THE EFFECT OF THE TOP QUARK ON THE $M_W - M_Z$ INTERDEPENDENCE AND POSSIBLE DECOUPLING OF HEAVY FERMIONS FROM LOW ENERGY PHYSICS

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Abstract

The resummation of the effects due to fermion doublets with a large mass splitting and their interplay with the photon vacuum polarization are discussed. The possibility of reexpressing the $M_W - M_Z$ interdependence in a form which is valid to all orders in perturbation theory is proposed. Some considerations on the possible decoupling of heavy fermions from low energy physics are presented.

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The largest quantum corrections in the Standard Model, at one-loop, are associated with the renormalization of the electric charge from zero momentum up to the vector boson scale and, potentially, with the presence of fermion isodoublets with a large mass splitting. This latter effect was discovered by Veltman [1] in his investigation on the ρ -parameter and later on identified [2] in the relation between the vector boson masses and the Fermi constant. More recently the two loop correction to ρ , due to fermion doublets, has been calculated in Ref.[3]. In this letter we want to address the question concerning the resummation of these effects, the relative size of the higher order reducible and the irreducible contributions and the possibility to reexpress the $M_W - M_Z$ interdependence in a form, which is valid to all orders in perturbation theory. We shall only discuss the interplay between the two potentially largest effects, the heavy fermion corrections and the renormalization of the electric charge, the remaining corrections having to be added perturbatively.

As a starting point we shall analyze first the problem of resumming the one-loop irreducible contributions due to light and heavy fermions. Both are contained only in the γ , W and Z propagators and $\gamma - Z$ mixing. As such they form a gauge invariant set which does not require any compensating terms coming from vertex corrections. As the derivation is not entirely trivial we shall discuss it in some detail.

At intermediate stage we shall use expressions which are only valid for three lepton and quark generations. The final result, however, will not have this limitation.

We can express the bare mass counterterms for W and Z up to two-loop reducible contributions as

$$\begin{aligned}\frac{\delta M_W^2}{M_W^2} &= \left(\frac{\delta M_W^2}{M_W^2}\right)^{light} + \left(\frac{\delta M_W^2}{M_W^2}\right)^{heavy} \\ \frac{\delta M_Z^2}{M_Z^2} &= \left(\frac{\delta M_Z^2}{M_Z^2}\right)^{light} + \left(\frac{\delta M_Z^2}{M_Z^2}\right)^{heavy}\end{aligned}\quad (1)$$

with $\left(\frac{\delta M_{W,Z}^2}{M_{W,Z}^2}\right)^{light}$ referring to the fermion contributions independent of the top quark mass. By retaining only the ultraviolet divergent part, we obtain

$$(\Delta = -\frac{2}{n-4} + \gamma - \ln(4\pi))$$

$$\left(\frac{\delta M_W^2}{M_W^2}\right)^{light} = \frac{\alpha_0}{\pi s_0^2} \Delta \equiv \Sigma'^{(o)} \quad (2)$$

$$\left(\frac{\delta M_Z^2}{M_Z^2}\right)^{light} = \frac{\alpha_0}{\pi s_0^2 c_0^2} \{c_0^2 - s_0^2 + \frac{8}{3}s_0^4\} \Delta + \frac{s_0^2}{c_0^2} [\Pi_{\gamma\gamma}(M_Z^2) - \Sigma'^{(o)}]^2 \quad (3)$$

and the last term in Eq.(3) takes into account the effect of $\gamma - Z$ mixing in the Z self-energy. In the above expressions α_0 is the bare fine structure constant up to two-loop reducible terms, i. e.

$$\alpha_0 = \alpha \left\{1 - \frac{\delta\alpha}{\alpha}\right\} \quad (4)$$

with

$$\frac{\delta\alpha}{\alpha} = \Pi_{\gamma\gamma}(0) - \Pi_{\gamma\gamma}^2(0) \quad (5)$$

and $q^2 \Pi_{\gamma\gamma}(q^2)$ represents the vacuum polarization function of the photon. In terms of this quantity the usual running coupling constant at a scale q^2 is expressed as

$$\alpha(q^2) = \frac{\alpha}{1 - [\Pi_{\gamma\gamma}(q^2) - \Pi_{\gamma\gamma}(0)]} \quad (6)$$

s_0 (c_0) represents the bare sine (cosine) of the weak mixing angle and it is related to the physical quantity

$$\sin^2 \Theta_W = 1 - \frac{M_W^2}{M_Z^2} \equiv s^2$$

through the relation

$$s_0^2 = 1 - \frac{M_{W0}^2}{M_{Z0}^2} = 1 - \frac{M_W^2 [1 - \frac{\delta M_W^2}{M_W^2}]}{M_Z^2 [1 - \frac{\delta M_Z^2}{M_Z^2}]} \equiv s^2 [1 - \frac{\delta s^2}{s^2}] \quad (7)$$

The quantities $(\frac{\delta M_{W,Z}^2}{M_{W,Z}^2})^{heavy}$ are related to the W and Z self-energy parts at $q^2 = 0$ and can be expressed in terms of the quantity

$$\delta \bar{\rho} \equiv \frac{3\alpha}{16\pi s^2} \frac{m_t^2}{M_W^2} \quad (8)$$

through the relations

$$(\frac{\delta M_W^2}{M_W^2})^{heavy} = 2\delta \bar{\rho} \{1 - \frac{\delta \alpha}{\alpha} + \frac{\delta s^2}{s^2}\} (\Delta + 1/2) \quad (9)$$

$$(\frac{\delta M_Z^2}{M_Z^2})^{heavy} = 2\delta \bar{\rho} \{1 - \frac{\delta \alpha}{\alpha} + \frac{c^2 - s^2}{s^2} \frac{\delta s^2}{s^2}\} \Delta \quad (10)$$

By using the above definition of $\frac{\delta s^2}{s^2}$ in Eq.(7) we find (up to two loops)

$$\begin{aligned} \frac{\delta s^2}{s^2} &= (\Pi_{\gamma\gamma}(M_Z^2) - \Sigma') (1 - \frac{\delta \alpha}{\alpha} + \frac{c^2}{s^2} \delta \bar{\rho}) \\ &\quad - \frac{c^2}{s^2} \delta \bar{\rho} [1 + \Pi_{\gamma\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(0) - \frac{c^2}{s^2} \delta \bar{\rho}] \end{aligned} \quad (11)$$

and Σ' is defined as

$$\Sigma' = \frac{\alpha}{\pi s^2} \Delta \quad (12)$$

related to $\Sigma'^{(o)}$ in Eq.(2) through

$$\Sigma'^{(o)} = \Sigma' \{1 - \frac{\delta \alpha}{\alpha} + \frac{\delta s^2}{s^2}\} \quad (13)$$

By collecting the above results we can calculate the quantum corrections to μ -decay and the relation with the Fermi constant G_μ . We find

$$\begin{aligned} \frac{G_\mu}{\sqrt{2}} &= \frac{e_0^2}{8s_0^2} \frac{1}{M_{W0}^2 - \Sigma_{WW}^{(o)}(0)} = \\ &= \frac{e^2}{8s^2 M_W^2} \{1 - \frac{\delta \alpha}{\alpha} + \frac{\delta s^2}{s^2} + \frac{\Sigma'^{(o)}}{\Sigma'} - \frac{\delta \alpha}{\alpha} \Sigma' + \frac{\delta s^2}{s^2} \Sigma' \\ &\quad - \frac{\delta \alpha}{\alpha} \frac{\delta s^2}{s^2} + \Sigma'^2 + (\frac{\delta s^2}{s^2})^2\} \end{aligned} \quad (14)$$

After a little algebra we find

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8s^2 M_W^2} \{1 + \epsilon_\alpha + \epsilon_\alpha^2 - \frac{c^2}{s^2} \delta\bar{\rho} (1 + 2\epsilon_\alpha - 2\frac{c^2}{s^2} \delta\bar{\rho})\} \quad (15)$$

and ϵ_α is defined as

$$\epsilon_\alpha = \Pi_{\gamma\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(0) \quad (16)$$

The same result as in Eq.(15) can be obtained from the expressions for the renormalized propagators and Δr given in Ref.[4]. Before going further we can relate our Eq.(15) to the results of Ref.[5]. By defining $\tilde{s}^2$ through the relation ($A = \frac{\pi\alpha}{\sqrt{2}G_\mu}$)

$$M_Z^2 = \frac{A}{\tilde{s}^2 \tilde{c}^2} \quad (17)$$

($\tilde{c}^2 = 1 - \tilde{s}^2$) and using Eq.(15), together with $M_W^2 = c^2 M_Z^2$, we can obtain the $M_W - M_Z$ relation

$$M_W^2 = \tilde{c}^2 M_Z^2 \left\{ 1 - \frac{\tilde{s}^2}{\tilde{c}^2 - \tilde{s}^2} \left(\epsilon_\alpha - \frac{\tilde{c}^2}{\tilde{s}^2} \delta\bar{\rho} \right) - \frac{\tilde{s}^2(1 - 3\tilde{s}^2 + 3\tilde{s}^4)}{(\tilde{c}^2 - \tilde{s}^2)^3} \epsilon_\alpha^2 \right. \\ \left. + \frac{2\tilde{c}^2 \tilde{s}^4}{(\tilde{c}^2 - \tilde{s}^2)^3} \epsilon_\alpha \delta\bar{\rho} + \frac{\tilde{c}^4(1 - 3\tilde{s}^2)}{(\tilde{c}^2 - \tilde{s}^2)^3} (\delta\bar{\rho})^2 \right\} \quad (18)$$

derived in Ref.[5].

Going back to Eq.(15) we note that by defining the one-loop quantity

$$\Delta r = \epsilon_\alpha - \frac{c^2}{s^2} \delta\bar{\rho} \quad (19)$$

the relation [6]

$$M_W^2 = \frac{A}{s^2} \frac{1}{1 - \Delta r} \quad (20)$$

correctly takes into account the terms ϵ_α^2 and $\epsilon_\alpha \delta\bar{\rho}$ but not the $(\delta\bar{\rho})^2$. We can improve on this situation by defining

$$\delta\rho \equiv \frac{3m_t^2 G_\mu}{8\pi^2 \sqrt{2}} = \delta\bar{\rho} \left(1 + \epsilon_\alpha - \frac{c^2}{s^2} \delta\bar{\rho} \right) \quad (21)$$

In this case we find

$$M_W^2 = \frac{A}{s^2} \left\{ 1 + \epsilon_\alpha + \epsilon_\alpha^2 - \frac{c^2}{s^2} \delta\rho (1 + \epsilon_\alpha) + \frac{c^4}{s^4} (\delta\rho)^2 + \dots \right\} \quad (22)$$

and the dots represent higher order terms not taken into account in our analysis. Eq.(22) is suggestive of the presence of two independent geometrical series which would produce the result

$$c^2 M_Z^2 = M_W^2 = \frac{A}{s^2} \frac{1}{1 - \epsilon_\alpha} \frac{1}{1 + \frac{c^2}{s^2} \delta\rho} \quad (23)$$

It is interesting to exploit the consequences of Eq.(23) on the $M_W - M_Z$ interdependence. In this case by defining

$$\rho = \frac{1}{1 - \delta\rho} \quad (24)$$

we find the quadratic equation

$$M_W^4 - \rho M_W^2 M_Z^2 + \rho M_Z^2 \frac{A}{1 - \epsilon_\alpha} = 0 \quad (25)$$

whose (experimentally) acceptable solution for $\rho \sim 1$ is

$$M_W^2 = \frac{\rho M_Z^2}{2} \left\{ 1 + \sqrt{1 - \frac{4A}{\rho M_Z^2} \frac{1}{1 - \epsilon_\alpha}} \right\} \quad (26)$$

to be compared with the alternative relation [7]

$$M_W^2 = \frac{M_Z^2}{2} \left\{ 1 + \sqrt{1 - \frac{4A}{M_Z^2} \frac{1}{1 - \Delta r}} \right\} \quad (27)$$

Δr being defined in Eq. (19). We note that Eq.(24) (where $\delta\rho$ represents the one-loop irreducible contribution) is in agreement with the complete two-loop result [3] on the ρ -parameter where the structure

$$\frac{G_{NC}}{G_{CC}} = \rho = 1 + (\delta\rho)_{irred} + (\delta\rho)_{irred}^2 + \dots = \frac{1}{1 - (\delta\rho)_{irred}} \quad (28)$$

was discovered, $(\delta\rho)_{irred}$ being given by [3]

$$(\delta\rho)_{irred} = N_{cf} [x_f - (2\pi^2 - 19)x_f^2] \quad (29)$$

with

$$x_f = \frac{\Delta m_f^2 G_\mu}{8\pi^2 \sqrt{2}} \quad (30)$$

and $N_{cf} = 1$ for leptons, $N_{cf} = 3$ for quarks. Eq.(26) was also suggested in Ref.[8] to be the right resummation procedure as being the correct $M_W - M_Z$ relation in an underlying theory where $\rho \neq 1$ at the tree level. We thus conclude that Eq.(26) should be used, in connection with eqs.(28), (29) and (30), to evaluate the effect of heavy fermions in the $M_W - M_Z$ interdependence, in place of Eq.(27).

We do not agree with Ref.[5] concerning the negligibility of the two-loop irreducible contributions to ρ as compared to the two-loop reducible ones. For leptons we find

$$\rho = 1 + x_f + x_f^2 + (19 - 2\pi^2) x_f^2 + \dots = 1 + x_f + 0.261 x_f^2 + \dots \quad (31)$$

and for quarks

$$\rho = 1 + 3 x_f + 9 x_f^2 + 3 (19 - 2\pi^2) x_f^2 + \dots = 1 + 3 x_f + 6.782 x_f^2 + \dots \quad (32)$$

Only in a theory with large N_{cf} the two and higher-loop irreducible contributions can be consistently neglected. By the way, this is only true for small x_f . In fact, regardless of the magnitude of N_{cf} , we see from Eq.(29) that as x_f becomes large the negative x_f^2 contribution to $(\delta\rho)_{irred}$ will start to cancel a large fraction of the one-loop result. This is, in our opinion, an interesting feature which has to be traced back to the Yukawa coupling with the scalar sector that only shows up at the two-loop level. As also noticed by analyzing the radiative corrections to the decay width [9] $\Gamma(Z \rightarrow b\bar{b})$ the effect of

the scalar particles is to cancel a large part ($\sim -15 \text{ MeV}$ for $m_t = 230 \text{ GeV}$ and $M_Z = 92 \text{ GeV}$) of the positive correction to the total width ($+50 \text{ MeV}$). This means that heavy physics could be effectively screened by two- and higher-loop contributions in such a way that at low energy we do not see large effects. In this case one could still get $\rho \sim 1$ up to small corrections but with completely different physical consequences. We note an interesting difference between the lepton and the quark case as far as Eq.(29) is concerned.

At the one-loop level, in both cases, ρ will blow up as the mass splitting in a doublet is made very large. At the two-loop level, however, one gets $\rho \leq 1.511$ for all values of x_f in the case of leptons. This means that the present experimental data (say $0.99 \leq \rho \leq 1.01$), in principle may be compatible with an overall mass splitting of 3.5 TeV , as far as only leptons are concerned. Yet these are only speculations and only a closer inspection of the higher order contributions can clarify this very important point.

We take them only as an indication that apart from summing the leading logs, which is controlled by the renormalization group, summing up one-loop contributions may produce misleading results.

Finally we conclude with a technical remark. We note that Eq.(26) is only valid for leading contributions to ρ , directly proportional to the mass of the particles. In this case the use of the Fermi constant prevents the occurrence of the running coupling constant at the vector boson scale at any order in the loop expansion. Nonleading one-loop self-energy effects can be included by using Eq.(26) together with the replacements:

$$\begin{aligned} \epsilon_\alpha &\rightarrow \Pi_{\gamma\gamma}(0) + \frac{\Sigma_W(0) - \Sigma_W(M_W^2)}{M_W^2} - \frac{c}{s} \frac{\Sigma_{\gamma Z}(0) - \Sigma_{\gamma Z}(M_Z^2)}{M_Z^2} \\ \delta\rho &\rightarrow \frac{\Sigma_Z(M_Z^2)}{M_Z^2} - \frac{\Sigma_W(M_W^2)}{M_W^2} + \frac{s}{c} \frac{\Sigma_{\gamma Z}(M_Z^2) + \Sigma_{\gamma Z}(0)}{M_Z^2}, \end{aligned}$$

where Σ_Z includes $\gamma - Z$ mixing terms as given in Ref.[4]. We have checked that the above substitution reproduces correctly all self-energy terms up to $O(\alpha^2)$. Such a resummation could make sense for the fermion contributions, which form a gauge invariant subset. However, since terms like the irreducible contribution $\propto \frac{\alpha}{4\pi} \sqrt{2} G_\mu m_t^2 \ln(m_t^2/M_Z^2)$ are unknown, non leading terms and the vertex and box corrections, (contributing to Eq.(27)) should be added perturbatively.

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