CERN-TH-5380/89
WU B 89-9

Non-Singlet Axial Vector Couplings of the Baryon Octet in Lattice QCD*

S. Güsken, U. Löw, K.-H. Mütter, R. Sommer

Physics Department, University of Wuppertal, 5600 Wuppertal 1, Fed. Rep. Germany

A. Patel and K. Schilling**

Theory Division, CERN, CH-1211 Genève 23, Switzerland

Abstract

We calculate in the quenched approximation the F and D parameters characterising the axial vector matrix elements of the baryon octet. The use of an improved fermion action and smeared interpolating fields enables us to go down in quark mass to the $SU(3)$ symmetric point. Both the F/D ratio and g_A are in good agreement with the experimental values within the statistical error of $\approx 10\%$.

* Work supported in part by Deutsche Forschungsgemeinschaft grant Schi 257/1-2.

** On leave of absence from Physics Department, University of Wuppertal.

Several years ago, a first lattice calculation of the axial vector couplings of the baryon octet was attempted by Fucito, Parisi and Petrarca [1]. They evaluated the matrix elements of interest from the rate of change of the nucleon mass as a function of an externally applied constant axial vector field. The calculation was performed on 9 configurations of a 5^4 lattice doubled to the size $5^3 \times 10$ at $\beta = 6/g^2 = 6.0$. Although large finite size effects are to be expected for such parameters (the linear dimension of the lattice is about 0.5 fm) and the non-perturbative renormalisation of the axial vector current was neglected among other systematic errors, the results [1] were found to be almost independent of the quark mass and in reasonable agreement with experiment.

Here we report on a calculation on a much larger system (in physical units) and with higher statistical accuracy. We have gained further improvements from (i) the use of an improved lattice action [2] which allows us to get close to the chiral limit [3], (ii) the calculation of zero 4-momentum insertion of the axial current in the nucleon correlator instead of perturbations in the presence of an external field and (iii) efficient smeared hadron operators.

We use Wilson fermions in our calculation [4]. Due to explicit breaking of chiral symmetry inherent in this approach, a precondition for the calculation of axial vector matrix elements is the knowledge of the non-perturbative renormalisation of the axial vector current. Following the suggestion by Bocchichio et al. [5] we have studied this [6] for our improved lattice fermion action. (For a discussion of the action we direct the reader to ref.[2].) Let us briefly recall the results of ref.[6] that are needed here. We define the local bare non-singlet axial vector current on the lattice as

$$A_\mu^{(c)L}(x) = \bar{q}(x) \frac{\lambda^c}{2} \gamma_\mu \gamma_5 q(x) . \quad (1)$$

For a non-zero lattice spacing a in the scaling region, this operator undergoes a finite multiplicative renormalisation relative to the continuum one:

$$A_\mu^{(c)}(x) = Z_A^L A_\mu^{(c)L}(x) . \quad (2)$$

The renormalisation constant Z_A^L was calculated as a function of the Wilson lattice quark mass (κ is the hopping parameter of the improved action [2]):

$$m_q = \ln[1 + \frac{3}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_c})] . \quad (3)$$

The results for Z_A^L , determined by imposing current algebra commutation relations on the lattice, are well parametrised by the fit

$$Z_A^L = 0.78 + 0.26 m_q , \quad m_q \leq 0.4 . \quad (4)$$

The statistical uncertainty of Z_A^L decreases from about 10% for small quark masses to about 2% for large ones. In the case of the standard Wilson action [4] at $\beta = 6.0$, it has been observed that the renormalisation of the local lattice vector current comes out different for

matrix elements in different states [7]. This effect is a violation of scaling and implies that physical results must be obtained by going to weaker gauge couplings. A similar analysis for Z_A^L and with our action has not been carried out. The lattice renormalisation of the local axial vector current is not as strong as that of the local vector current. Therefore it is natural to believe that the scaling violations would be smaller in the case of the axial vector current; for the time being we assume that Z_A^L is the same for all hadron states.

Let us now define the axial vector couplings of the baryon octet states. Throughout this work we assume exact isospin symmetry. To leading order in flavour $SU(3)$ symmetry and using Lorentz and parity invariances of the strong interactions, the matrix elements of the axial vector current between two baryon states at rest, B_i and B_f , are parametrised as

$$\langle B_f(s_f) | A_\mu^{(c)} | B_i(s_i) \rangle = \bar{\psi}_f(s_f) \gamma_\mu \gamma_5 \psi_i(s_i) \cdot (F_A \text{Tr}(B_f^\dagger [\lambda_c, B_i]) + D_A \text{Tr}(B_f^\dagger \{\lambda_c, B_i\})) . \quad (5)$$

Here s_i, s_f label the spins of the initial and final baryon states. Experimental information on the size of the $SU(3)$ reduced matrix elements F_A and D_A comes from the spin correlations and lifetime of neutron decay [8]

$$g_A = F_A + D_A = 1.259(4) , \quad (6)$$

and the relative strength of the two couplings has been determined from a fit to all the semi-leptonic hyperon decay rates [9]

$$F_A/D_A = 0.59(5) . \quad (7)$$

We also know the answers in the static ($m_q = \infty$) limit; the non-relativistic quark model provides the numbers:

$$g_A = 5/3 , \quad F_A/D_A = 2/3 . \quad (8)$$

In the chiral limit, the fact that $g_A \neq 1$ is a demonstration of the strong interactions causing a spontaneous breakdown of the axial vector symmetry. On the other hand, the departures from the static limit tell us about the importance of relativistic motion of the quarks bound into hadrons [10].

In our lattice calculation we measure the following ratio of three-point to two-point correlation functions:

$$\frac{\sum_{\vec{x}} \langle \mathcal{P}_\alpha(\vec{x}, t) \sum_z \bar{q}(z) \sum_{\mu=1}^3 \gamma_\mu \gamma_5 q(z) \mathcal{P}_\beta^\dagger(\vec{0}, 0) \rangle}{\frac{1}{2} \sum_{\delta=1}^2 \sum_{\vec{x}} \langle \mathcal{P}_\delta(\vec{x}, t) \mathcal{P}_\delta(\vec{0}, 0) \rangle} = \sum_{\mu=1}^3 (\gamma_\mu \gamma_5)_{\alpha\beta} R_q^A(t) . \quad (9)$$

Here $\mathcal{P}_\alpha(\vec{x}, t)$ is an interpolating field for the proton with spin s_α . Just as in the case of the scalar density matrix elements [11], one can relate $R_q^A(t)$ to the desired matrix elements:

$$\begin{aligned} \frac{1}{2} R_u^A(t) &\xrightarrow{t \text{ large}} \text{const.} + f t F_A/Z_A^L , \\ \frac{1}{2} R_u^A(t) - R_d^A(t) &\xrightarrow{t \text{ large}} \text{const.} + f t D_A/Z_A^L . \end{aligned} \quad (10)$$

These relations are valid in the range of single particle dominance in both the correlation functions that form the ratio in eq.(9). The extra factor f in front of the matrix elements is required in order to get rid of the lattice artifacts in the matrix elements for very heavy quarks. It has to obey the constraints

$$f = \begin{cases} 1, & \text{for } m_q \rightarrow 0; \\ 2\kappa/3, & \text{for } m_q \rightarrow \infty. \end{cases} \quad (11)$$

As a smooth interpolation between these two limits, we use

$$f = \frac{\partial m_q}{\partial(3/2\kappa)} = [1 + \frac{3}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_c})]^{-1}. \quad (12)$$

However, we emphasise that this choice is by no means unique¹.

There are two contractions of the three-point correlation function. Only the contraction with all quark propagators connected (cf. fig. 1a of ref.[12]) contributes to the non-singlet matrix elements. This is the part which we can consistently calculate while ignoring the effects of sea quarks (quenched approximation). The results presented below are based on 47 configurations of an $8^3 \times 16$ lattice that we have used previously [3][12][6]. We used anti-periodic boundary conditions for the quarks in the time direction, and improved our signal by using the finite size reduction technique in the spatial directions [13]. The matrix inversions were performed using an overrelaxed conjugate residual algorithm at $\kappa = 0.1320, 0.1345, 0.13525, 0.1359$ and 0.1361 . This range of values allows us to observe the crossover from the static limit towards the chiral limit, coming as close as $M_{PS}^2/M_V^2 = 0.29$ to the latter in terms of the pseudoscalar to vector meson mass ratio. We find $\kappa_c = 0.13650(2)$, and our estimate for the lattice spacing varies from $a^{-1} = 1.15 \text{ GeV}$ to 1.30 GeV , depending upon the choice of the nucleon or the rho mass to fix the scale [3]. Instead of the usual point quark source, we used a gauge-covariantly smeared quark source

$$S(\vec{x}) = \Delta_{3d}^{-1} \delta_{\vec{x},0} = (D^2 + m_{sc}^2)_{3d}^{-1} \delta_{\vec{x},0}. \quad (13)$$

Δ_{3d} is the interaction matrix for a scalar particle propagating in the three-dimensional subspace (one time-slice) of the same background gauge field. The parameter m_{sc} was tuned so as to give the quark source an r.m.s. radius of about 3 lattice spacings. This will be discussed at length in a forthcoming publication containing details of the hadron mass spectrum [14]. The three-point correlators were computed in the manner explained in ref.[11].

In fig. 1, we show the “raw data” for the ratios R_u^A and R_d^A for $\kappa = 0.1361$ corresponding to $M_{PS}^2/M_V^2 = 0.29$. The statistical precision of our data is higher for all other κ -values. Already at $t = 1$ and for all quark masses, we observe single particle dominance. This early onset of single particle dominance is due to our use of an efficient smearing operator. For $t \geq 6$ there appear to be deviations from the linear behaviour in t . This might be

¹An analogous choice for staggered fermions would be $m_q = \sinh^{-1}(m)$, $f = \partial m_q / \partial m = (1 + m^2)^{-1/2}$, where m denotes the mass parameter appearing in the action.

a statistical “accident”, influence of finite size effects or contamination from the $N^*(\frac{1}{2}^-)$ state propagating backwards in time in these correlation functions. The slopes versus t were determined from fits in the range $1 \leq t \leq 5$ at all values of κ with good statistical precision. They are shown in fig. 2 as functions of the quark mass.

In order to compare our results to the experimental values for the couplings, we plot the universal ratio F_A/D_A as a function of M_{PS}^2/M_V^2 in fig. 3. This ratio is not affected by the uncertainties in the renormalisation constant Z_A^L . The experimental value is included at the $SU(3)$ symmetric point:

$$m_q = (m_u + m_d + m_s)/3 \Leftrightarrow M_{PS}^2/M_V^2 = 3(2M_K^2 + M_\pi^2)/(2M_{K^*}^2 + M_\rho^2) \approx 0.234. \quad (14)$$

The variation between the experimental data point and the static limit is small, and our Monte Carlo data nicely interpolate between the two. In fact we have been able to calculate F_A/D_A with a precision similar to the experimental number and in good agreement with it.

Figure 3 also shows our results for g_A . The plotted error bars for g_A do not include the errors in Z_A^L which are comparable but correlated. The experimental data point in this case is plotted at the $SU(2)$ symmetric point, $M_{PS}^2/M_V^2 = 0.032$. Our results again provide a good interpolation between the experimental value and the static limit. However, our data is farther away from the experiment here and an extrapolation that is in agreement with the experimental value is plausible but not conclusive.

What have we learnt from this exercise? First, compared to the scalar density matrix elements of the baryon octet [12], the axial vector matrix elements show a weaker dependence on the quark mass. This gives us confidence that the corrections to the leading order flavour $SU(3)$ analysis are smaller here—a good indication for the parametrisation of the semi-lepton hyperon decays. Second, our data, for both F_A/D_A and g_A , beautifully interpolate between the chiral and the static limits. Indeed they allow us to judge how the relativistic corrections modify the non-relativistic quark model picture as the quark masses are decreased.

We finally note that there remain two major sources of unknown systematic errors in these results. It is necessary to check that the results found here are not destroyed by going to weaker gauge couplings and by including dynamical quarks. This is the direction in which future improvements of this calculation should focus.

Acknowledgements : We are grateful to Philippe de Forcrand for his configurations. The computations were performed on the CRAY X-MP/48 at the HLRZ in Jülich and on the CRAY X-MP/48 at CERN. We appreciate the enthusiastic support of the staff of these institutions.

References

- [1] F. Fucito, G. Parisi and S. Petrarca, Phys. Lett. 115B (1982) 148.
- [2] S. Güsken, K. Schilling, R. Sommer, R. Gupta, K.-H. Mütter and A. Patel, Nucl. Phys. B314 (1989) 63.
- [3] Ph. de Forcrand, R. Gupta, S. Güsken, K.-H. Mütter, A. Patel, K. Schilling and R. Sommer, Phys. Lett. B200 (1987) 143.
- [4] K. Wilson, in *New Phenomena in Subnuclear Physics*, Erice 1975, Plenum, New York (1977).
- [5] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi, and M. Testa, Nucl. Phys. B262 (1985) 331.
- [6] S. Güsken, R. Sommer, K. Schilling, K.-H. Mütter and A. Patel, CERN-TH-5304/89, submitted to Nucl. Phys. B (1989).
- [7] C. Sachrajda, in proceedings of the 1988 International Symposium “*Lattice 88*”, Nucl. Phys. B (Proc. Suppl.), to appear.
- [8] Particle Data Group, *Review of Particle Properties*, Phys. Lett. B204 (1988).
- [9] J.-M. Gaillard and G. Sauvage, Ann. Rev. Nucl. Part. Sci. 34 (1984) 351, and private communication;
J. Donoghue, B. Holstein and S. Klimt, Phys. Rev. D35 (1987) 934.
- [10] See for example: F. Close, *An Introduction to Quarks and Partons*, Academic Press, London (1979).
- [11] L. Maiani, G. Martinelli, M. L. Paciello and B. Taglienti, Nucl. Phys. B239 (1987) 420.
- [12] S. Güsken, R. Sommer, K. Schilling, K.-H. Mütter and A. Patel, Phys. Lett. B212 (1988) 216.
- [13] R. Sommer, in proceedings of the 1987 International Symposium “*Field Theory on the Lattice*”, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 147.
- [14] S. Güsken, U. Löw, K.-H. Mütter, K. Schilling, R. Sommer and A. Patel, in preparation.

Figure Captions

1. The ratios R_u^A and R_d^A defined in eq.(9) at $\kappa = 0.1361$.
2. The unrenormalised F_A and D_A couplings as a function of the quark mass. The dashed lines on the right-hand side indicate their values in the static limit.
3. g_A and F_A/D_A versus M_{PS}^2/M_V^2 . Also shown are the experimental results (open symbols) and the static limits ($\diamond$).

Fig. 1 :

Fig. 2 :

Fig. 3