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THE PROTON SPIN AT HIGH ENERGIES

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ABSTRACT

It is conceivable that a large part of the proton spin is carried by gluons. We show how the gluon component can be determined by a measurement of the polarized structure functions at very high energies.

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Recently, the European Muon Collaboration has measured the polarized proton structure function g_1 using a polarized muon beam on a polarized target. They found [1]

$$\int_0^1 dx g_1(x, Q^2) = 0.114 \pm 0.012 \pm 0.026 \quad (1)$$

at $Q^2 = 10.7 \text{ GeV}^2$.

In the naive parton model the first moment $\langle g_1 \rangle$ of g_1 is given by [2]

$$\langle g_1 \rangle = \int_0^1 dx g_1(x) = \frac{1}{2} \sum_{q=u}^d Q_q^2 \langle \Delta q \rangle \quad (2)$$

where Q_q is the electric charge of the quark q . $\Delta q = q_+ - q_-$ is the difference of the densities of quarks with helicities $+\frac{1}{2}$ and $-\frac{1}{2}$ in a proton with helicity $+\frac{1}{2}$ and $\langle \Delta q \rangle$ is the first moment of Δq .

This naive parton result can in principle get corrections from several sources. There may be a sea quark contribution [2], a gluon contribution [3-5] and a contribution from the orbital angular momentum [6] to the spin of the proton. The effect of the sea quarks can be included by extending the sum in Eq. (2) to n_f excited flavours. Possible effects of the orbital angular momentum have been studied in the framework of generalized Skyrme models [7]. In this letter we will concentrate on the gluon contribution and on the question of how it can be determined by a measurement at high energies.

The gluon helicity content Δg in the polarized proton is given by the difference of the gluon densities with helicities $+1$ and -1 . It can be shown that to leading order the product $\alpha_s \langle \Delta g \rangle$ is scale-independent [8], which means that it does not decrease with increasing energy like a typical $O(\alpha_s)$ contribution. This has led the authors of Ref. [3] to conjecture that $\alpha_s \langle \Delta g \rangle$ is large (of order one), rather than of order α_s . In this case, a consistent lowest-order calculation should contain the contributions from the diagram in Fig. 1, which are of order $\alpha_s \cdot \Delta g$, whereas contributions of order $\alpha_s \Delta q$, $\alpha_s^2 \Delta g$, etc., can be safely neglected.

One can calculate the contribution from Fig. 1 to the first moment of g_1 [3-5,9]. The net outcome is that the first moments $\langle \Delta q \rangle$ should be replaced by the differences $\langle \Delta q - (\alpha_s/2\pi) \Delta g \rangle$. It is conceivable that the bulk of the proton spin is carried by the valence quarks and the gluon. However, it is clear that a measurement of $\langle g_1 \rangle$ does not give sufficient information to check this possibility

or to determine $\langle \Delta g \rangle$ separately^{*)}. The main aim of this letter is to show that by going to HERA energies and including weak effects, $\langle \Delta g \rangle$ can be uniquely determined.

Before I discuss ep scattering, let me say a few words about neutrino proton scattering, because there the situation is very simple. In Fig. 2, I have drawn the relevant lowest order process and defined my notation for the momenta. The process can be described in terms of an effective interaction between a left-handed lepton and a left-handed quark current

$$\mathcal{L} = \frac{G}{\sqrt{2}} j_\mu \bar{j}^\mu \quad (3)$$

$$j_\mu = \bar{e} \gamma_\mu (1 - \gamma_5) \nu \quad \bar{j}_\mu = \bar{u} \gamma_\mu (1 - \gamma_5) d \quad (4)$$

At the high energies in which we are interested we can neglect all fermion masses. Therefore the only mass in the problem is the W mass m_W . The coupling in Eq. (3) is the Q^2 -dependent "Fermi coupling"

$$\frac{G}{\sqrt{2}} = \frac{e^2}{8 \sin^2 \theta_W} \frac{1}{Q^2 + m_W^2} \quad (5)$$

The cross-section

$$d\sigma = \frac{2\pi G^2}{p_{p1}} L_{\mu\nu} W^{\mu\nu} \frac{d^3 p_2}{(2\pi)^3 p_2^0} \quad (6)$$

factorizes into a lepton tensor

$$L_{\mu\nu} = \{p_1 p_2\}_{\mu\nu} - i\varepsilon(\mu\nu p_1 p_2), \quad (7)$$

$$\{p_1 p_2\}_{\mu\nu} := p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu} - g_{\mu\nu} p_1 p_2 \quad (8)$$

and a hadron tensor $W_{\mu\nu}$, which in general depends on eight structure functions [10]. Here we are interested only in that part of the hadron tensor which depends on the polarization vector s of the proton, and for simplicity we will assume a longitudinally polarized proton beam, $s = \lambda_p P$, with average helicity λ_p . The hadron

^{*)} One may get some information by sweeping over a heavy quark threshold [4], or from jet production in polarized deep inelastic scattering [5].

tensor is then of the form

$$W_{\mu\nu} = W_{\mu\nu}(\lambda_p = 0) + \lambda_p H_{\mu\nu} \quad (9)$$

The polarization part $H_{\mu\nu}$ of this tensor has the following decomposition into three structure functions g_1 , g_3 and g_4 :

$$H_{\mu\nu} = -i\varepsilon(\mu\nu P q) \frac{g_1}{P q} + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) g_3 + (P_\mu - \frac{P q}{q^2} q_\mu)(P_\nu - \frac{P q}{q^2} q_\nu) \frac{g_4}{P q} \quad (10)$$

Formally this looks identical to the decomposition of the unpolarized hadron tensor into the well-known structure functions F_1 , F_2 and F_3 . For simple photon exchange where one has no γ_5 coupling, the structure functions g_3 and g_4 are zero. However, for the W exchange and the neutrino scattering of Fig. 2 one gets

$$g_1 = \Delta d \quad (11a)$$

$$g_3 = -\Delta d \quad (11b)$$

It is clear that in antineutrino scattering one would measure Δu instead of Δd . One can get g_4 from g_3 by a Callan-Gross-like relation [11]

$$g_4 = 2 \times g_3 \quad (12)$$

which means that, instead of the two independent tensors $-g_{\mu\nu} + (q_\mu q_\nu)/q^2$ and $(P_\mu - (Pq/q^2)q_\mu)(P_\nu - (Pq/q^2)q_\nu)$, only their linear combination $\{Pp'\}_{\mu\nu}$ appears. We will see that the gluon contributions respect this relation. This is in contrast to the unpolarized case, where the gluons are well known to lead to deviations from the Callan-Gross relation [12].

According to the Altarelli-Ross presumption, Eq. (11) has to be modified by adding the gluon contributions drawn in Fig. 3, where also the particle momenta and indices are defined. The quark trace which enters the calculation is a sum of two terms, one with and one without a γ_5 :

$$\begin{aligned} \Gamma_{(5)} = & \text{tr } p'(\gamma_5) \left[\gamma_\mu \frac{p - \not{k}}{2pk} \gamma_\nu + \gamma_\nu \frac{p' - \not{p}}{2pp'} \gamma_\mu \right] \\ & \not{k} \left[\gamma_\sigma \frac{p - \not{k}}{2pk} \gamma_\nu + \gamma_\nu \frac{p' - \not{p}}{2pp'} \gamma_\sigma \right] \end{aligned} \quad (13)$$

To get the contribution proportional to Δg one has to calculate the difference of the cross-sections $\sigma(Wg+) - \sigma(Wg-)$ for a gluon with helicity $+1$ and -1 . Therefore one has to contract the traces (13) with

$$(\varepsilon_g \varepsilon_g^*)_+ - (\varepsilon_g \varepsilon_g^*)_- \sim i \varepsilon(g \sigma p q) / p q \quad (14)$$

where $\varepsilon_{\pm}$ are the polarization vectors of the gluon for helicity ± 1 . The parton-level hadron tensor is then

$$h_{\mu\nu} \sim \frac{\alpha_s}{2\pi} \frac{n_f}{2} i \varepsilon(g \sigma p q) [\Gamma(\mu\nu, g\sigma) + \Gamma_5(\mu\nu, g\sigma)] \quad (15)$$

I have included a factor of $n_f/2$ for the number of generations which may run in the quark loop of Fig. 3. The product of the ε -tensor and Γ_5 is symmetric in μ and ν . Therefore it can give contributions only to g_3 and g_4 . Similarly, the product of the ε -tensor and Γ gives the contribution to g_1 , which is known from the photon exchange calculations [3-5]. If one adds the two contributions, one gets

$$\begin{aligned} h_{\mu\nu} = & \frac{n_f}{2} \frac{\alpha_s}{2\pi} \Delta P_{qg} \left\{ -i \varepsilon(\mu\nu p q) \left[\frac{\tau}{\eta} + \frac{\eta}{\tau} + p^2 \frac{\tau^2}{Q^2} \left(\frac{1}{\tau^2} + \frac{1}{\eta^2} \right) + \dots \right] \right. \\ & \left. + \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) p q + 2z \left(p_\mu - \frac{p q}{q^2} q_\mu \right) \left(p_\nu - \frac{p q}{q^2} q_\nu \right) \left[\frac{\tau}{\eta} - \frac{\eta}{\tau} + p^2 \frac{\tau^2}{Q^2} \left(\frac{1}{\tau^2} - \frac{1}{\eta^2} \right) + \dots \right] \right] \right\} \end{aligned} \quad (16)$$

where $\eta = y - p^2 z^2 / Q^2$ and $\tau = 1 - y - p^2 z^2 / Q^2$.

Here $y = \frac{1}{2}(1 - \cos \theta)$ is a measure of the scattering angle θ in the W - g centre-of-mass system and $z = Q^2 / 2pq$ is the parton level x . To regulate the infra-red singularities at $y \rightarrow 0$ and $y \rightarrow 1$ the gluon has been put off-shell, $p^2 \neq 0$. The ellipses indicate irrelevant p^2 contributions.

$$\Delta P_{qg} = \frac{1}{2} (2z - 1) \quad (17)$$

is the polarized splitting function [8].

To get the contributions to the first moments of the structure functions g_1 , g_3 and g_4 , one has to integrate this expression with the measure $\int_0^1 dz \int_0^1 dy$. We take the calculation for g_1 from Ref. [4], and what we get is that the $\langle \Delta d \rangle$ in the expression for the first moment of g_1 is effectively replaced by $\langle \Delta d - 3\Delta g(\alpha_s/2\pi) \rangle$. The infra-red singularities cancel, essentially due to the antisymmetry of ΔP_{qg} under the exchange $z \leftrightarrow 1-z$. On the other hand, for g_3 and g_4 , not only the

infra-red singularities, but the whole contribution drops out due to the antisymmetry of the integrand under the exchange $y \leftrightarrow 1-y$. Therefore Eqs. (11b) and (12) remain unchanged and the conclusion is that from a measurement of $\langle g_1 \rangle$ and $\langle g_3 \rangle$ one can determine $\langle \Delta g \rangle$. A reasonable procedure is to use g_3 or g_4 to define the quark densities and to calculate $\langle \Delta g \rangle$ from $\langle g_1 \rangle$.

The problem is that for small energies the cross-section (6) is of the order $(Q^2/m_W^2)^2$, which is too small for the neutrino energies feasible in the near future. The situation is better for ep scattering where, as we shall see, the relevant terms are of the order Q^2/m_Z^2 . So let me say a few words about ep scattering. For longitudinally polarized electrons (with average helicity λ_e), one can formally maintain the cross-section formula (6) and the general decomposition of the polarized hadron tensor (10) with the proviso that the structure functions will depend on the electron charges and degree of polarization (but not on the electron momenta). One gets

$$\frac{G}{\sqrt{2}} = \frac{e^2}{Q^2} \quad (18)$$

$$\langle g_1 \rangle = \frac{1}{2} \sum_{q=1}^{n_f} b_q \langle \Delta q - \frac{\alpha_s}{2\pi} \Delta q \rangle \quad (19)$$

$$b_q = Q_q^2 \lambda_e - 2 Q_q v_q (a_e + v_e \lambda_e) \frac{Q^2}{Q^2 + m_Z^2} + (a_q^2 + v_q^2) (2 a_e v_e + \lambda_e (a_e^2 + v_e^2)) \left(\frac{Q^2}{Q^2 + m_Z^2} \right)^2 \quad (20)$$

$$\langle g_3 \rangle = - \frac{Q^2}{Q^2 + m_Z^2} \sum_{q=1}^{n_f} a_q c_q \langle \Delta q \rangle \quad (21)$$

$$c_q = - Q_q (v_e + a_e \lambda_e) + v_q (v_e^2 + a_e^2 + 2 \lambda_e v_e a_e) \frac{Q^2}{Q^2 + m_Z^2} \quad (22)$$

$$g_4 = 2 \times g_3 \quad (23)$$

The weak charges v_e , a_e , v_q and a_q are defined with the normalization $a_e = (4 \sin^2 \theta_w \cos \theta_w)^{-1}$ [13]. We see that the effect of the Z in g_1 is only to change the factor in front of the difference $\langle \Delta q - \Delta g(\alpha_s/2\pi) \rangle$. For $\langle g_3 \rangle$ and $\langle g_4 \rangle$ we again find no gluon contribution at all.

To get a qualitative feeling of what the effect of the gluon may be, I have calculated the first moments of g_1 and g_3 for two cases: (a) with a large gluon component and (b) with a small gluon component and a large contribution from the sea quarks. It is clear from Fig. 4 that there is a large difference between the two cases for large enough electron polarization. One should note, however, that a large antiquark component would complicate the analysis.

Finally, I would like to make some comments on the regularization scheme dependence of the result. It is in fact well known from unpolarized deep inelastic scattering that different regularization schemes lead to different QCD corrections to the structure functions. Only the differences between these corrections are scheme independent [14]. Physical results must be independent of the regularization scheme, and indeed only those differences are physically meaningful. One is free to redefine the quark densities in such a way that the very simple relation between the structure function F_2 and these densities which holds in lowest order is maintained also in higher order.

Now we come back to the polarized case. The null result which I have derived for g_3 and g_4 is in fact independent of the regularization scheme. On the other hand, the correction to g_1 is scheme dependent [5]. However, among all possible results the result of the off-shell regularization is preferred, because it is in agreement with what is dictated by the operator product expansion and the anomaly. We have seen that this result is finite. Therefore it is possible to attribute it solely to the gluon.

Let me now come to the conclusions. We have seen how by a measurement of the polarized structure functions at high energies the gluon component of the proton spin can be determined. Since HERA is intended to work at some point with polarized jet targets, it is possible that this can be achieved in the near future.

We have collected here only the leading contributions to the structure functions. It would be interesting to know also the terms of the form $\alpha_s^2 \Delta g$ and $\alpha_s \Delta q$ and the scale corrections (anomalous dimensions) beyond the leading order [15].

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Fig. 1

Fig. 2

Fig. 3

Fig. 4 : The first moments of the structure functions g_1 and g_3 as a function of the electron degree of polarization λ_e at $\sqrt{Q^2} = 60$ GeV.
(a) $\Delta u = 0.97$, $\Delta d = -0.28$, $\Delta s = \Delta c = \Delta b = 0$,
 $(\alpha_s/2\pi)\Delta g = 0.23$.
(b) $\Delta q = \Delta q_{(a)} - 0.23$, $\Delta g = 0$.
Note that g_1 is the same for both cases.