CERN-TH.5342/89

TPI-MINN-89/6-T

PARITY VIOLATION IN LIGHT ATOMS AND THE STRANGE
SPIN CONTENT OF THE PROTON

B.A. Campbell

CERN - Geneva

and

Physics Dept., Univ. of Alberta, Edmonton, Canada

John Ellis

CERN - Geneva

and

R.A. Flores

School of Physics and Astronomy and
Theoretical Physics Institute, Univ. of Minnesota
Minneapolis, U.S.A.

ABSTRACT

We re-evaluate Standard Model predictions for nuclear spin-dependent parity-violating effects in Hydrogen and Deuterium, in the light of the recent EMC measurement of polarized muon-proton scattering. This yields a new estimate of the isoscalar axial Z^0 coupling to hadrons, whose compatibility with $\nu N \rightarrow \nu \pi N$ cross-section data we discuss. The EMC data indicate that, contrary to previous analyses, nuclear spin-dependent parity violation in Deuterium is not dominated by electroweak radiative corrections, but rather by polarized strange quarks in the nucleon.

Atomic physics parity violation experiments [1] offer the prospect of precision tests of the neutral currents in the Standard Model [2]. Initial experiments with Bismuth [3] were plagued by large atomic physics uncertainties. These are greatly reduced in Cesium and Thallium, where precision measurements have now been made [4,5]. They contribute significantly to tests of the electroweak theory [6,7], improving substantially the upper bound on the top quark mass inferred from a combined analysis of low- and high-energy experiments [6-8]. However, the best atoms to test the electroweak theory are likely to be light ones such as Hydrogen and Deuterium, where atomic physics uncertainties are negligible [2]. Indeed, it has been claimed [2] that radiative corrections [9,10] make the leading-order contribution to one parity-violating matrix element in Deuterium, which would therefore be a particularly sensitive probe of the Standard Model.

The matrix element in question is that of an effective operator

$$\mathcal{L}_{eff}^P = \frac{G_F}{\sqrt{2}} (\bar{e} \gamma_\mu e) \left(\sum_f C_{2f} \bar{q} \gamma_\mu \gamma_5 q \right) \quad (1)$$

with a vector current coupling to the electron and axial vector current couplings to nucleons. The latter cannot as yet be calculated reliably using QCD, and are therefore model-dependent. When estimating parity-violating effects in atoms, it has normally been assumed [2,9,10] that only the $\bar{u} \gamma_\mu \gamma_5 u$ and $\bar{d} \gamma_\mu \gamma_5 d$ currents have non-zero matrix elements, and that $\bar{s} \gamma_\mu \gamma_5 s$ matrix elements can be neglected. However, this assumption is contradicted by recent EMC data [11] on polarized muon-proton scattering which indicate that $\langle p | \bar{s} \gamma_\mu \gamma_5 s | p \rangle \neq 0$. Denoting $\langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle \equiv \Delta q \cdot \Sigma_\mu(p)^*$, where $\Sigma_\mu(p)$ is the proton spin, the EMC data combined with previous neutron and hyperon β -decay data indicate that [12]

$$\Delta u = 0.74 \pm 0.08, \quad \Delta d = -0.51 \pm 0.08, \quad \Delta s = -0.23 \pm 0.08 \quad (2)$$

Support for this conclusion that $\Delta s \neq 0$ is provided [13,14] by recent data on νp and $\bar{\nu} p$ elastic scattering [15], which are sensitive to the same combination of hadronic axial currents as that appearing in the atomic physics parity violation experiments. It is apparent from Eq. (2) that [12]

$$\sum_{q=u,d,s} \Delta q = 0.00 \pm 0.24 \quad (3)$$

*) In the quark-parton model, the Δq can be interpreted as the fractions of the proton spin (strictly, helicity in the infinite-momentum frame) carried by quarks of flavour q . This intuitive interpretation is not needed for our subsequent analysis.

This result would not be expected in the naive non-relativistic quark model (NQM), but finds a natural interpretation [16] in the Skyrme model that incorporates the ideas of chiral symmetry, namely that the u,d and s quarks are light. We will see that the central value $\Delta u + \Delta d + \Delta s = 0$ plays a special rôle in our analysis of parity violation in atomic physics.

The structure of this paper is as follows. We first give general expressions for the parity-violating interaction coefficients C_{2p} and C_{2n} :

$$\mathcal{L}_{eff}^N = \frac{G_F}{\sqrt{2}} (\bar{e} \gamma_\mu e) (C_{2p} \bar{p} \gamma_\mu \gamma_5 p + C_{2n} \bar{n} \gamma_\mu \gamma_5 n) \quad (4)$$

in terms of the C_{2q} . We next comment on the nuclear matrix elements of $\mathcal{L}_{eff}^N$ (4) in Deuterium and Cesium. Then we use the radiatively-corrected [9,10] values of C_{2u} and C_{2d} to give numerical values of C_{2p} , C_{2n} and C_{2D} in the Standard Model. These are quoted for arbitrary values of Δs . In the particular case where the Δq take their central values in Eqs. (2) and (3), we find that C_{2D} is not given by the radiative corrections to the tree-level vanishing of $C_{2u} + C_{2d}$, as had previously been thought, but is instead given by a term $\propto \Delta s (C_{2d} - C_{2u})$ which does not vanish at the tree level. We also note in passing that the $s \gamma_\mu \gamma_5 s$ neutral axial current makes in principle important contributions to the cross-sections for $(\bar{\nu})_V + N \rightarrow (\bar{\nu})_V + \pi + N$ which have been neglected in previous analyses [17,18]. The magnitude (2) of Δs indicated by the EMC is barely compatible with the present data, which should be checked in future experiments.

The parity-violating interaction coefficient C_{2p} in (4) is given by

$$C_{2p} = C_{2u} \Delta u + C_{2d} \Delta d + C_{2s} \Delta s \quad (5)$$

The values of C_{2d} and C_{2s} are equal at the tree level and indistinguishably different at the one-loop level, so we set $C_{2s} = C_{2d}$ to obtain

$$C_{2p} = C_{2u} \Delta u + C_{2d} (\Delta d + \Delta s) \quad (6)$$

This can be rewritten in the form

$$C_{2p} = C_{2d} (\Delta u + \Delta d + \Delta s) + (C_{2u} - C_{2d}) \Delta u \quad (7)$$

which points up the special rôle of the central value of (2): $\Delta u + \Delta d + \Delta s = 0$. Two combinations of the Δq were known with fair precision before the EMC experiment: from neutron β -decay

$$\Delta u - \Delta d = g_A = F + D \approx 1.25 \quad (8)$$

and from hyperon β -decay [19]

$$\Delta u + \Delta d - 2\Delta s = 3F - D \approx 0.68 \quad (9)$$

We have taken the central values of the reduced SU(3) matrix elements: $F/D = 0.63$ from the standard global analysis of hyperon data [19], which is consistent with more recent data [20] at the $1-\sigma$ level. We can rewrite (6) in the form

$$\begin{aligned} C_{2p} &= \left(\frac{C_{2u} - C_{2d}}{2} \right) (g_A) + \left(\frac{C_{2u} + C_{2d}}{2} \right) (3F - D) + (C_{2u} + 2C_{2d}) \Delta s \\ &\approx 0.63 (C_{2u} - C_{2d}) + 0.34 (C_{2u} + C_{2d}) + (C_{2u} + 2C_{2d}) \Delta s \end{aligned} \quad (10)$$

By isospin symmetry, the corresponding expressions for C_{2n} are

$$C_{2n} = C_{2u} \Delta d + C_{2d} (\Delta u + \Delta s) \quad (6')$$

$$= C_{2d} (\Delta u + \Delta d + \Delta s) + (C_{2u} - C_{2d}) \Delta d \quad (7')$$

$$\begin{aligned} &= \left(\frac{C_{2d} - C_{2u}}{2} \right) g_A + \left(\frac{C_{2u} + C_{2d}}{2} \right) (3F - D) + (C_{2u} + 2C_{2d}) \Delta s \\ &\approx 0.63 (C_{2d} - C_{2u}) + 0.34 (C_{2u} + C_{2d}) + (C_{2u} + 2C_{2d}) \Delta s \end{aligned} \quad (10')$$

We will shortly put numerical values of C_{2u} and C_{2d} into these formulae, but first we discuss nuclear structure effects.

If the Deuteron were a pure S-wave n - p bound state, C_{2D} would just be

$$C_{2p} + C_{2n} = C_{2u} (\Delta u + \Delta d) + C_{2d} (\Delta u + \Delta d + 2\Delta s) \quad (6'')$$

$$= (C_{2u} + C_{2d}) (\Delta u + \Delta d + \Delta s) + (C_{2d} - C_{2u}) \Delta s \quad (7'')$$

$$\begin{aligned} &= (C_{2u} + C_{2d}) (3F - D) + (2C_{2u} + 4C_{2d}) \Delta s \\ &\approx 0.68 (C_{2u} + C_{2d}) + (2C_{2u} + 4C_{2d}) \Delta s \end{aligned} \quad (10'')$$

However, we know that the Deuteron is in fact a mixture of S- and D-wave components:

$$|D; J=1\rangle = a |L=0\rangle + b |L=2\rangle \quad (11)$$

where $|a|^2 + |b|^2 = 1$. The values of a and b can be determined by fitting the experimental value of the Deuteron magnetic moment using the proton and neutron magnetic moments as inputs to obtain [21]

$$|a|^2 \simeq 0.962, \quad |b|^2 \simeq 0.038 \quad (12)$$

Using the standard composition of the $L = 2$ component of a $J_z = 1$ Deuteron state:

$$|D; J_z=1\rangle = \sqrt{\frac{3}{5}} |L_z=2, S_z=-1\rangle - \sqrt{\frac{3}{10}} |L_z=1, S_z=0\rangle + \sqrt{\frac{1}{10}} |L_z=0, S_z=1\rangle \quad (13)$$

we find that [21]

$$C_{2d} = (|a|^2 - \frac{1}{2} |b|^2) (C_{2p} + C_{2n}) \quad (14a)$$

$$\simeq 0.943 (C_{2p} + C_{2n}) \quad (14b)$$

This nuclear structure effect is small, but is potentially significant for future precision measurements of parity violation in Deuterium.

Precision measurements [4] of parity violation in $^{55}_{133}\text{Cs}$ are starting to be sensitive to effects beyond those due to the coherent electroweak vector charge coupling of the Z^0 to the nucleus. The next largest effect [22] is that due to the nuclear anapole moment, about which we have nothing to add. Then comes a nuclear spin-dependent effect which has been estimated (22) in the nuclear shell model. It is proportional to C_{2p} , to estimate which one should use the numerical value of Eq. (10).

We now turn to the numerical evaluation of C_{2p} , C_{2n} and C_{2d} for arbitrary values of Δs , including electroweak radiative corrections. Reference [9] gives complete expressions for C_{2u} and C_{2d} :

$$C_{2u} = \frac{1}{2} S_{PV} \left[1 - 4 K_{PV}(0) \sin^2 \hat{\theta}_w(m_w) \right] - \frac{\alpha}{9\pi} (1 - 4s^2) + \frac{\alpha}{9\pi} \left[\ln \frac{m_w^2}{m^2} + \frac{1}{6} \right] - \frac{\alpha}{9\pi} (1 - \frac{8}{3}s^2) \left[\ln \frac{m_w^2}{m^2} + \frac{1}{6} \right] \\ + \frac{\alpha}{2\pi} \left[\frac{1}{s^2} + (1 - \frac{8}{3}s^2) \left[\ln \frac{m_w^2}{M^2} + \frac{3}{2} \right] + \frac{3}{32s^2c^2} (1 - 4s^2) [1 + (1 - \frac{8}{3}s^2)^2] \right] \quad (15)$$

and

$$C_{2d} = -\frac{1}{2} S_{PV} \left[1 - 4 K_{PV}(0) \sin^2 \hat{\theta}_w(m_w) \right] + \frac{\alpha}{36\pi} (1 - 4s^2) - \frac{2\alpha}{9\pi} \left[\ln \frac{m_w^2}{m^2} + \frac{1}{6} \right] + \frac{\alpha}{18\pi} (1 - \frac{4}{3}s^2) \left[\ln \frac{m_w^2}{m^2} + \frac{1}{6} \right] \\ + \frac{\alpha}{2\pi} \left[\frac{1}{4s^2} + \frac{1}{2} (1 - \frac{4}{3}s^2) \left[\ln \frac{m_w^2}{M^2} + \frac{3}{2} \right] + \frac{3}{32s^2c^2} (1 - 4s^2) [1 + (1 - \frac{4}{3}s^2)^2] \right] \quad (16)$$

where

$$S_{PV} = 1 + \frac{\alpha}{4\pi} \left[\frac{3}{4S^4} \ln c^2 - \frac{7}{4S^2} + \frac{3}{4S^2} \frac{m_t^2}{m_w^2} + \frac{3}{4} \frac{\xi}{S^2} \left[\frac{\ln(c^2/\xi)}{c^2 - \xi} + \frac{1}{c^2} \frac{\ln \xi}{1 - \xi} \right] \right] \quad (17)$$

and

$$\chi_{PV}(0) = 1 - \frac{\alpha}{2\pi S^2} \left[\frac{7}{9} - \frac{S^2}{3} + \frac{1}{6} \sum_i (C_{3i} Q_i - 4S^2 Q_i^2) \ln(m_i^2/m_w^2) \right] \quad (18)$$

In the above expressions,

$$\alpha \equiv \frac{1}{137.036}, \quad S^2 \equiv \sin^2 \hat{\theta}_w(m_w), \quad c^2 \equiv 1 - S^2, \quad \xi \equiv \frac{m_H^2}{m_z^2} \quad (19)$$

m is an effective u or d quark mass, M is a hadronic mass scale which we follow Ref. [9] in identifying with m , Q_i and I_{3i} are the electromagnetic charge and electroweak isospin of the fermion i . The definition (19) of s^2 is related to

$$s_w^2 \equiv 1 - \frac{m_w^2}{m_z^2} \quad (20)$$

by [23]

$$S^2 = 0.991 S_w^2 \quad (21)$$

We take $s_w^2 = 0.230 \pm 0.005$ from a recent global analysis of neutral current data [6,17,24]. As for the other unknowns in Eqs. (15)-(19) and (21), we take

$$m_H = m_{z^0}, \quad m_t = m_{w^\pm}, \quad m = 0.1 \text{ GeV} = M \quad (22)$$

The radiatively-corrected expression for C_{2s} is identical to (16) except that m and M could be somewhat larger. However, we believe this difference to be negligible and hence take $C_{2s} = C_{2d}$ as mentioned earlier.

For the central value $s_w^2 = 0.230$ corresponding via (21) to $s^2 = 0.228$, we evaluate

$$C_{2u} = 0.042 \quad (23a)$$

and

$$C_{2d} = C_{2s} = -0.028 \quad (23b)$$

Inserting these values into the expressions (10), (10') and (14b), we find

$$C_{2p} = 0.070 \frac{g_A}{2} + 0.013 \frac{(3F-D)}{2} - 0.015 \Delta S \quad (24a)$$

$$C_{2n} = -0.070 \frac{g_A}{2} + 0.013 \frac{(3F-D)}{2} - 0.015 \Delta S \quad (24b)$$

and

$$C_{2D} = 0.012 (3F-D) - 0.028 \Delta S \quad (24c)$$

The figure shows these three quantities graphed as functions of ΔS in the range $[-0.4, 0.2]$. For each quantity, the central line is for $s_w^2 = 0.230$ (24) and $F/D = 0.63$, and the dotted lines are for s_w^2 differing by ± 0.005 but the same value of F/D . The dashed lines for C_{2p} are for $s_w^2 = 0.230$ and $F/D = 5/9, 2/3$, which are the values favoured by the Skyrme and the naïve quark model respectively. The dash-dotted line for C_{2D} is the tree-level value without radiative corrections. The error bars at the top of the figure correspond to three independent determinations of ΔS . One [12] is the EMC [11] value (2) that we discussed earlier, and the second is

$$\Delta S = -0.15 \pm 0.09 \quad (25)$$

from an analysis [13,14] of $(\bar{\nu})p \leftrightarrow (\bar{\nu})p$ scattering [15]. The third is from a re-analysis of incoherent single-pion production by neutral currents, which we now present.

The experimental data on $(\bar{\nu})N \rightarrow (\bar{\nu})\pi N$ have been discussed extensively [17,18] with a view to constraining the general neutral current parameters of Hung and Sakurai [25], especially the isoscalar (u,d) axial current coefficient δ . In our notation the quantity bounded directly by the experimental data is in fact

$$\bar{\delta} \equiv \frac{\delta(\Delta u + \Delta d) + 2\delta'\Delta S}{g_A} \quad ; \quad g_A = \Delta u - \Delta d \quad (26)$$

and present data indicate that $|\bar{\delta}| = 0.007^{+0.103}_{-0.102}$. In the past [17,18], it has either been assumed that $(\Delta u + \Delta d)/g_A = 3/5$ as in the naïve quark model, or that $(\Delta u + \Delta d)/g_A = 1$, and it has always been assumed that $\Delta S = 0$. We see from (2) that $(\Delta u + \Delta d)/(\Delta u - \Delta d) \approx 0.18$ now seems to be a better value, and the bound on δ quoted in Ref. [18] and obtained by neglecting the second term in the numerator of (26) should be relaxed accordingly. In fact, in the Standard Model we have $\delta = 0$ and $\delta' = -\frac{1}{2}$ at the tree level, and we should rather use (26) to bound ΔS :

$$|\Delta S| = \left| \frac{(\Delta u - \Delta d)}{2\delta'} \bar{\delta} \right| = |-1.25 \bar{\delta}| = -0.01 \pm 0.13 \quad (27)$$

In our view, the simple isobar model used [18] to derive the bound $|\bar{\delta}| < 0.1$ is less reliable than the form factor models used [15] to analyze $(\bar{\nu})p \rightarrow (\bar{\nu})p$ elastic

scattering, which is in turn less reliable than the QCD-improved parton model estimate (2) based on the EMC data. However, both the model and the $(\bar{\nu})_N \rightarrow (\bar{\nu})_{\pi N}$ data could perhaps be improved in the future.

We note one curiosity of the expression (7'') for $C_{2p} + C_{2n}$ and hence C_{2D} . When $\Delta u + \Delta d + \Delta s = 0$ as suggested (2) by the EMC, corresponding to $\Delta s = -0.23$, all sensitivity is lost to $(C_{2u} + C_{2d})$ which is directly proportional to the radiative corrections in (18). In this case, C_{2D} is entirely given by Δs , with a coefficient $(C_{2d} - C_{2u})$ which is only weakly sensitive to the radiative corrections. We therefore conclude that parity violation in Deuterium is more a probe of the strange spin content of nucleon than a test of electroweak radiative corrections and a precision technique for measuring $\sin^2 \theta_w$. We see from the figure that if $\Delta s \sim -0.2$, changing s_w^2 by ± 0.005 corresponds to changing Δs by ~ 1 (~ 0.1) in Hydrogen (Deuterium). This means that measuring C_{2p} could be a good way to determine $\sin^2 \theta$ precisely, whereas measuring C_{2D} might only turn out to give a more precise measure of Δs .

ACKNOWLEDGEMENTS

The research of one of us (B.A.C.) was supported in part by the National Sciences and Engineering Research Council of Canada. Two of us (B.A.C. and J.E.) thank the Lake Louise Winter Institute for its warm hospitality at the start of this work.

REFERENCES

- [1] A. Bouchiat and C. Bouchiat, Phys. Lett. 48B (1974) 111; J. Phys. 35 (1974) 899 and 36 (1975) 493.
- [2] E.N. Fortson and L.L. Lewis, Physics Reports C113 (1984) 289.
- [3] J. Taylor, Oxford Thesis (unpublished) plus references therein (1984);
J.H. Hollister et al., Phys. Rev. Lett. 46 (1981) 683;
L.M. Barkov and M.S. Zolototев, Sov. Phys. JETP 52 (1980) 360 plus references therein.
- [4] M.A. Bouchiat et al., J. Phys. 47 (1986) 1175, 1709 and references therein;
M.C. Noecker, B.P. Masterson and C.E. Wieman, Phys. Rev. Lett. 61 (1988) 310.
- [5] P.H. Buchsbaum et al., Phys. Rev. D24 (1981) 1134;
P.S. Drell and E.D. Commins, Phys. Rev. A32 (1985) 2196.
- [6] U. Amaldi et al., Phys. Rev. D36 (1987) 1385.
- [7] G. Costa et al., Nucl. Phys. B297 (1988) 244.
- [8] J. Ellis and G.L. Fogli, Phys. Lett. B213 (1988) 526.
- [9] W.J. Marciano and A. Sirlin, Phys. Rev. D27 (1983) 552.
- [10] J.F. Wheeler, Phys. Lett. 105B (1981) 483;
B.W. Lynn, in 1983 Trieste Conference on Radiative Corrections in $SU(2) \times U(1)$,
Eds. B.W. Lynn and J.F. Wheeler, (World Scientific, Singapore) 1984, p. 311.
- [11] EMC Collaboration, J. Ashman et al., Phys. Lett. B206 (1988) 364.
- [12] J. Ellis, R.A. Flores and S. Ritz, Phys. Lett. B194 (1987) 493.
- [13] D. Kaplan and A. Manohar, Nucl. Phys. B310 (1988) 527.
- [14] J. Ellis and M. Karliner, Phys. Lett. B213 (1988) 73.
- [15] L. Ahrens et al., Phys. Rev. D35 (1987) 785.
- [16] S.J. Brodsky, J. Ellis and M. Karliner, Phys. Lett. B206 (1988) 309.
- [17] R.M. Barnett, Phys. Rev. D14 (1976) 2990;
L.J. Abbott and R.M. Barnett, Phys. Rev. Lett. 40 (1978) 1303;
E. Monsay, Phys. Rev. D18 (1978) 2277.
- [18] G.L. Fogli, Nucl. Phys. B207 (1982) 322.
- [19] M. Bourquin et al., Zeit. Phys. C21 (1983) 27.
- [20] R.L. Jaffe and A. Manohar, MIT preprint, MIT-CTP-1706 (1989).
- [21] J. Ellis and R.A. Flores, Nucl. Phys. B307 (1988) 883.
- [22] V.N. Novikov et al., Sov. Phys. JETP 46 (1977) 420.
- [23] A. Sirlin, Phys. Rev. D22 (1980) 971;
W.J. Marciano and A. Sirlin, Phys. Rev. D22 (1980) 971, 2695; Phys. Rev. Lett. 46 (1981) 163; Nucl. Phys. B189 (1981) 442.
- [24] G.L. Fogli, Bari preprint BA-TH/40-89 (1989).
- [25] P.Q. Hung and J.J. Sakurai, Phys. Lett. 63B (1976) 295.

FIGURE CAPTION

Values of the nuclear spin-dependent parity violating interaction coefficients $C_{2p,n,D}$ as functions of Δs . The solid lines are for $\sin^2\theta_w = 0.230$ and the dotted lines are $\sin^2\theta_w = 0.235, 0.225$. The dashed lines for C_{2p} are for $F/D = 2/3$ and $5/9$. The dash-dotted line marked L.O. is C_{2D} for $\sin^2\theta_w = 0.230$ without radiative corrections. The horizontal error bars at the top of the figure are the experimental constraints on Δs discussed in the text.