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(0,2) HETEROTIC STRING COMPACTIFICATIONS FROM
N=2 SUPERCONFORMAL THEORIES

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ABSTRACT

We study the construction of (0,2) heterotic string compactifications built by tensoring of $N = 2$ superconformal theories. These models are obtained by embedding discrete symmetries of the superconformal blocks into the gauge degrees of freedom and using orbifold techniques. Some of the resulting models can be understood as the addition of Wilson lines on an original (2,2) background whereas others cannot. Phenomenological implications of these new constructions are briefly discussed.

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Starting from the work of Gepner [1], much effort has been devoted to the study of string vacua based on tensoring of $N = 2$ superconformal theories (SCT's). Although these constructions are very general, they have explicit $(2, 2)$ world-sheet supersymmetry. It is natural to try to find an extension to the less restrictive $(0, 2)$ case. From the phenomenological point of view, $(0, 2)$ models are more interesting since they allow for many possibilities for gauge group and matter content. In this note we start exploring the construction of $(0, 2)$ models built by tensoring of $N = 2$ SCT's. For simplicity we concentrate on $N = 2$ minimal models but the techniques described can also be applied to the class of $N = 2$ coset theories recently discussed by Kazama and Suzuki [2]. The $(0, 2)$ models are obtained by embedding discrete symmetries of the $N = 2$ building blocks into the gauge degrees of freedom. Some of the resulting models can be understood as the product of adding Wilson lines on a $(2, 2)$ background whereas others cannot. Phenomenological prospects of these new constructions are briefly discussed.

We start by reviewing some features of 4-dimensional heterotic compactifications with space-time supersymmetry. The basic idea is to use an internal $N = 2$ SCT with $c = 9$ together with the space-time bosonic coordinates and NSR fermions to match $c_{total} = 15$ in the right sector. In the left sector we use the same SCT plus the space-time bosonic coordinates and 13 bosons living on an $SO(10) \times E_8$ torus thus saturating $\bar{c}_{total} = 26$. In particular, the $c = 9$ $N = 2$ SCT can be built by tensoring $N = 2$ minimal theories. Gepner has shown how the above ingredients lead to a modular invariant string theory with space-time supersymmetry [1].

We now turn into a brief discussion of $N = 2$ minimal theories. These theories are labelled by an integer k related to the central charge by $c = 3k/(k + 2)$. Primary states of the right (left) $N = 2$ algebra are denoted by $|l, q, s\rangle$ ($|\bar{l}, \bar{q}, \bar{s}\rangle$) where $0 \leq l \leq k$, $0 \leq |q - s| \leq l$, $l + q + s = 0 \mod 2$. s takes the values $s = 0, 2$ in the Neveu-Schwarz (NS) sector and $s = \pm 1$ in the Ramond (R) sector. q is defined $\mod (2(k + 2))$ and $s \equiv s(\mod 4)$. The conformal dimension and $U(1)$ charge of an $|l, q, s\rangle$ state are given by

$$h = \frac{[l(l + 2) - q^2]}{4(k + 2)} + \frac{s^2}{8} \quad ; \quad Q = -\frac{q}{k + 2} + \frac{s}{2} \quad (1)$$

Similar expressions hold for $|\bar{l}, \bar{q}, \bar{s}\rangle$. A number of states of the $N = 2$ algebra are identified under the automorphism $l \rightarrow k - l$, $q \rightarrow q + k + 2$, $s \rightarrow s + 2$. This automorphism implies the equivalence relation

$$|l, q, s\rangle \otimes |\bar{l}, \bar{q}, \bar{s}\rangle \equiv |k - l, q + k + 2, s + 2\rangle \otimes |k - \bar{l}, \bar{q} + k + 2, \bar{s} + 2\rangle \quad (2)$$

The $N = 2$ models at level k possess additional discrete symmetries. Of particular relevance here is the Z_{k+2} symmetry under which fields acquire a phase $e^{2\pi iz/(k+2)}$ where $z = (q + \bar{q})/2$ is referred to as the Z_{k+2} charge [1,3].

Let us now explain how modding a minimal theory by a sub-group of Z_{k+2} gives rise to a new orbifolded theory. Given $\gamma, M \in Z$ with M the least integer for which $M\gamma = 0 \bmod (k+2)$ we can construct a Z_M orbifold of the original level k theory. In the orbifold theory there are twisted sectors in which fields satisfy boundary conditions closed up to phases. The partition function of the orbifold theory can be schematically written as

$$Z = \frac{1}{M} \sum_{g, f=0, \dots, M-1} Z(g, f) \quad (3)$$

where (g, f) refers to boundary conditions closed up to $e^{2\pi i g \gamma z / (k+2)}$ and $e^{2\pi i f \gamma z / (k+2)}$ in the σ and t directions respectively. For the $(0, 0)$ sector we can write

$$Z(0, 0) = \frac{1}{2} \sum_{l_{qs}, \bar{l}_{\bar{q}\bar{s}}} \mathcal{M}_{l_{qs}, \bar{l}_{\bar{q}\bar{s}}} \chi_q^{ls} \chi_{\bar{q}}^{*\bar{l}\bar{s}} \quad (4)$$

The structure of the characters χ_q^{ls} and the properties of the coefficients $\mathcal{M}$ have been discussed in refs. [1,4]. For a generic (g, f) sector $Z(g, f)$ is given by [1,3]

$$Z(g, f) = \frac{1}{2} \sum_{l_{qs}, \bar{l}_{\bar{q}\bar{s}}} e^{2\pi i f \gamma z(g) / (k+2)} \mathcal{M}_{l_{qs}, \bar{l}_{\bar{q}\bar{s}}} \chi_{q+2g\gamma}^{ls} \chi_{\bar{q}}^{*\bar{l}\bar{s}} \quad (5)$$

where $z(g) = (q + 2g\gamma + \bar{q})/2$.

Eqs. (3) and (5) show the appearance of ‘ g twisted sectors’. Keeping g fixed and summing over f effectively implements a projection into invariant states in the g sector. Since the conformal dimensions of the states are independent of f we see that the projector takes the form

$$P(g) = \frac{1}{M} \left\{ 1 + e^{2\pi i \Delta(g)} + \dots + e^{2\pi i (M-1) \Delta(g)} \right\} \quad (6-a)$$

$$\Delta(g) = \frac{\gamma z(g)}{k+2} \quad (6-b)$$

Thus, the invariance condition is $\Delta(g) = \text{integer}$.

To obtain a $N = 2$ SCT with $c = 9$ we tensor r minimal models satisfying $\sum_{i=1}^r 3k_i / (k_i + 2) = 9$. Primary states of the $N = 2$ algebra are given by $\bigotimes_{i=1}^r |l_i, q_i, s_i\rangle \otimes |\bar{l}_i, \bar{q}_i, \bar{s}_i\rangle$. States in the associated heterotic compactification carry other labels indicating their space-time and gauge properties. The mass levels of these states are given by

$$\frac{m_R^2}{8} = N_R + \frac{1}{2} w^2 + h_{int} - \frac{1}{2} \quad (7-a)$$

$$\frac{m_L^2}{8} = N_L + \frac{1}{2} \bar{w}^2 + \frac{1}{2} p^2 + \bar{h}_{int} - 1 \quad (7-b)$$

where $h_{int}, \bar{h}_{int}$ are the conformal dimensions associated with the internal tensor theory; N_R, N_L are oscillator numbers; w ($\bar{w}$) stands for an $SO(2)$ ($SO(10)$) weight and p is a

vector in the E_8 root lattice. States are often represented by $V_R \otimes V_L$ where $V_R = (w; q_1, \dots, q_r; s_1, \dots, s_r)$ and $V_L = (\bar{w}; \bar{q}_1, \dots, \bar{q}_r; \bar{s}_1, \dots, \bar{s}_r)$.

An important ingredient in Gepner's method is the generalized GSO projection that guarantees space-time supersymmetry by projecting out all states not having odd-integral total superconformal $U(1)$ charge. To impose this condition it is convenient to introduce the vector

$$\beta_0 = (\bar{s}; 1, \dots, 1; 1, \dots, 1) \quad (8)$$

where $\bar{s}$ is an anti-spinor $SO(2)$ weight. We also define the scalar product

$$V \cdot V' = \frac{1}{4} w \cdot w' + \frac{1}{4} \sum_{i=1}^r s_i \cdot s'_i - \frac{1}{2} \sum_{i=1}^r \frac{q_i \cdot q'_i}{k_i + 2} \quad (9)$$

The total $U(1)$ conformal charge of a state represented by a vector V (including the $SO(2)$ or $SO(10)$ contribution) is given by

$$Q_{TOT}(V) = 2\beta_0 \cdot V \quad (10)$$

The generalized GSO projection consists of imposing $Q_{TOT} = \text{odd integer}$. Notice that Q_{TOT} and the total Q_{int} are related by $Q_{TOT}(V) = Q_{int}(V) + 2w \cdot \bar{s}$. To be sure that the fermions in the various sectors have aligned boundary conditions (either NS or R) another type of projection is also required. This is achieved by imposing $Q_i(V) \equiv 2\beta_i \cdot V = \text{even integer}$, where β_i is given by

$$\beta_i = (v; 0, \dots, 0; 0, \dots, 2, \dots, 0) \quad (2 \text{ in the } i^{th} \text{ position}) \quad (11)$$

where v here is an $SO(2)$ vector weight.

The above projections are done independently on the right and left states represented by V_R and V_L . In addition, these two vectors are constrained to differ by an element of the lattice generated by β_0 and the β_i , i.e. $V_L - V_R = m_0\beta_0 + m_i\beta_i$. It is also necessary, when constructing the Neveu-Schwarz sector, to add a vector $(v; 0, \dots, 0; 0, \dots, 0)$ to V_L , where here v stands for an $SO(10)$ vector weight. Note, however, that in computing the Q charge the effect of this vector is not to be included. Gepner has shown that the construction just described leads to a modular invariant string theory with space-time supersymmetry [1]. The right and left $l, \bar{l}$ indices are correlated by one of the $SU(2)$ affine invariants. For simplicity we assume the diagonal invariant $\prod_i \delta_{l_i, \bar{l}_i}$.

The $N = 2$ tensor theory possesses a discrete symmetry group $G = Z_{k_1+2} \times \dots \times Z_{k_r+2}$ together with permutations of identical blocks. We can obtain an orbifoldized theory by modding by a sub-group of G . Consider a generator $\gamma = (\gamma_1, \dots, \gamma_r)$, $\gamma_i \in Z$, of order M , i.e M is the least integer for which $M\gamma = (0, \dots, 0) \text{ mod } (k_1 + 2, \dots, k_r + 2)$. To construct a Z_M orbifold of the original tensor model we can follow the same

steps explained above for a single minimal model. The orbifoldized theory can then be converted to an heterotic model via the Gepner construction. The new feature is the appearance of g twisted sectors in which $V_L - V_R$ lies on an enlarged lattice given by

$$V_L - V_R = g(2\gamma) + m_0\beta_0 + m_i\beta_i \quad ; \quad g = 0, \dots, M-1 \quad (12)$$

where it is understood that $\gamma = (0; \gamma_1, \dots, \gamma_r; 0, \dots, 0)$. Invariant states in the g sector satisfy

$$\Delta = -\gamma \cdot (V_R + V_L) = \text{integer} \quad (13)$$

with V_R and V_L related as in eq. (12) and with the scalar product defined in eq. (9). Actually, not all γ 's are allowed. To maintain space-time supersymmetry the discrete sub-group specified by γ should commute with supersymmetry, corresponding to the condition

$$2\beta_0 \cdot \gamma = -\sum_{i=1}^r \frac{\gamma_i}{k_i + 2} = \text{integer} \quad (14)$$

Notice also that the element $(2, 2, \dots, 2)$ acts trivially. Thus, the discrete symmetry group of a tensor model is G/H , where H is the sub-group generated by $(2, 2, \dots, 2)$. The modding by discrete symmetries as just described usually leads to $(2, 2)$ models with reduced number of generations [5,6]. The analysis immediately generalizes to modding by permutation symmetries amongst repeated factors in the tensor product.

We now wish to explain how embedding the internal SCT modding into the gauge degrees of freedom leads to a new class of models. We recall that the gauge degrees of freedom are associated to the 13 free left bosons, generically denoted by X , living on an $SO(10) \times E_8$ torus. We choose to embed the action of γ on these bosons by a shift A . This means that in the (g, f) sector we have boundary conditions of the form

$$\begin{aligned} X(\sigma + 2\pi, t) &= X(\sigma, t) + L + gA \\ X(\sigma, t + 2\pi) &= X(\sigma, t) + L' + fA \end{aligned} \quad (15)$$

where L, L' belong to the $SO(10) \times E_8$ root lattice $\Gamma = \Gamma_{SO(10)} \times \Gamma_{E_8}$. Notice that consistency requires

$$MA \in \Gamma \quad (16)$$

where M is the order of the generator γ .

The contribution to the partition function due to the free bosons X with boundary conditions (15) can be evaluated as a path-integral on a world-sheet torus (with modular parameter τ) or by operator methods. For the $(0, 0)$ sector the result can be written as

$$\frac{1}{\bar{\eta}^{13}} \sum_{P \in \Gamma^* \bmod \Gamma} e^{-i\pi\tau P^2} \quad (17)$$

where η is the Dedekind function and Γ^* is the lattice dual to Γ , i.e. the weight lattice (recall $\Gamma_{E_8} = \Gamma_{E_8}^*$). For a generic sector the path-integral result leaves an undetermined phase that can be fixed by requiring modular invariance. In a (g, f) sector we then have the following contribution to the partition function

$$e^{i\pi f g A^2} \frac{1}{\eta^{13}} \sum_{P \in \Gamma^* \bmod \Gamma} e^{-i\pi \tau (P+gA)^2} e^{-2\pi i (P+gA) \cdot f A} \quad (18)$$

From the above expression we can deduce the modular invariance constraint

$$MA^2 = 0 \bmod 2 \quad (19)$$

The embedding by a shift A that we have described is a standard procedure in the orbifold formalism. For more details on the steps leading to eqs. (18) and (19) we refer the reader to ref. [7]. Similar issues have been considered in ref. [8].

Eq. (18) shows that in presence of the shift A we must replace $P \rightarrow P + gA$, i.e. $\bar{w} \rightarrow \bar{w} + A_{SO(10)}$, $p \rightarrow p + A_{E_8}$, in the left mass relation eq. (7-b). Eq. (19) then guarantees left-right level matching. Eq. (18) also shows that the invariance condition for states in the g twisted sector is now given by

$$\Delta = -\gamma \cdot (V_R + V_L) - [(P + gA) \cdot A - \frac{g}{2} A^2] = \text{integer} \quad (20)$$

with V_R, V_L related as in eq. (12). The above invariance condition together with the mass formulae and the constraints implicit in the Gepner construction can be used to find the spectrum of models obtained by a modding plus embedding action denoted generically by $\mathcal{A} = \gamma \otimes A$. As we will see, a general feature of models found in this way is the breaking of the gauge group.

The simplest example to illustrate the previous discussion is the model obtained by tensoring 9 copies of the $k = 1$ minimal model. It is most convenient to display the states in the form $|w; \phi_R\rangle \otimes |P; \phi_L\rangle$ where $\phi_R = \bigotimes_{i=1}^9 (l_i, q_i, s_i)$ and similarly for ϕ_L . Here w is an $SO(2)$ weight and P belongs to the shifted lattice $\Gamma^* + gA$. For chiral supermultiplets we choose to exhibit the 4-dimensional scalar components, $w = 0$. In the 1^9 model there are 84 generations of E_6 . The states transforming as the 10 of $SO(10) \in 27$ of E_6 have $P = v$, an $SO(10)$ vector weight, and are associated with the 84 states $\phi_R = \phi_L = \underline{(1, 1, 0)^3(0, 0, 0)^6}$ (where underlining means permutations). The 1^9 model has a group of discrete symmetries $Z_3^8 \times P_9$ where P_9 is the permutation group of 9 elements. Hence, there are several (some equivalent) possibilities for modding this model. For example, one can obtain a new model by modding by the symmetry generated by

$$\rho = (1, 1, 1, -1, -1, -1, 0, 0, 0) \quad (21)$$

The massless spectrum found is that of the standard Z_3 orbifold as observed in ref. [9]. The 27 'twisted' generations of the Z_3 orbifold correspond to the states $\phi_R =$

$\phi_L = \underline{(1,1,0)(0,0,0)^2} \underline{(1,1,0)(0,0,0)^2} \underline{(1,1,0)(0,0,0)^2}$, i.e. they are untwisted in the SCT sense with identical ϕ_R and ϕ_L . Here we have grouped the minimal blocks in three sets with three factors each (each set corresponds to one of the $SU(3)$ sub-lattices of the orbifold torus). There are three more generations corresponding to the states $\phi_R = \phi_L = \underline{(1,1,0)^3(0,0,0)^3(0,0,0)^3}$. In addition, there are states such as $\phi_R = (1,1,0)^3(0,0,0)^3(0,0,0)^3$, $\phi_L = (1,3,2)^3(0,-2,2)^3(0,0,0)^3$ that differ by elements of the enlarged lattice containing ρ, β_0, β_i . They thus appear in twisted sectors of the modding. We can check that all states mentioned above are invariant under the modding. Altogether we recover the expected 36 generations present in the massless spectrum of the $(2,2)$ version of the standard Z_3 orbifold with gauge group $E_6 \times SU(3) \times U(1)^6 \times E_8$. Notice that from the initial 9 $U(1)$'s associated to each $k=1$ block one diagonal linear combination is now contained in E_6 whereas two other $U(1)$'s are promoted to a full $SU(3)$ group. The charged $SU(3)$ generators come from states of the type $\phi_R = (0,0,0)^9$ ($w=v$), $\phi_L = \underline{(0,2,2)^3(0,-2,2)^3(0,0,0)^3}$ that differ by elements of the enlarged lattice. Finally, there are also E_6 singlet states that complete the massless spectrum.

Consider now a non-trivial embedding of a discrete symmetry in the gauge degrees of freedom giving rise to $(0,2)$ models. We start with the 36 generation twisted $1^9/\rho$ model described above and further mod out by $A_1 = \gamma \otimes A_1$ where

$$\gamma = (2, 2, 2, 0, 0, 0, 0, 0, 0) \quad (22-a)$$

$$A_1 = \frac{1}{3}(0, 0, 0, 0, 0, 0) \times (1, 1, -2, 0, 0, 0, 0, 0, 0) \quad (22-b)$$

Notice that the order of γ is 3 and that A_1 fulfills conditions (16) and (19). The gauge group turns out to be $[E_6 \times SU(3)]^2 \times U(1)^6$. The second $E_6 \times SU(3)$ arises from states of the form $|v; (0,0,0)^9\rangle_R \otimes |p; (0,0,0)^9\rangle_L$ where $p \in \Gamma_{E_8}$, $p^2 = 2$, and $\Delta = -p \cdot A_1 = \text{integer}$. There are matter generations in the untwisted sector of the γ modding, namely the states

$$|0; (1,1,0)^3(0,0,0)^3(0,0,0)^3\rangle_R \otimes |W; \underline{(1,1,0)^3(0,0,0)^3(0,0,0)^3}\rangle_L \quad (23)$$

where $W \in \Gamma_{SO(10)}^*$, $W^2 = 1$ (from the masslessness condition), survive projection since $\Delta = -W \cdot A_1 = 0$. Adding $\beta_0, 2\beta_0$ to V_L in (23) and changing W appropriately we find the remaining states to complete a $(27, 3)$ of the first $E_6 \times SU(3)$. Notice that these fields come in three copies since we can permute the $(1,1,0)^3$ block in V_R in (23).

There are also twisted sectors. For instance, states of the form

$$|0; (1,1,0)^3(0,0,0)^3(0,0,0)^3\rangle_R \otimes |P - A_1; (1,3,2)^3(0,0,0)^3(0,0,0)^3\rangle_L \quad (24)$$

are massless provided $\frac{1}{2}(P - A_1)^2 = 1$. The invariance condition for the state (24) is given by

$$\Delta = -P \cdot A_1 + \frac{1}{2}A_1^2 = \text{integer} \quad (25)$$

We can check that necessarily $P \in \Gamma_{E_8}$ and that E_8 root lattice vectors satisfying (25) as well as the masslessness condition give rise to a $(27', 3')$ of the second $E_6 \times SU(3)$. There are further twisted generations from states of the form

$$\begin{aligned} &|0; (1, 1, 0)(0, 0, 0)^2(1, 1, 0)(0, 0, 0)^2(1, 1, 0)(0, 0, 0)^2\rangle_R \otimes \\ &|P - A_1; (1, 3, 2)(0, 2, 2)^2(1, 1, 0)(0, 0, 0)^2(1, 1, 0)(0, 0, 0)^2\rangle_L \end{aligned} \quad (26)$$

Since here $\bar{h}_{int} = 2/3$ we require $(P - A_1)^2 = 2/3$ for massless states and the invariance condition

$$\Delta = -\frac{1}{3} - P \cdot A_1 + \frac{1}{2}A_1^2 = \text{integer} \quad (27)$$

These conditions are fulfilled by $3(P - A_1) = (-1, -1, 2, 0, 0, 0, 0, 0) \in \Gamma_{E_8}$. Considering all possible permutations in (26) we obtain altogether 27 fields transforming as $(\bar{3}, \bar{3}')$. The non-singlet chiral matter content of this model is thus

$$3(27, 3) + 3(27', 3') + 27(\bar{3}, \bar{3}') \quad (28)$$

This coincides with the massless spectrum of one of the three $(0, 2)$ embeddings of the Z_3 orbifold, the one obtained by acting with a shift $V = \frac{1}{3}(1, 1, -2, 0, \dots, 0) \times (1, 1, -2, 0, \dots, 0)$ on the $E_8 \times E_8$ gauge lattice. This is not surprising since the shift in (22) is just completing the standard $(2, 2)$ embedding action to reproduce V above.

Remarkably, all (rank 22) $(0, 2)$ Z_3 orbifold constructions can in fact be obtained from the $1^9/\rho$ model. For instance, to reproduce the $(0, 2)$ Z_3 with gauge group $SU(9) \times SO(14) \times U(1)$ we act with $\mathcal{A}_2 = \gamma \otimes A_2$ where γ is given in (22 - a) and

$$A_2 = \frac{1}{3}(1, 1, 0, 0, 0) \times (2, 0, \dots, 0) \quad (29)$$

It is a tedious but straightforward exercise to recover the massless spectrum of the corresponding orbifold as given in ref. [10]. Care has to be taken to include all twisted states and to recognize the way in which $SU(9)$ representations are decomposed. Finally, the third $(0, 2)$ Z_3 model with gauge group $SO(14) \times U(1) \times E_7 \times U(1)$ can be found by acting with $\mathcal{A}_3 = \gamma \otimes A_3$ where

$$A_3 = \frac{1}{3}(-2, 0, 0, 0, 0) \times (1, 1, 0, \dots, 0) \quad (30)$$

Both (28) and (29) can be seen as completing the standard embedding shift to reproduce the non-standard shifts associated to each model. Notice that in the above examples the $\mathcal{A}$ action does not simply break the $E_6 \times SU(3)$ observable symmetry. Some gauge generators are projected out but also new ones appear in twisted sectors giving rise to completely different groups like $SU(9)$ or $SO(14)$. Thus, the $(0, 2)$ models obtained do not resemble at all the original $(2, 2)$ model from which we started.

Sometimes the $\gamma \otimes A$ action admits an interpretation in terms of turning on Wilson lines in the associated compact (internal) variety. Adding Wilson lines is a powerful tool in attempts to break the gauge symmetries down to the Standard Model group. However, let us remark an important point. Usually γ is not freely acting, i.e there are γ twisted sectors. Thus, the effect of Wilson lines is not just to break the gauge group but also to change the massless spectrum (e.g. the number of generations as is well known happens in the orbifold case [10]). We now turn to describe an example of this type based on $1^9/\rho$. This time we want to consider an action $\mathcal{A}' = \gamma' \otimes A_1$ where A_1 is given in (22 - b) and

$$\gamma' = (2, -2, 0, 0, 0, 0, 0, 0, 0) \quad (31)$$

The gauge group again turns out to be $[E_6 \times SU(3)]^2 \times U(1)^6$. In the untwisted sector we find $3(27, 3)$ states of the form (23). There are further massless untwisted states of the form

$$\begin{aligned} &|0; (0, 0, 0)^2(1, 1, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_{R\otimes} \\ &|W; (0, 0, 0)^2(1, 1, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_L \end{aligned} \quad (32)$$

with $W^2 = 1$, $W \in \Gamma_{SO(10)}^*$. Adding $\beta_0, 2\beta_0$ to V_L in (32) and adjusting W appropriately we find $9(27, 1)$ states that survive projection. We also find $27(1, \bar{3})$ states. Concerning the twisted sectors, we have states

$$\begin{aligned} &|0; (1, 1, 0)(0, 0, 0)(0, 0, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_{R\otimes} \\ &|P + A_1; (1, -1, 0)(0, 2, 2)(0, 0, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_L \end{aligned} \quad (33)$$

with $(P + A_1)^2 = 2/3$ and

$$\Delta = \frac{4}{3} - P \cdot A_1 - \frac{1}{2}A_1^2 = \text{integer} \quad (34)$$

The solutions are given by $3(P + A_1) = (1, 1, -2, 0, \dots, 0) \in \Gamma_{E_8}$. Including permutations we find $9(\bar{3}, 3')$. Similarly, we find $9(\bar{3}, \bar{3}')$ states of the form

$$\begin{aligned} &|0; (0, 0, 0)(1, 1, 0)(0, 0, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_{R\otimes} \\ &|P - A_1; (0, 2, 2)(1, -1, 0)(0, 0, 0) \underline{(1, 1, 0)(0, 0, 0)^2} \underline{(1, 1, 0)(0, 0, 0)^2}\rangle_L \end{aligned} \quad (34)$$

Altogether the massless spectrum of the model is

$$3(27, 3) + 9(27, 1) + 27(1, \bar{3}) + 9(\bar{3}, 3') + 9(\bar{3}, \bar{3}') \quad (35)$$

which is rather different from that in (28). The model obtained by the $\mathcal{A}'$ action may be interpreted in terms of the (rank 22) $(2, 2)$ Z_3 orbifold in the presence of a Wilson line parametrized by the shift $a = \frac{1}{3}(0, \dots, 0) \times (1, 1, -2, 0, \dots, 0)$ in the $E_8 \times E_8$ lattice. This type of models has been constructed and discussed in detail in refs. [7,10]. Notice how the number of E_6 generations of the $(2, 2)$ model has decreased from 36 to 18. This is a

generic feature of Wilson line models that turns out to be very useful in the construction of 3-generation orbifold models.

We might try to naively understand the difference between the $\mathcal{A}$ and $\mathcal{A}'$ actions as follows. In orbifolds, Wilson line shifts are associated to translations in the torus lattice which is an action purely on the bosonic coordinates. Now, in the 1^9 model each set of 3 $k = 1$ blocks can be interpreted as a set of 2 free bosons on an $SU(3)$ torus together with two (real) fermions ψ, γ' can be regarded as acting only on the bosons, thus A_1 rather corresponds to a Wilson line. Clearly, this sort of interpretation cannot be given for a random tensor model. In general, it will be difficult to distinguish models with Wilson line backgrounds from other $(0, 2)$ compactifications.

The construction of $(0, 2)$ models that we have described and exemplified should be applicable to any model obtained through tensoring of $N = 2$ SCT's. We have not presented examples of $(0, 2)$ models based on general cosets because their massless spectrum is cumbersome to derive. It is clear however, that the construction allows the generalization of well-known orbifold techniques such as the addition of Wilson lines. In particular, these techniques allow to vary the number of quark-lepton generations while breaking the gauge symmetries down to groups of the form $SU(3) \times SU(2) \times U(1) \times G'$. Within the present formalism it is possible to construct both the three-generation orbifold models of refs. [7,10] (based on 1^9) and the three-generation model of ref. [5] (based on $1 \ 16^3$). Thus, we can directly compare these models and search for general features. Other three-generation examples with Standard Model gauge group could be obtainable and comparison of their properties should be feasible. We hope to report on these matters soon.

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