CERN-TH.5388/89
DTP/89/16

Double Higgs boson production in Z decay

E. W. N. Glover

CERN, CH-1211 Geneva 23, Switzerland
and
Department of Physics, University of Durham, England
and

A. D. Martin

Department of Physics, University of Durham, England

ABSTRACT

We calculate the rate for the double Higgs decay of the Z boson, $Z \rightarrow HHf\bar{f}$, and find that it is within reach of the present Z factory experiments if $M_H \lesssim 10$ GeV. We show the sensitivity of the rate to the $ZZHH$ and H^3 couplings. We discuss briefly other modes of double Higgs production and also other Higgs decay modes of the Z boson.

CERN-TH.5388/89
DTP/89/16
May 1989

In the minimal standard model, electroweak symmetry breaking is achieved by a neutral Higgs boson, H with gauge boson and self couplings given by,

$$\mathcal{L} = \left(\frac{g^2}{4} W^+ W^- + \frac{g_Z^2}{8} Z Z \right) (H^2 + 2Hv) - \frac{M_H^2}{2v} H^3 - \frac{M_H^2}{8v^2} H^4, \quad (1)$$

with couplings $g = e/\sin\theta_W$, $g_Z = g/\cos\theta_W$ and where the Higgs vacuum expectation value $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV. There are very few constraints on the possible mass, M_H , of a standard model Higgs boson. Studies of K and B meson decays appear to rule out masses below $2m_\tau$. In principle the absence of $\Upsilon \rightarrow H\gamma$ decays could exclude much of the range below M_Υ , but theoretical uncertainties from QCD and relativistic corrections preclude a definitive constraint at present. At one time it was believed that there was a theoretical lower limit $M_H \gtrsim 7$ GeV arising from the requirement that the symmetry breaking vacuum be the absolute minimum of the effective Higgs potential [1]. However with the possible existence of a heavy top quark, $m_t \sim M_W$, this limit vanishes altogether. In considering the Higgs decay modes of the Z we therefore explore the entire mass range $M_H < M_Z$.

In the near future, experiments at the LEP and SLC Z factories should be able to detect whether or not the Higgs boson exists with a mass up to $M_H \sim 50$ GeV via the process [2],

$$e^+e^- \rightarrow Z \rightarrow HZ^* \rightarrow H\mu^+\mu^-, \quad (2)$$

which is mediated by the HZZ coupling of eq. (1). The rate for this process is compared with that for $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$ as a function of M_H in Fig. 1. To probe the H^3 and $HHZZ$ couplings it has been suggested [3,4,5] that it may be possible to identify double Higgs production at very high luminosity e^+e^- [3,5] and pp [4] supercolliders. Here we wish to point out that the rate for the double Higgs process

$$e^+e^- \rightarrow Z \rightarrow HHZ^* \rightarrow HHf\bar{f}, \quad (3)$$

is sufficient for it to be observed at the present Z factories if M_H is less than about 10 GeV.

The Feynman diagrams for $Z \rightarrow HHf\bar{f}$ are shown in Fig. 2 and the amplitude for a Z boson of momentum p_Z and polarisation ϵ_μ decaying into two Higgs of momenta p_1, p_2 and a massless fermion pair of momenta k_1, k_2 and helicities λ is, up to an overall phase,

$$\mathcal{M} = g_Z^3 c_\lambda^f D_Z \epsilon_\mu \bar{u}_\lambda(k_2) \gamma_\nu u_\lambda(k_1) (D_1 p_1^\mu p_2^\nu + D_2 p_2^\mu p_1^\nu + D_3 g^{\mu\nu}), \quad (4)$$

where c_λ^f is the $Zf\bar{f}$ coupling and,

$$D_Z = \frac{1}{M_{f\bar{f}}^2 - M_Z^2 + i\Gamma_Z M_Z}, \quad (5)$$

$$D_i = \frac{1}{M_H^2 - 2p_Z \cdot p_i + i\Gamma_Z M_Z} \quad \text{for } i = 1, 2, \quad (6)$$

$$D_3 = \frac{1}{2} + \frac{3M_H^2}{2((p_1 + p_2)^2 - M_H^2)} + M_Z^2 (D_1 + D_2). \quad (7)$$

The first two terms of eq. (7) originate from the Feynman diagrams shown in Fig. 2a, 2b respectively and are proportional to the $ZZHH$ and H^3 couplings. The remaining terms in eqs. (7) and (4) arise from the double Higgs Bremsstrahlung of Fig. 2c and are proportional to the square of the ZZH coupling. From eq. (4) we find,

$$\Gamma(Z \rightarrow HHf\bar{f}) = \frac{g_Z^6}{6M_Z} (c_L^f + c_R^f) \int |D_Z|^2 \left(\sum_{i,j=1}^3 D_i D_j^* T_{ij} \right) dLips(4). \quad (8)$$

$dLips(4)$ is the four particle phase space while c_L^f and c_R^f are the left- and right-couplings of the fermions, which are related to the standard $c_V^f + c_A^f \gamma^5$ notation for the $Zf\bar{f}$ couplings by the relation,

$$c_L^2 + c_R^2 = 2 (c_V^2 + c_A^2). \quad (9)$$

The kinematic quantities T_{ij} are symmetric under the interchange of i and j ,

$$T_{ij} = T_{ji}, \quad (10)$$

and are defined by the following relations;

$$\begin{aligned} T_{11} &= \left(M_H^2 - \frac{(p_1 \cdot p_Z)^2}{M_Z^2} \right) (k_1 \cdot k_2 M_H^2 - 2k_1 \cdot p_2 k_2 \cdot p_2) \\ T_{12} &= \left(p_1 \cdot p_2 - \frac{p_1 \cdot p_Z p_2 \cdot p_Z}{M_Z^2} \right) (p_1 \cdot p_2 k_1 \cdot k_2 - p_1 \cdot k_2 p_2 \cdot k_1 - p_1 \cdot k_1 p_2 \cdot k_2) \\ T_{13} &= k_1 \cdot k_2 \left(p_1 \cdot p_2 - \frac{p_1 \cdot p_Z p_2 \cdot p_Z}{M_Z^2} \right) - k_1 \cdot p_2 \left(k_2 \cdot p_1 - \frac{k_2 \cdot p_Z p_1 \cdot p_Z}{M_Z^2} \right) \\ &\quad - k_2 \cdot p_2 \left(k_1 \cdot p_1 - \frac{k_1 \cdot p_Z p_1 \cdot p_Z}{M_Z^2} \right) \\ T_{33} &= k_1 \cdot k_2 + \frac{2 k_1 \cdot p_Z k_2 \cdot p_Z}{M_Z^2}. \end{aligned} \quad (11)$$

The remaining T_{ij} are obtained by permuting p_1 and p_2 ,

$$\begin{aligned} T_{22} &= T_{11}(p_1 \leftrightarrow p_2), \\ T_{23} &= T_{13}(p_1 \leftrightarrow p_2). \end{aligned} \quad (12)$$

We perform a Monte Carlo integration over the 4-particle phase space to obtain the $Z \rightarrow HHf\bar{f}$ decay rate. The result is shown, relative to $Z \rightarrow \mu^+ \mu^-$, as a function of M_H in Fig. 1. The rate is summed over only the leptonic $f\bar{f}$ pairs (i.e. $\ell^+ \ell^-$, $\nu \bar{\nu}$) since these should lead to an identifiable signal as by then single Higgs detection would have

been fully investigated (c.f. eq. (3)). For M_H less than about 10 GeV we see that the rate is within reach of the existing Z factories. The distribution of events on the $M_{f\bar{f}}$ plot is characteristically peaked close to the $M_Z - 2M_H$ kinematic limit due to the effect of the Z pole. Similarly, the Higgs bosons are preferentially produced with small transverse momentum and the invariant mass of the Higgs pair is close to threshold.

If a light Higgs exists, the double production rate offers a chance to probe the $ZZHH$ and H^3 couplings. From eq. (7) we see that diagrams 2a and 2b contribute with approximately equal strength. We therefore parameterise the effect of deviations from standard model couplings by multiplying both the $ZZHH$ and H^3 couplings by a factor K . Fig. 3 illustrates the effect of varying K from its standard model value, $K = 1$. Reducing K tends to reduce the $Z \rightarrow HHf\bar{f}$ rate, while increasing K has the opposite effect.

In Table 1 we compare the cross section for double Higgs production at a Z factory with that for production at higher energy e^+e^- colliders for various values of M_H . For collider energies above M_Z , Higgs pair production can proceed via a virtual Z , i.e. $e^+e^- \rightarrow Z^* \rightarrow ZHH$. The matrix elements are given by the ‘crossed’ version of eq. (4) with $p_Z \rightarrow -p_Z$ in D_1 and D_2 , and T_{ij} unchanged. The resulting cross section decreases with increasing $\sqrt{s}$ due to the s -channel Z exchange and by supercollider energies WW fusion is found to be the dominant production mechanism, i.e. $e^+e^- \rightarrow HH\nu_e\bar{\nu}_e$ [5], with a cross section which increases with $\sqrt{s}$. However, we see from Table 1 that if $M_H \lesssim 20$ GeV, the simpler way to probe HH production would be to increase the existing Z factories luminosity, and even if $20 \lesssim M_H \lesssim 40$ GeV, the cross section via a virtual Z is larger at $\sqrt{s} = 200$ GeV (LEP II) than that due to WW fusion at a 1 TeV e^+e^- supercollider.

Other possible Higgs decay modes of the Z boson are listed in Table 2, first for the minimal standard model and second for the minimal supersymmetric model [7] with three neutral Higgs bosons, H_1 , A and H_2 . H_i and A are states with CP even and odd respectively, where H_1 , and possibly also A , are lighter than the Z boson. Due to the CP properties of the A boson, there are no AZZ or AWW couplings in the SUSY lagrangian.

For comparison, Fig. 1 also shows the rate for the standard model decay [8],

$$Z \rightarrow H\gamma, \tag{13}$$

which to lowest order proceeds dominantly via a W loop. Fermion loops also contribute, but with an amplitude proportional to m_f^2/M_W^2 for $m_f \ll M_Z$. Even for large m_f , the fermion loops give relatively small contributions as can be seen from the sensitivity of the $Z \rightarrow H\gamma$ rate to the mass, m_t , of the top quark. Only if $M_H > 60$ GeV will the $Z \rightarrow H\gamma$ rate exceed that for $Z \rightarrow H\mu^+\mu^-$. Even for small M_H the $Z \rightarrow H\gamma$ signal is only just within reach of the existing Z factory experiments.

In the minimal SUSY model, the $Z \rightarrow H_1\gamma$ rate [9] may be larger (or smaller) than the standard model $Z \rightarrow H\gamma$ decay, depending on the choice of the supersymmetric parameters

and due to loop contributions from the supersymmetric partners of the standard model particles. The same is also true for the $Z \rightarrow A\gamma$ decay [9]. The observation of $H\gamma$ production at the standard model rate therefore serves to severely constrain the SUSY parameters. Similarly the double Higgs production rate could, in principle, give evidence of SUSY since $\Gamma(Z \rightarrow H_1 H_1 f \bar{f})$ is smaller than $\Gamma(Z \rightarrow H H f \bar{f})$ for $M_{H_1} = M_H$ by a factor β^2 which depends on the values of the SUSY parameters, and where $\Gamma(Z \rightarrow H_1 f \bar{f}) \sim \beta \Gamma(Z \rightarrow H f \bar{f})$.

For very small M_H we expect $\Gamma(Z \rightarrow H H \gamma)/\Gamma(Z \rightarrow H H \mu \mu)$ to be approximately equal to $\Gamma(Z \rightarrow H \gamma)/\Gamma(Z \rightarrow H \mu \mu)$ and so, the $Z \rightarrow H H \gamma$ rate to be $O(10^{-7} - 10^{-6})$ that for $Z \rightarrow \mu^+ \mu^-$ (cf. Fig. 1). Even though for larger M_H the $Z \rightarrow H H \gamma$ rate does not decrease as rapidly as $Z \rightarrow H H f \bar{f}$, it is clearly out of reach of present experiments.

We conclude that if a relatively light Higgs boson is discovered, presumably via $Z \rightarrow H f \bar{f}$ decay, then double Higgs production could well be within reach of present Z decay experiments and yield valuable information on the Higgs sector. In fact we found that present e^+e^- colliders, and the extension to LEP II, offer a better chance to study double Higgs production than TeV e^+e^- machines provided $M_H \lesssim 40$ GeV.

Acknowledgements

We thank U. Baur for stimulating discussions.

Table 1. Cross sections (in fb $\equiv 10^{-6}$ nb) for double Higgs production in e^+e^- collisions of c.m. energy $\sqrt{s}$.

	$\sqrt{s} = M_Z$ (Z factory)	$\sqrt{s} = 200$ GeV (LEP II)		$\sqrt{s} = 1$ TeV (e^+e^- supercollider)	
M_H (GeV)	HHZ^*	HHZ	$HH\nu_e\bar{\nu}_e$	HHZ	$HH\nu_e\bar{\nu}_e$
10	50	8	0.02	0.22	0.8
20	3	5	0.01	0.20	0.7
30	0.1	3	0.005	0.19	0.6

HHZ^* , HHZ and $HH\nu_e\bar{\nu}_e$ denote final states produced via a real Z , a virtual Z and WW fusion respectively. The $HH\nu_e\bar{\nu}_e$ cross sections are taken from [5]. The radiatively corrected $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$ cross section at $\sqrt{s} = M_Z$ is taken to be 1.1 nb [6].

Table 2. Possible Higgs decay modes of the Z boson.

Standard model	Minimal SUSY model [†]
$Z \rightarrow H f \bar{f}$	$Z \rightarrow H_1 f \bar{f}$
$Z \rightarrow H \gamma$	$Z \rightarrow H_1 \gamma, A \gamma$
	$Z \rightarrow H_1 A$
$Z \rightarrow HH f \bar{f}$	$Z \rightarrow H_1 H_1 f \bar{f}, AA f \bar{f}$
$Z \rightarrow HH \gamma$	$Z \rightarrow H_1 H_1 \gamma, AA \gamma$

[†] Although one of the two CP -even Higgs bosons (denoted H_1) must satisfy $M_{H_1} \lesssim M_Z$, the condition $M_A < M_Z$ depends on the choice of SUSY parameters. The decay $Z \rightarrow H_1 A$ may well be kinematically forbidden.

References

- [1] S. Weinberg, Phys. Rev. Lett. **36**, 294 (1976);
A. D. Linde, JETP Lett. **23**, 64 (1976), Phys. Lett. **70B**, 306 (1977).
- [2] J. D. Bjorken, in *Weak Interactions at High Energy and the Production of New Particles*, Proceedings of the SLAC Summer Institute on Particle Physics, 1976, edited by M. Zipf, p1 (1976);
J. Finjord, Physica Scripta **21**, 143 (1980).
- [3] G. J. Gounaris, D. Schildknecht and F. M. Renard, Phys. Lett. **83B**, 191 (1979);
erratum **89B**, 437 (1980);
V. Barger, T. Han and R. J. N. Phillips, Phys. Rev. **D38**, 2766 (1988);
K. J. F. Gaemers and F. Hoogeveen, Z. Phys. **C26**, 249 (1984).
- [4] W.-Y. Keung, Mod. Phys. Lett. **2A**, 765 (1987);
D. A. Dicus, K. J. Kallianpur and S. S. D. Willenbrock, Phys. Lett. **B200**, 189 (1988);
O. J. P. Eboli, G. C. Marques, S. F. Novaes and A. A. Natale, Phys. Lett. **B197**, 269 (1988);
D. A. Dicus, C. Kao and S. S. D. Willenbrock, Phys. Lett. **B203**, 457 (1988);
E. W. N. Glover and J. J. van der Bij, Nucl. Phys. **B309**, 282 (1988);
see also the second reference of Ref. [2].
- [5] V. Barger and T. Han, Madison preprint MAD/PH/430 (1988).
- [6] G. Altarelli and G. Martinelli, in Physics at LEP, CERN yellow report 86-02, vol 1, p. 47.
- [7] J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986); Nucl. Phys. **B278**, 449 (1986).
- [8] R. N. Cahn, M. S. Chanowitz and N. Fleishon, Phys. Lett. **82B**, 113 (1979);
L. Bergstrom and G. Hulth, Nucl. Phys. **B259**, 137 (1985).
- [9] G. Gamberini, G. F. Giudice and G. Ridolfi, Nucl. Phys. **B292**, 237 (1987);
T. Weiler and T. C. Yuan, Vanderbilt preprint VAND-TH88-5 (1988).

Figure Captions

- Fig. 1: The decay rate, normalised to $\Gamma(Z \rightarrow \mu^+\mu^-)$, for (a) $Z \rightarrow H\mu^+\mu^-$, (b) $Z \rightarrow H\gamma$ for $m_t = 60$ and 200 GeV, and (c) $Z \rightarrow HH\ell\bar{\ell}$ where $\ell = e, \mu, \tau$ or ν as a function of the Higgs boson mass, M_H . We use $M_Z = 92$ GeV, $\sin^2\theta_W = 0.23$ and $\alpha = \frac{1}{128}$. The scale on the right shows the number of such decays expected for 10^7 Z bosons produced.
- Fig. 2: The Feynman diagrams for the $Z \rightarrow HHf\bar{f}$ decay.
- Fig. 3: The total $Z \rightarrow HHf\bar{f}$ decay rate for non-standard model $ZZHH$ and H^3 couplings relative to the Standard Model rate. The non-standard model couplings are K times the standard model values.

Fig. 1

Fig.2

Fig. 3