that the sea quarks do not influence the nucleon and the rho in an identical manner, and the ratio M_N/M_ρ decreases as the sea quark mass is decreased. We also find a large value for the sea quark contribution to the matrix element $\langle N | \bar{u}u + \bar{d}d | N \rangle$.

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During the past few years, many lattice QCD results concerning hadronic properties have been obtained using numerical simulations. Most of these simulations have ignored the contributions of the quark sea by using the “quenched” or valence approximation. The most optimistic interpretation of this approximation is that for certain observables the quark sea serves mainly to renormalize the gauge coupling. Attempts to go beyond this approximation have observed some features characteristic of dynamical fermions, such as the flattening of the static $\bar{q}q$ potential. But as far as the hadron spectrum itself is concerned, no significant signature of dynamical quarks has been found beyond a renormalization of the gauge coupling [1] [2]. This is worrying, since several quantities—nucleon to rho mass ratio, pion-nucleon sigma-term, vector meson decay constant—have values in the valence approximation that deviate substantially from experiment. Part of the problem is certainly that calculations have not yet reached the scaling region. Also, many of the calculations have been done in small volumes, and all use relatively large quark masses. Another contributing factor could be the use of approximate algorithms for simulating the quark determinant.

In order to search for indications that full QCD will resolve the problems of the quenched approximation, we have gone beyond previous spectrum calculations in two ways. First, we have used an exact algorithm to simulate the quark determinant. This is probably not a major factor, but it relieves us from having to worry about one type of systematic error. Second, and more important, we have specifically looked for indications that as the dynamical quark mass is reduced from infinity (the valence approximation), the ratio M_N/M_ρ and the sigma-term vary in the correct direction to become consistent with experiment.

We incorporate the effect of dynamical quarks into our simulation of the functional integral using the Hybrid Monte Carlo algorithm [3]. The details of our methods and checks have been published separately [4], and we follow the same notation here.

Our results [5] are obtained with two flavours of $r = 1$ dynamical Wilson quarks at $\beta = 5.3$ and for $\kappa \equiv \kappa_{sea} = 0.162, 0.165$ and 0.167 on an 8^4 lattice. Boundary conditions are periodic in all directions for the gauge field, antiperiodic in space and periodic in time for the quarks. Each new run begins from a thermalised configuration at the next lowest value of κ (a pure gauge configuration in the case of $\kappa = 0.162$). Table 1 lists the parameters we use (separation between configurations and thermalisation are quoted in units of molecular dynamics trajectories).

To study the hadron spectrum, we calculate valence quark propagators on $8^3 \times 24$ lattices obtained by periodically extending the background gauge field configurations in the time direction. The inversions are carried out to higher accuracy than in the configuration update, and the convergence criteria R for both cases are listed in table 1. Since we use local hadron operators, rather than extended sources, we have significant contamination from

excited states at short times. We find that the $8^3 \times 24$ lattices are long enough to observe the asymptotic fall-off of the two-point correlators. (A comparison of nucleon correlators on $8^3 \times 16$ and $8^3 \times 24$ lattices is given in ref.[5].) We also observe that asymptotic behaviour sets in earlier for lighter quark masses, indicating a larger mass difference between the lowest and the first excited state.

κ	0.162	0.165	0.167
Trajectory length	0.8	0.75	0.75
Step size	0.04	0.0375	0.03
Metropolis acceptance	0.62(1)	0.47(1)	0.31(1)
$\langle \exp(-\delta H) \rangle$	1.03(4)	0.95(5)	0.84(7)
Thermalisation	150	180	190
No. of configurations	30	30	60
Configuration separation	50	60	40
R_{update}	10^{-10}	10^{-11}	10^{-12}
R_{hadron}	10^{-12}	10^{-12}	10^{-14}

Table 1: Statistical parameters

Dynamical and valence quark masses, characterised by κ_{sea} and κ_{val} respectively, are independent parameters in the lattice calculation. We make use of this freedom by extending the hadron spectrum calculation to $\kappa_{sea} \neq \kappa_{val}$ at the lightest ($\kappa = 0.167$) and the heaviest ($\kappa = 0.162$) sea quark masses. This allows us to separately study the variation of observables with valence and sea quark masses. Results for $\kappa_{sea} > \kappa_{val}$ are useful for charmed and bottom hadron systems. It is the results with $\kappa_{sea} = \kappa_{val}$ that must be extrapolated towards the chiral limit to obtain numbers relevant for hadrons made up of light quarks.

Our measured values for π , ρ , N and Δ masses are given in table 2. All the results come from stable double exponential fits to the correlators in the range $3 \leq t \leq 12$. The quoted errors are statistical only. The non-zero momentum contribution to the baryon masses due to anti-periodic spatial boundary conditions has been subtracted off using the lattice dispersion relation :

$$M^2 = E^2 - \sin^2 p_x - \sin^2 p_y - \sin^2 p_z \quad . \quad (1)$$

From the values for M_π , we estimate the critical hopping parameter to be $\kappa_c = 0.1686(3)$. Linear extrapolation of M_ρ and M_N values for the two lighter quark masses to κ_c gives

κ_{val}	κ_{sea}	π	ρ	N	Δ	M_π/M_ρ	M_N/M_ρ
0.167	0.162	0.89(1)	1.03(1)	1.75(6)	1.80(8)	0.86	1.70
0.165	0.162	0.96(1)	1.08(1)	1.80(6)	1.84(7)	0.89	1.67
0.162	0.162	1.05(1)	1.15(1)	1.87(5)	1.89(6)	0.91	1.63
0.165	0.165	0.79(1)	0.93(1)	1.52(7)	1.61(8)	0.85	1.63
0.167	0.167	0.49(2)	0.68(2)	1.12(7)	1.22(7)	0.72	1.65
0.165	0.167	0.58(1)	0.74(2)	1.19(7)	1.26(7)	0.78	1.61
0.162	0.167	0.71(1)	0.83(1)	1.29(7)	1.34(7)	0.86	1.55

Table 2: Hadron masses and mass ratios

the lattice scale to be $a_\rho^{-1} = 1.6$ GeV and $a_N^{-1} = 1.2$ GeV; our combined estimate for the lattice spacing is $a^{-1} = 1.4(2)$ GeV.

We display our results for mass ratios in fig. 1. The figure also shows the result for infinite quark mass, and the experimental data point. It is important to note that the curve for M_N/M_ρ for two dynamical quark flavours *need not* pass through the experimental point. The strange quark loops, which are being left out in the two flavour theory, can have a substantial effect. Unfortunately, we do not know how to reliably estimate either the sign or magnitude of this effect. The best we can do is to estimate the change in M_N/M_ρ from its physical value of 1.22 to the case when m_s is set to zero. By extending the chiral quark model calculation of Nelson [6], we find that the ratio drops to about 1.0. Thus 20% changes in M_N/M_ρ are quite possible when simulating a theory different from the real world.

All of our results correspond to fairly heavy quarks. Using the experimental vector/pseudoscalar mass splittings we estimate that $\kappa = 0.167$ corresponds roughly to the strange quark mass. For these heavy quark masses, and for $\kappa_{val} = \kappa_{sea}$, we find that the ratio M_N/M_ρ is greater than 1.5, having increased from its infinite quark mass value. This result is consistent with previous studies. There is no evidence of the curvature needed to reliably extrapolate close to the experimental point. We know, however, that there are large systematic errors in our results. The most important is due to our use of strong gauge coupling: the nucleon correlation length never reaches one lattice spacing. In addition, the lattice volume is small, and the statistical errors may have been underestimated due to correlations between the lattices at the largest value of κ_{sea} .

There is one feature of the results which we find encouraging. The three points at $M_\pi/M_\rho \approx 0.85$ show that, at a fixed value of M_π/M_ρ , lowering the the sea quark mass decreases M_N/M_ρ . This result is more significant than the statistical errors would lead one to believe because the points are highly correlated. The quenched theory ($m_{sea} = \infty$) has

consistently produced too large a value for M_N/M_ρ compared to experiment. Our result indicates that adding sea quarks reduces this discrepancy, at least for the same coupling and lattice size.

We can also use the results of table 2 to estimate the connected matrix elements of $(\bar{u}u + \bar{d}d)$; for example

$$\frac{\partial M_N}{\partial m} = \langle N | \bar{u}u + \bar{d}d | N \rangle . \quad (2)$$

We can take the derivatives numerically using $m = \ln[1 + (1/\kappa - 1/\kappa_c)/2]$. In fact, by taking derivatives separately with respect to m_{sea} and m_{val} , we can separate out the valence and sea quark parts of these matrix elements. We find

$$\begin{aligned} \frac{\partial M_\rho}{\partial m_{sea}} &= (2.5 \pm 1 \pm ?) \frac{\partial M_\rho}{\partial m_{val}} , \\ \frac{\partial M_N}{\partial m_{sea}} &= (4 \pm 1 \pm ?) \frac{\partial M_N}{\partial m_{val}} , \end{aligned} \quad (3)$$

where the quoted error is from taking numerical derivatives and the question mark reemphasises unknown systematic errors. This result indicates that the sea quark contribution to the scalar density matrix element is larger than that of the valence quarks, and has a larger impact on the nucleon mass than on the rho mass.

To explain the significance of this result, we must provide a brief discussion of the status of the $\pi - N$ sigma-term. First, we define the matrix elements

$$A = \langle p | \bar{u}u | p \rangle_{val} , \quad B = \langle p | \bar{d}d | p \rangle_{val} , \quad C = \langle p | \bar{u}u | p \rangle_{sea} = \langle p | \bar{d}d | p \rangle_{sea} . \quad (4)$$

Assuming exact isospin symmetry ($m_u = m_d = m$), the sigma-term is

$$\sigma = \sigma_{val} + \sigma_{sea} = m(A + B) + 2mC . \quad (5)$$

Experimental data on $\pi - N$ scattering, together with a somewhat involved theoretical extrapolation, give $\sigma = 50 - 60$ MeV [7]. This is to be compared with the standard first order flavour $SU(3)$ breaking analysis of the octet baryon masses which gives $\sigma_{val} = 26$ MeV.

There are two options to explain the experimental results. The first is simply that $\sigma_{sea} \sim \sigma_{val}$. Such a large sea contribution is in conflict with a naive interpretation of the non-relativistic quark model. Moreover, if one assumes $SU(3)$ symmetry, it implies a significant dependence of the nucleon mass on the strange quark mass [8] [9].

The alternative is that higher order $SU(3)$ breaking terms, i.e., terms of $\mathcal{O}(m_s^2)$, are important. The first order analysis then fails, and it is possible that σ_{val} is close to the experimental result with σ_{sea} much smaller. Since $SU(3)$ breaking is large, the dependence of M_N on m_s cannot be calculated from experimental data, and could be weak.

A difficulty with this second approach is that one must maintain the successful prediction of the first order analysis, namely the Gell-Mann–Okubo (GMO) mass formula. Violation of the GMO relation is caused by an effective operator transforming as the 27-plet of flavour $SU(3)$ [10]. The effective interaction between two valence quarks in a flavour antisymmetric state, no matter what its mass dependence, has no contribution from this operator. In the language of the quark model, a flavour antisymmetric state of a diquark is also its $S = 0$ state (for the ground state $L = 0$ baryons). Therefore, if the spin-0 part of the effective interaction between two valence quarks dominates over all other non-linear interactions, then the GMO relation will be well satisfied. This turns out to be possible in a number of models. Successful non-linear models include the constituent quark model [11], the chiral bag model [8], the chiral quark model [6], the Skyrme model [12], and an instanton model [13]. Some evidence for non-linear dependence of the nucleon scalar density matrix elements on the quark masses has been found in quenched lattice simulations [14].

Our numerical results clearly provide support for a large value of $\sigma_{sea}/\sigma_{val}$, and are thus consistent, qualitatively, with the first option. The large value arises mainly from the change of the scale of the theory (i.e. Λ_{QCD}), when the sea quark mass is varied holding the bare gauge coupling fixed. We would like to stress, however, that sea quark effects on different hadrons are unequal. Thus one cannot account for the sea quarks by a trivial global rescaling. This point, and its relation to the scale anomaly of QCD, has been further developed in a separate paper [15].

As a check on our results we calculate the scalar density matrix elements directly from three-point functions. For the valence part of the matrix elements we use the method of refs. [16] [14]. In case of baryons, we actually measure $\partial E/\partial m$ because of our anti-periodic spatial boundary conditions. Multiplication by E/M then converts the numbers to $\partial M/\partial m$. All our results are summarised in table 3. They are consistent with those obtained by taking numerical derivatives of hadron masses. In fact, the three-point function method gives a much better determination of the valence quark matrix elements.

For the baryon octet we can calculate A and B separately, and construct the two independent combinations

$$F = A/2 \quad , \quad D = (A - 2B)/2 \quad . \quad (6)$$

Both are included in table 3. The values of D/F are similar to those obtained in the quenched approximation.

We have also attempted to measure C directly, using pseudofermions to calculate the disconnected fermion loop (cf. fig. 1b of ref.[14]). The measured quantity is the ratio of the three-point to two-point correlation functions, e.g.

$$R_u(t) = \frac{\langle \pi^+(t) | \sum \bar{u}u | \pi^+(0) \rangle - \langle \pi^+(t) | \pi^+(0) \rangle \langle \sum \bar{u}u \rangle}{\langle \pi^+(t) | \pi^+(0) \rangle} \quad . \quad (7)$$

κ_{val}	κ_{sea}	F	D	π^+	ρ^+	Δ^{++}
0.167	0.162	0.69(3)	-0.080(5)	0.92(2)	0.62(2)	1.65(9)
0.165	0.162	0.69(2)	-0.067(4)	0.87(2)	0.63(1)	1.73(6)
0.162	0.162	0.68(2)	-0.051(3)	0.82(1)	0.63(1)	1.81(5)
0.165	0.165	0.77(3)	-0.13(1)	1.06(4)	0.66(2)	1.93(14)
0.167	0.167	0.53(14)	-0.24(2)	1.20(9)	0.5(1)	1.5(3)
0.165	0.167	0.56(10)	-0.19(1)	1.04(6)	0.57(9)	1.4(2)
0.162	0.167	0.61(6)	-0.15(1)	0.93(3)	0.62(6)	1.6(2)

Table 3: Valence quark contribution to scalar density matrix elements. The last three columns list $\langle \bar{u}u \rangle_{val}$ for the corresponding particle.

Here $\bar{u}$ and u are contracted with each other and the summation represents a zero 4-momentum operator. With 500 pseudofermions on each configuration, we see a signal for the π , consistent with the value obtained from table 2 by taking a numerical derivative. We display the data in fig. 2 for $\kappa_{sea} = \kappa_{val} = 0.165$. Unfortunately, the signal for other channels is too poor to extract a number. Since this is a calculation more akin to measuring a glueball than a hadron correlator, we consider it significant that we can see any signal at all. Nevertheless, for the present, the numerical derivative method is superior to using three-point functions when measuring sea quark matrix elements.

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Figure Captions

- [1] Monte Carlo results depicting the effect of dynamical quarks on the nucleon to rho mass ratio. Error bars are omitted, but the hadron masses are listed in table 2.
- [2] The sea quark contribution to the scalar density matrix element of the pion for $\kappa_{sea} = \kappa_{val} = 0.165$. The slope of R_u versus t is the desired matrix element.

HADRON MASS RATIOS

Fig. 1 :

Fig. 2 :