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CONCERNING THE MEASUREMENT OF THE GRAVITATIONAL ACCELERATION
OF CHARGED PARTICLES

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A B S T R A C T

Measurement of the gravitational acceleration of antimatter is of fundamental importance as a test of the weak equivalence principle and may also test some theories of quantum gravity. A new idea how to perform such a measurement is presented (at a theoretical level). It may lead to a measurement of gravitational acceleration of antiprotons and positrons as well as ordinary particles, protons, electrons and certain ions.

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As was realized by Newton, there are two quite distinct concepts of mass involved in gravitation [1]: the inertial mass, m_I , of a freely falling body, which is a kinematic quantity determining the proportionality of force and acceleration and the gravitational mass, m_G , which measures the body's coupling to the gravitational field. The weak equivalence principle states that

$$m_I = m_G \quad (1)$$

for all objects, with the result that the mass of the object factors out of the gravitational equations of motion.

Up to now, the weak equivalence principle has only been tested for ordinary matter. In the Eötvös experiment [2], for bulk matter in the Earth's gravitational field, the principle has been verified with a precision of five parts in 10^9 for a variety of materials. (Let us note that Fishbach et al. [3], in a presumably incorrect reanalysis of the Eötvös experiment, concluded that Eötvös data in fact "support the existence of an intermediate-range coupling to the baryon number or the hypercharge".) For bulk matter in the Sun's gravitational field, Dicke [4] and Braginski [5] achieved much higher precision to several parts in 10^{12} . For free particles such as atoms [6], neutrons [7] and photons [8] the accuracy achieved has only been one part in 10^2 or 10^3 .

It is clear that such a fundamental principle (weak equivalence) which has implications beyond any particular class of theories has to be tested also for antimatter. Besides the weak equivalence principle, a measurement of the gravitational acceleration of antimatter is important in the context of certain attempts to formulate a theory of quantum gravity. These theories predict that there will be spin-1 (graviphoton) and spin-0 (graviscalar) partners of the spin-2 graviton [9]. These new partners are expected to couple with approximately gravitational strength to some conserved charge (such as baryon-quark and/or lepton number), and to be massive (produce Yukawa potentials). Phenomenologically, under certain conditions [10] the static potential energy between two point masses m_1 and m_2 is

$$V(r) = -G \frac{m_1 m_2}{r} (1 \mp a e^{-\frac{r}{v}} + b e^{-\frac{r}{s}}) \quad (2)$$

where a and b are the moduli of products of the vector and scalar charges (in units $Gm_1 m_2$) and v and s are the inverse graviphoton and graviscalar masses (ranges). In general the "charge" in question will be some linear combination of baryon and lepton numbers. This force will therefore be composition-dependent. The $-(+)$ sign in front of a in Eq. (2) is chosen for the interaction between matter and matter (antimatter). A general prediction of this type of theory is, then, that antimatter would experience a greater gravitational acceleration towards the earth than

matter. This clearly violates the weak equivalence principle. Let us note that there are attempts to interpret geophysical and laboratory experimental data on anomalous gravitational effects by Eq. (2), i.e. in terms of broken-supersymmetry quantum-gravity models [11]. In fact, it may be that some experiments [12] with contradictory results, when interpreted with a single Yukawa term (as people usually do when considering a "fifth force" [3,13]), are consistent when interpreted by Eq. (2).

In any case, experimental tests of the weak equivalence principle with antimatter are of fundamental importance. Such an experiment, LEAR PS 200 (a measurement of the gravitational acceleration of the antiproton) was proposed and approved at CERN three years ago [14].

The first (and technically very difficult) step in the proposed experiment is to trap and store antiprotons in a Penning trap at a temperature as low as the temperature of liquid helium. After that, the particles will be released vertically, a few at a time, into a drift tube (with a coaxial magnetic field for radially confining the antiprotons) used to shield the particle orbits from stray electric fields. The particle time-of-flight spectrum through the drift tube is then measured. This spectrum, reflecting the initial distribution of velocities, will exhibit a cut-off corresponding to the equality of the initial kinetic energy and the gravitational potential energy change in the drift tube. This cut-off time is related to the gravitational acceleration a experienced by the particle by $t_c = \sqrt{2L/a}$ where L is the drift length. For more technical details and study of unwanted effects (electrostatic effects, magnetostatic effects, the patch effect, the electron and the lattice sag effect, thermal and vacuum effects) see Ref. [14].

One of the main troubles in this modern version of the Galileo experiment is that the gravitational potential energy change (for an acceptable drift tube of about $L = 1\text{m}$) is of order $U \sim 10^{-7} \text{ eV}$ ($\sim 10^{-3} \text{ }^\circ\text{K}$) which is small, even compared with the value $k_B T = 3,45 \cdot 10^{-4} \text{ eV}$ for $T = 4 \text{ }^\circ\text{K}$ (a temperature which has yet to be achieved). As a consequence there are a small number of such low-energy antiprotons in the initial distribution and the poor statistics pose a problem in a determination of the cut-off time. Because of this problem and some other technical problems there is growing doubt about the possibility of the proposed measurement without some new ideas [15]. So, new ideas are encouraged. We may assume the availability of antiprotons (or positrons) at room temperature [15]. Thus the question is how to use these particles in order to measure their gravitational acceleration.

In the present paper a simple method is studied by assuming ideal experimental conditions. A detailed study of the method under real experimental conditions will

be a subject of another publication in collaboration with my experimental colleagues.

In order to grasp the idea, let us start from the equation of motion of a charge q in an electromagnetic field

$$\frac{d\vec{p}}{dt} = q\vec{E} + q\vec{v} \times \vec{B} \quad (3)$$

The solution of this equation for a particle with non-relativistic velocity ($v \ll c$) in crossed, constant and uniform electric and magnetic fields may be found in any good book about electrodynamics (see for example Ref. [16]). The crucial point for further analysis is that in the direction perpendicular to the common plane of fields $\vec{E}$ and $\vec{B}$, a particle moves with a velocity which is a periodic function of time with an average value, drift velocity

$$\vec{v}_{\text{drift}} = \frac{\vec{E} \times \vec{B}}{B^2} \quad (4)$$

In the particular case $\vec{E} = 0$, $\vec{v}_{\text{drift}} = 0$ and the orbit of the particle is a helix, with its axis parallel to $\vec{B}$.

Let us consider the case when $\vec{E} = 0$, but instead there is a constant and uniform gravitational field. Then, instead of $q\vec{E}$ there is a term $m_I \vec{a}$ in Eq. (3), $\vec{a}$ being the gravitational acceleration. So, gravitation is equivalent to the existence of an electric field $\vec{E} = m_I \vec{a} / q$ in Eq. (3). The corresponding expression for drift velocity is

$$\vec{v}_{\text{drift}} = \frac{m_I}{qB^2} \vec{a} \times \vec{B} \quad (5)$$

Now, let us have a look at the Figure. The magnetic field is along the horizontal z axis and gravitational acceleration along the vertical y axis. The particle moves in the z direction with a uniform velocity. In the y direction the velocity is a periodic function of time with average value $v_y = 0$. So the particle is not really falling. We can think of it as confined in a horizontal plane (a plane parallel to the xz plane in the Figure). The motion in the x direction is also a periodic function of time, but with an average drift velocity given by Eq. (5). The drift velocity is very small (and much smaller than the velocity along the z axis). So it is necessary to have the possibility to confine the particle in the space between two vertical planes (planes A and B in the Figure) parallel to the xy plane. It may be done with an appropriate electric field collinear to the z axis. Because of such an electric field, instead of moving with uniform velocity,

the particle will oscillate along the z axis. The motion in the x and y directions will remain unchanged.

The real experiment would be performed with many particles. In the ideal case they would be launched one by one, say, from the left to the right in the Figure. A detector at plane B will measure time of arrival and position. (This may be done with a microchannel plate detector [17] or some other position-sensitive detector.)

It is appropriate (and realistic) that the electric field used is able to confine only particles with an energy less than E_c . Particles with higher energy will directly go to plane B and be detected, giving a set of points whose geometrical centre can be easily found. In fact, because of the small drift velocity, there is not any detectable drift along the x axis for such particles. So these particles give us information about the x co-ordinate when there is no drift. Particles with energy smaller than E_c will be confined for a time τ . The electric field could then be switched off allowing them to hit plane B, and be detected, which will give a new set of points with a geometrical centre at a distance x_{drift} from the first one. Then,

$$v_{\text{drift}} = |\vec{v}_{\text{drift}}| = \frac{x_{\text{drift}}}{\tau} \quad (6)$$

From (5) and (6), taking into account that $\vec{a}$ and $\vec{B}$ are perpendicular, we can calculate the gravitational acceleration

$$a = \frac{|q|B}{m_I} \frac{x_{\text{drift}}}{\tau} \quad (7)$$

(Let us note that m_I is experimentally known with high accuracy to be the same for particles and antiparticles [18], as theoretically predicted by the CPT theorem.) In addition, for known $\vec{B}$, q and the measured drift direction (positive or negative) it is easy to distinguish between expected acceleration down, and unexpected, but for antimatter perhaps possible, acceleration up.

After discussions with a number of people, I have understood that a majority are not familiar with the fact that a charged particle does not fall in crossed constant and uniform gravitational and magnetic fields. So, I have decided to give here the full solution of the equation of motion in crossed gravitational and magnetic fields including also an electrical field used for confining the particle between planes A and B in the Figure, and a small stray electric field. The equation of motion in this case is

$$\frac{d\vec{p}}{dt} = m_I \vec{a} + q \vec{v} \times \vec{B} + q \vec{E} \quad (8)$$

with the gravitational acceleration $\vec{a}$, velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ given by

$$\begin{aligned}\vec{a} &= a \vec{j} \\ \vec{v} &= \dot{x} \vec{i} + \dot{y} \vec{j} + \dot{z} \vec{k} \\ \vec{E} &= E_x \vec{i} + E_y \vec{j} + E_z \vec{k} \\ \vec{B} &= B \vec{k}\end{aligned}\quad (9)$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors along the corresponding rectangular axes (see also the Figure). Using (8) and (9) a very simple calculation leads to the system of equations

$$\begin{aligned}\frac{d}{dt}(\dot{x} + i\dot{y}) + i \frac{qB}{m_x}(\dot{x} + i\dot{y}) &= \frac{q}{m_x}(E_x + iE_y) + ia \\ \ddot{z} &= \frac{q}{m_x}E_z\end{aligned}\quad (10)$$

It is an ideal situation to have $E_x = E_y = 0$ in the system (10). However, the electric field which we use to confine the particle is always in a direction z' (with unit vector $\vec{k}'$) slightly different from z . In addition there are always small stray fields. (In a superconducting drift tube as demonstrated by Witteborn and Fairbank [19], stray electric fields could be controlled at the level of 10^{-11} v/m.)

In the real experimental conditions, for example, a confining electrostatic potential of the form

$$V_{\text{conf}} = V_0 \left(\frac{\vec{r} \cdot \vec{k}'}{d} \right)^2 \quad (11)$$

can be produced. Here d and V_0 are known parameters, $\vec{r}$ is the radius vector of a given point, and $\vec{k}'$ is the unit vector introduced above. So we have

$$\vec{E}_{\text{conf}} = - \frac{\partial V_{\text{conf}}}{\partial \vec{r}} \quad (12)$$

i.e.,

$$\begin{aligned}(E_x)_{\text{conf}} &= - \frac{2V_0}{d^2} (x \cos \alpha + y \cos \beta + z \cos \gamma) \cos \alpha \\ (E_y)_{\text{conf}} &= - \frac{2V_0}{d^2} (x \cos \alpha + y \cos \beta + z \cos \gamma) \cos \beta \\ (E_z)_{\text{conf}} &= - \frac{2V_0}{d^2} (x \cos \alpha + y \cos \beta + z \cos \gamma) \cos \gamma\end{aligned}\quad (13)$$

where $\cos \alpha = \vec{i} \cdot \vec{k}'$, $\cos \beta = \vec{j} \cdot \vec{k}'$, $\cos \gamma = \vec{k} \cdot \vec{k}'$. Because of the fact that $\cos \gamma$ is much larger than $\cos \alpha$ and $\cos \beta$, (13) can be simplified:

$$\begin{aligned}
 (E_x)_{\text{conf}} &\approx - \left(\frac{2V_0}{d^2} \cos \gamma \cos \alpha \right) z \equiv - \delta_x z \\
 (E_y)_{\text{conf}} &\approx - \left(\frac{2V_0}{d^2} \cos \gamma \cos \beta \right) z \equiv - \delta_y z \\
 (E_z)_{\text{conf}} &\approx - \left(\frac{2V_0}{d^2} \cos^2 \gamma \right) z \equiv - \delta_z z
 \end{aligned} \tag{14}$$

We shall also include some small, but not precisely known, constant stray fields, $\varepsilon_x, \varepsilon_y, \varepsilon_z$, so that for electric fields in system (10) we have

$$\begin{aligned}
 E_x &= -\delta_x z - \varepsilon_x \\
 E_y &= -\delta_y z - \varepsilon_y \\
 E_z &= -\delta_z z - \varepsilon_z
 \end{aligned} \tag{15}$$

Introducing (15) into (10) gives

$$\begin{aligned}
 \frac{du}{dt} + i\omega u &= -\frac{q}{m_I} (\delta_x + i\delta_y) z - \frac{q}{m_I} (\varepsilon_x + i\varepsilon_y) + i\alpha \\
 \ddot{z} + \Omega^2 z &= -\frac{q}{m_I} \varepsilon_z
 \end{aligned} \tag{16}$$

with

$$\omega = \frac{qB}{m_I}, \quad \Omega^2 = \frac{q}{m_I} \delta_z, \quad u = \dot{x} + i\dot{y} \tag{17}$$

The general solution of the second equation of system (16) is

$$z = A \sin(\Omega t + \phi) - \frac{\varepsilon_z}{\delta_z} \tag{18}$$

where A and ϕ depend on initial conditions. So, the particle oscillates along z and can be confined between planes A and B in the Figure. Using (18), the first equations of system (16) can be transformed into

$$\frac{du}{dt} + i\omega u = -A \sin(\Omega t + \phi) + B + i\alpha \tag{19}$$

with

$$\begin{aligned}
 A &= A \frac{q}{m_I} (\delta_x + i\delta_y) \\
 B &= \frac{q}{m_I} \left\{ \frac{\varepsilon_z}{\delta_z} \delta_x + i \frac{\varepsilon_z}{\delta_z} \delta_y - \varepsilon_x - i\varepsilon_y \right\}
 \end{aligned} \tag{20}$$

The general solution of (19) is

$$u = \dot{x} + i\dot{y} = - \frac{A}{\omega(1 - \frac{\Omega^2}{\omega^2})} \left\{ \frac{\Omega}{\omega} \cos(\Omega t + \phi) - i \sin(\Omega t + \phi) \right\} + \frac{a}{\omega} + \frac{B}{i\omega} + \text{const.} \cdot e^{-i\omega t} \quad (21)$$

As a consequence of (21), $\dot{x}$ and $\dot{y}$ are periodic functions of time with average values

$$\bar{\dot{x}} = \frac{a}{\omega} + \frac{q}{m_I \omega} \left(\frac{\epsilon_z}{\delta_z} \delta_y - \epsilon_y \right) \quad (22)$$

$$\bar{\dot{y}} = \frac{q}{m_I \omega} \left(\frac{\epsilon_z}{\delta_z} \delta_x - \epsilon_x \right) \quad (23)$$

In the ideal case when stray electric fields are zero ($\epsilon_x = \epsilon_y = \epsilon_z = 0$), we have simply $\bar{\dot{x}} = a/\omega$ and $\bar{\dot{y}} = 0$. With a good control of stray fields as in [19], drift along the x axis caused by electric fields is much smaller than that caused by gravitation. Drift along the y axis is also small (and even a larger drift along y is not important for our measurement which is in fact a measurement of drift along the x axis).

Finally, let us note that the essential part of the method is the existence of crossed gravitational and magnetic fields. The problem of confinement of the particle may be solved in a way which differs from the confining electrostatic potential (11). The measurement of the drift velocity (6) seems to be also a fundamental part of the method, but who knows? It may even end up like some gravitational version of the Hall effect.

Of course, the possibility of having a calibration with H^- ions exists here as well as in the original proposal, LEAR PS 200. In fact, in principle, this method is appropriate for all long-lived charged particles. The measurement with antiprotons and positrons is the most important, but because of poor experimental information about free particles, such measurements should also be performed with protons, electrons and different ions. However, no one can expect an easy experiment with particles and such a feeble force as gravity. A lot of work has to be done before we know for sure if it is possible with today's experimental facilities. We must not forget that from a theoretical point of view, the idea of the original PS 200 proposal is also quite simple and possible.

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FIGURE CAPTION

Schematic view of the proposed experiment. LT-launching trap, D-detector, $\vec{B}$ magnetic field, $\vec{a}$ gravitational acceleration, $\vec{E}$ electric field used for confining particle between planes A and B. For more information, see the text.